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High-speed train cooperative control based on fractional-order sliding mode adaptive algorithm

Junting Lin and Mingjun Ni
School of Automation and Electrical Engineering, Lanzhou Jiaotong University, Lanzhou, China, and
Huadian Liang
Electrical R&D Department, CRRC Nanjing Puzhen Co., Ltd, Nanjing, China

Abstract

Purpose – This study aims to propose an adaptive fractional-order sliding mode controller to solve the problem of train speed tracking control and position interval control under disturbance environment in moving block system, so as to improve the tracking efficiency and collision avoidance performance.

Design/methodology/approach – The mathematical model of information interaction between trains is established based on algebraic graph theory, so that the train can obtain the state information of adjacent trains, and then realize the distributed cooperative control of each train. In the controller design, the sliding mode control and fractional calculus are combined to avoid the discontinuous switching phenomenon, so as to suppress the chattering of sliding mode control, and a parameter adaptive law is constructed to approximate the time-varying operating resistance coefficient.

Findings – The simulation results show that compared with proportional integral derivative (PID) control and ordinary sliding mode control, the control accuracy of the proposed algorithm in terms of speed is, respectively, improved by 25% and 75%. The error frequency and fluctuation range of the proposed algorithm are reduced in the position error control, the error value tends to 0, and the operation trend tends to be consistent. Therefore, the control method can improve the control accuracy of the system and prove that it has strong immunity.

Originality/value – The algorithm can reduce the influence of external interference in the actual operating environment, realize efficient and stable tracking of trains, and ensure the safety of train control.

Keywords High-speed trains, Sliding mode control, Fractional-order differentiation, Adaptive law, Cooperative control

Paper type Research paper

1. Introduction

With the rapid development of railway communication technology, automatic driving technology and train positioning system, the train control mode has also changed from quasi-mobile block to mobile block (Long, Meng, Wang, Luan, & Zhang, 2020), which is inevitably affected by external interference in the actual operating environment of the train, so in such a high-speed and high-density tracking mode, a collaborative control algorithm is urgently needed to improve the control performance and immunity during train operation. Under the quasi-moving block mode, the target point of high-speed train tracking operation is the starting point of the block zone occupied by the front train, which limits the transportation efficiency to a certain extent; under the moving block mode, the train adopts the method of

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"hitting the soft wall" to realize tracking operation, which further shortens the tracking distance and increases the flexibility and autonomy of trains. With the rapid development of railway communication technology, automatic driving technology and train positioning system, conditions are created for multi-train cooperative control. During the tracking operation of high-speed train, real-time and high-quality information interaction is realized between trains through Radio Block Center (RBC) and other ground equipments, and the cooperative control between trains is realized through intelligent control algorithm based on automatic driving technology, so as to achieve smaller tracking interval and accurate tracking of the expected speed curve, thus improving traffic efficiency, safety and comfort (Tian, 2020). However, today's control algorithms ignore the influence of interference on the control system, so improving the control algorithm and enhancing the immunity of the system is of far-reaching significance to the research of multi-train collaborative control.

At present, scholars at home and abroad mainly study the multi-train cooperation problem from the perspective of operation scheduling and control algorithms. In terms of operational scheduling, the key is to shorten the departure interval, optimize the adjustment of the running chart and improve the efficiency of the line transportation of cooperative operation between trains (Pan, Mei, & Zheng, 2015; Sun, 2019; Zeng, Zhang, & Chen, 2019). On the other hand, the automatic train driving control algorithm based on fuzzy control and particle swarm optimization was studied (Cao, Ma, & Zhang, 2018; Xu, Yang, Tu, & Wu, 2021; Zhang & Wu, 2021). Chen, Dang, and Hu (2014) established multi-agent system (MAS) interaction mechanism between the train and RBC based on multi-agent theory and realized real-time train ground communication and safe distance control of multi-train tracking operation. Liu (2020) proposes a virtual coupling train group control strategy based on MAS, and builds a control model according to the control rules to achieve the goal of stable and coordinated operation of train groups. Therefore, in an ideal operating environment, status information can be received in real time through the communication mechanism between trains and grounds, so as to ensure stable and safe tracking between trains.

However, in the actual line operation, it is inevitably affected by external interference, resulting in system uncertainty, and sliding mode control has strong robustness to uncertain parameters and external interference, so the accuracy of the model can be ensured by introducing the controller. Zhao *et al.* (2022) artificially eliminate the influence of parameter uncertainty and external interference on the longitudinal cruise control of intelligent trains, and propose a longitudinal cruise control method based on adaptive dynamic sliding mode, which finally realizes stable and accurate tracking of vehicle speed. In order to achieve accurate end trajectory control of the multi-robotic arm system, Li, Xu, and Gui (2021) used time delay estimation and adaptive fuzzy sliding mode controller to eliminate interference and realize the operation of collaborative handling of target objects. The action mode of the hook and slow device on adjacent trains is clarified, the strong coupling model of the high-speed EMU is established, and the distributed neural network sliding mode control strategy is designed to carry out speed tracking control for the high-speed EMU (Li, Jin, Yang, Tan, & Fu, 2020). However, due to the existence of the inertia of the sliding mode control system, the system will inevitably lag in the switching process, so it is easy to cause jitter, and some scholars have used fractional-order to weaken jitter. Yu, Zhang, and Jiang (2020) studied that in the case of actuator failure of quadrotor UAV, combined with neural network technology and fractional sliding mode control method, it can still follow the trajectory of the virtual long machine and maintain the ideal relative position. Under the condition of parameter change and external disturbance in the system, Wei, Wang, Ji, and Fang (2021) designed an adaptive fuzzy fractional sliding mode control method to improve the robustness of the system and ensure the tracking accuracy of the system. Aiming at sensorless remote control robot system with uncertainties such as time delay and external interference, variable structure control based on neural networks and optimized fractional-order selection strategy are proposed

(Ma, Liu, Huang, & Kuang, 2022). Dong, Yang, and Basin (2022) focuses on the problem of fast position tracking while reducing jitter under logarithmic sliding mode control signals in permanent magnet linear motor systems.

In order to obtain the accurate parameters of the train dynamics model in time-varying environment and further improve the robustness and fault tolerance, an adaptive mechanism is proposed. He, Yang, and Lv (2019) designed a controller based on tracking error according to the sliding mode control theory for the problem of accurate pit and parking of the automatic driving system of high-speed train, and then introduced adaptive and fuzzy reasoning rules to further weaken the jitter phenomenon, so as to achieve accurate parking. In the vehicle-to-vehicle communication mode, Wang (2018) proposed the multi-train adaptive collaborative control algorithm based on sampling feedback and nonlinear gain, and finally realized the steady-state tracking of trains. Wang, Wu, Feng, and Zhang (2016) applied the terminal sliding mode control principle to design the train stop control algorithm, and introduced the parameter adaptive mechanism to further enhance the adaptability of the control system. Based on Lyapunov stability theory, Li, Meng, Xu, and Yin (2018) designed the automatic sliding mode adaptive robust controller for high-speed trains, and used adaptive control to approximate the system input coefficient with uncertain characteristics in real time, thereby eliminating the system jitter phenomenon. Train collaborative tracking operation is a multi-train coupling control system, and the mutual influence between trains, in the complex nonlinear and disturbance environment, the study of multi-train speed tracking and interval control is of great significance to realize the overall collaborative stability of the queuing train.

The main contributions of this paper are:

1. During the operation of the train, considering the influence of external interference on the tracking accuracy of the queue train, a sliding mode controller based on state error is designed, which can improve the robustness and response speed of the algorithm in complex nonlinear environment to a certain extent (Liu, 2019).
2. In view of the problem of jitter in traditional sliding mode control, the fractional difference is added on the sliding mode surface by introducing fractional calculus, so as to reduce the inherent jitter of the sliding mode controller and improve the accuracy of the controller.
3. In the actual line operation environment, there are system parameters that change with time, so the parameter adaptation law can be used to approximate the real train drag coefficient in real time, avoiding the model error caused by the use of traditional empirical parameters.

This paper studies the collaborative control problem of trains based on the theory of multiple agents. Firstly, the information interaction model between train and ground equipment is constructed based on matrix graph theory, the system state equation is determined according to the train operation mode, and the sliding surface function is established based on the train state error. Secondly, in order to suppress the jitter phenomenon of the sliding mode controller, a fractional derivative is added to the sliding surface, and in order to reduce the influence of time-varying parameters on the accuracy of the model, the parameter adaptive law is introduced to approach the time-varying resistance coefficient on the basis of the fractional sliding mode control algorithm. Finally, the experimental simulation using MATLAB software verifies the effectiveness of the proposed algorithm.

2. Mathematical description of train operation process

2.1 Multi-train interaction topology model in adjacent communication mode

In this paper, the high-speed rail multi-train system operates under the mobile blocking system, and the train and the ground system in the automatic train control (ATC) system exchange information through the GSM-R wireless communication network, and the train-to-ground wireless structure is used to achieve information exchange, as shown in Figure 1.

In the process of high-speed railway multi-train tracking operation, the following information interaction mechanism is defined: taking train *i* as an example, train *i* carries out bidirectional information interaction with its “topology adjacent” train through RBC, that is, trains are kept at the minimum safe interval from the forward train according to their respective braking curves. At the same time, each train transmits its own state information to the ground equipment RBC through the GSM-R network, from which it is transferred to other trains to realize the information interaction between trains. This train-ground-train communication mechanism is the basis of train expected state calculation and cooperative control. Algebraic graph theory is used to establish a high-speed rail multi-train information interaction topology model as the information basis for train collaborative control. The research object of this paper is multi-train tracking in a single line, so the topology of the high-speed railway multi-train communication network remains constant in the process of train operation, which is a fixed topology.

The scenario of multi-train operation in high-speed railway is regarded as the communication network topology of multi-agent system, and each running train and RBC are regarded as an individual agent. The mathematical model established based on matrix and graph theory can be represented by a simple directed graph $G = (V, E)$, where $V = \{a_0, a_1, \ldots, a_n\}$ represents the train set in the graph, $E \subset V \times V$ represents the edge set of a directed graph, and (a_i, a_j) is defined as the edge of agent a_i to agent a_j (representing the communication relationship between the agents). If there is a communication relationship between the two trains, then (a_i, a_j) exists, it is called that agent a_i and agent a_j are a neighbor nodes. The neighbor set that interacts with a_i is represented by N_i. Define $A = \{a_{ij}\}$ as the adjacency matrix of G. If (a_i, a_j) exists, then $a_{ij} = 1$, if there is no communication relationship between them, then $a_{ij} = 0$. $L_p = \{l_{ij}\}$ is the Laplace matrix of G, there is $l_{ij} = -a_{ij}$ and $l_{ii} = \sum_{j=0}^{n} a_{ij}$ when $i \neq j$. Considering that the train control system has high real-time requirements for the state information of each train, in order to simplify the complexity of the communication model, the information interaction model does not take the effect of time delay into account. In order to give the mathematical model of multi-train communication relationship, formula (1) is established.

$$l_{ij} = \begin{cases} -a_{ij}, i \neq j \\ \sum_{i=1}^{n} a_{ij}, i = j \end{cases} i = 1, 2, \cdots, N \tag{1}$$

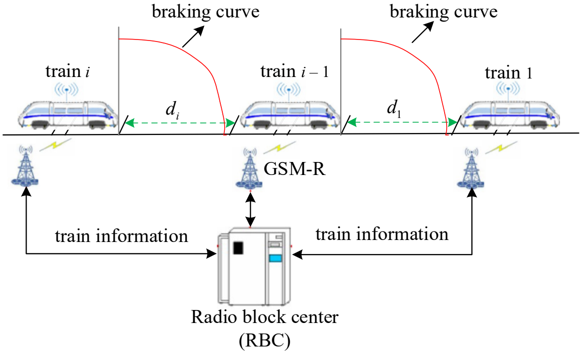

Figure 1. Multi-train communication mechanism in adjacent communication mode

2.2 Dynamic equation of train operation

Considering that multi-vehicle collaboration is realized through the ATC of a single train, the train is modeled as a rigid nature point, and the dynamic model of the single particle point of the train is obtained as follows:

$$\begin{cases} \dfrac{dx}{dt} = v \\ v' = \xi(u - w) = a_c \\ \xi = \dfrac{0.0098}{1 + \gamma} \\ w = a + bv + cv^2 \end{cases} \tag{2}$$

Where x is the displacement; v is the real-time speed of train; u is the traction/braking force received by the train; w is the basic resistance of the train; a, b and c are the rolling mechanical resistance coefficients, friction resistance coefficient and air resistance coefficient of the train, respectively; ξ is the acceleration coefficient of the train, among them, 0.098 is the reference value defined in the technical regulations of high-speed trains (Yang & Zhou, 2018); γ is the wheel rotation mass coefficient of the train. In actual train operation, the resistance of the train includes additional resistance and basic resistance. The basic resistance is affected by the train speed and mechanical wear, but the additional resistance only appears in the fixed part of the line. In this paper, the additional resistance and uncertain disturbance are combined for a convenient computing (Lian, Liu, & Li, 2020).

According to the train model of equation (2), the train operation state-space equation is established as:

$$\begin{cases} x_i(t)' = v_i(t) \\ m(1 + \gamma)v_i(t)' = f_i(t) - w_i(t) - d(t) \end{cases} \tag{3}$$

The physical meanings of the above variables are explained as follows,

m_i is the mass of train i, $v_i(t)$ is the real-time speed of train i, $x_i(t)$ is the real-time position of train i, $f_i(t)$ is traction/braking force of train i, $w_i(t)$ is real-time basic resistance, $d(t)$ is additional resistance and external disturbance. Referring to the technical regulations of high-speed railways, the basic running resistance of the train is affected by environmental factors, such as wind speed, rail surface conditions, etc. The coefficients are obtained through multiple test fitting in the project. Therefore, the error in formula used in the model is inevitably caused by external environmental factors in the actual running of the train. The parameter structure of the basic running resistance is $w_i = a_i + b_i v_i + c_i v_i^2$. Considering the uncertainty and time-varying characteristics of parameters under the action of the complex environment during train's operation, the coefficients a_i, b_i, c_i are set to a variable parameter structure with a reference constant term and an unknown time variant. These parameters use adaptive law to approach the actual value under a complex external environment in real-time, they are $a_i(t) = a_i^* + \Delta a_i(t), b_i(t) = b_i^* + \Delta b_i(t), c_i(t) = c_i^* + \Delta c_i(t)$, including standard constant terms a_i^*, b_i^* and c_i^*. Terms $\Delta a_i(t), \Delta b_i(t)$ and $\Delta c_i(t)$ represents the time-varying characteristics under external disturbance and internal variation of the system. The above three parameters can be approximated in real time using the adaptive law to approximate the actual values in complex external environments (Li & Hou, 2015). Comparing adaptive sliding mode control with traditional sliding mode control in following text, it can be found that the former is a fixed parameter, while the latter is a parameter of adaptive law fitting, and the improved algorithm is more in line with the actual characteristics of the train.

3. Controller design

The control object of this paper is multi-trains operation in a single track. In the whole tracking process, the biggest difficulty for the controller lies in the accurate tracking of the speed curve. Secondly, the complexity of the resources and environment in the whole line determines high nonlinearity of the train's additional resistance. Therefore, if the controller has a good disturbance suppression ability, the tracking accuracy in the operation process will be guaranteed and it is conducive for the train to realize the coordination of operation state. According to the operation characteristics of high-speed train, the basic resistance parameters of the system are highly susceptible to time-varying characteristics such as wear of wind speed locomotive components. Therefore, the uncertainty of the basic resistance coefficients of the train will affect the stable operation of the train. So the uncertainty of system model parameters should be considered in the design process of the controller. The designed controller has a good robustness so that it can overcome the unknown interference outside the system and the uncertainty of braking system parameters, thus achieving a fast and stable online control, and ensuring high-precision speed tracking during the tracking of the train, stable and compact train distance and smooth control input during train tracking operation. The structural block diagram of train cooperative control is shown in Figure 2.

3.1 Error model extraction

The sliding mode controller responds quickly to the target requirements, which can make the system state converge to the desired trajectory in a limited amount of time. It also has a strong parameter adaptability and robustness, so as to ensure that there will be no overshoot when the system has parameter uncertainty, and avoid adverse impact on the system (Zhang, 2019). Therefore, in multi-train collaborative control, sliding mode control can suppress the interference of the external environment and provide a solution for the uncertainty of the model caused by time-varying parameters.

The state equation of velocity and position error are defined as,

$$\begin{cases} e_i = x_i - x_{ir} \\ e_i{}' = v_i - v_{ir} \end{cases} \tag{4}$$

Where e_i is the position error of the train; $e_i{}'$ is the speed error of train; x_i is the actual position of the train and x_{ir} is the referential position; v_{ir} is the referential speed. Considering the relative position between trains, the expression of position error according to the actual situation is,

$$e_i = x_i - x_{i-1} - (L_b + L_s + L_d) \tag{5}$$

Where L_b is the rear car braking distance, L_s is the safety envelope, L_d is the train length.

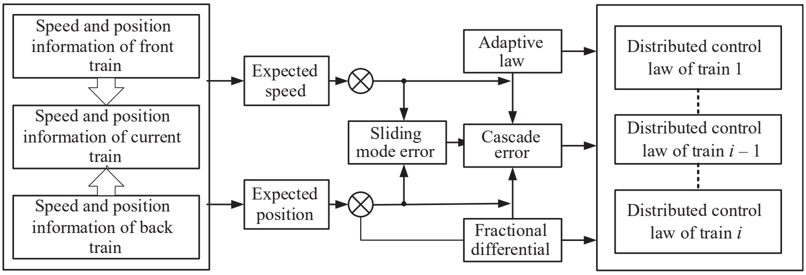

Figure 2.
Structure diagram of multi-train cooperative controller

3.2 Fractional calculus

Due to the inevitable switching phenomenon of sliding mode control, in order to suppress its chattering, fractional calculus is introduced to improve this problem. Based on the advantages of fractional calculus in softening discontinuous switching (Deng, 2014), the fractional calculus of Caputo form adopted in this paper is defined as follows:

$$ {}_{\alpha}D_t^{\alpha} f(\mathrm{t}) = \begin{cases} \dfrac{1}{\Gamma(\mathrm{m}-\alpha)} \displaystyle\int_0^t \dfrac{f^{(\mathrm{m})}(\tau)}{t-\tau} d\tau, m-1 < \alpha < m \\ \dfrac{d^m}{dt^m} f(\mathrm{t}), \alpha = m \end{cases} \tag{6} $$

Where d^m/dt^m is the differential in the traditional sense, m is the minimum integer not less than the fractional-order α, and t is the time; τ is the integral variable; when $\alpha < 0$, it is a fractional-order differential, and when $\alpha > 0$, it is a fractional-order integral; $\Gamma(\mathrm{x})$ is the gamma function $\Gamma(\mathrm{x}) = \int_0^\infty e^{-t} t^{x-1} dt$, and e is the position tracking spacing error.

Since the controlled object in this paper has a large operating range and is relatively flexible, it needs to have a good dynamic processing effect, so the fractional calculus parameters are selected. The performance of the adjustment process under different fractional orders is different, the appropriate fractional calculus operator can be selected according to the actual operation status of the site, so that the system can meet different dynamic and static performance. In summary, the sliding mode control based on fractional-order can make the error system converge faster, the control accuracy is higher and the control process is smoother (Fang, 2021).

3.3 Fractional-order sliding mode controller design

In the process of train tracking operation, it is necessary to accurate track the reference position and the reference speed curve at the same time, and by adding the fractional differentiation on the sliding hyperplane (Zhou, 2021), that is, introducing the train position error e_i and train speed error $e_i{}'$ into the sliding hyperplane, it can not only effectively suppress the inherent jitter phenomenon of sliding mode control, but also ensure the rapid synchronous convergence of the error. The fractional-order sliding surface is designed as,

$$ s_i(t) = \lambda e_i + D^{\alpha-1} e_i{}' \tag{7} $$

Where λ is the gain coefficient of sliding mode surface, $\lambda > 0$.

The above formula only considers the speed error and position error of the train i, but does not consider the role of neighboring trains in the coordinated formation control, so it is difficult to accurately describe the stability of the multi-train queue. Therefore, in order to realize the cooperative control characteristics of multiple trains, a weighted error N_i is introduced to couple the status information of adjacent vehicles:

$$ N_i = \frac{a_{ij}}{\Delta j + \beta_i} S_j - S_i. \tag{8} $$

Where Δj and β_i characterize the regulatory parameters of the relationship between the train i and the train j.

Next, the sliding mode controller is designed to realize the online tracking of train reference speed and reference position curve:

$$u_i = \frac{k_1}{\beta_i + 1} N_i + \frac{1}{\beta_i + 1} f_i + \widehat{w}_i - k_2 \mathrm{sgn}(S_i) d_i, \tag{9}$$

Where $\widehat{w}_i$ is the estimation term of train basic resistance; $k_2 \mathrm{sgn}(S_i)\mathrm{d}_i$ is the nonlinear switching control term of the system, which is used to deal with external disturbances and uncertainties, and k_1, k_2 is the control gain, where $k_1 > 0, k_2 > 0$.

The uncertainty and time variation of the basic resistance depend on its resistance coefficients, so the control law is

$$u_i = \frac{k_1}{\beta_i + 1} N_i + \frac{1}{\beta_i + 1} f_i + \widehat{a}_i + \widehat{b}_i x_i' + \widehat{c}_i x_i^{2'} + k_2 \mathrm{sgn}(S_i) d_i \tag{10}$$

Where $\widehat{a}_i, \widehat{b}_i, \widehat{c}_i$ are the estimated values of the resistance coefficients. In order to eliminate the influence of time-varying and uncertain factors of the basic resistance, the following parameter adaptive laws are designed:

$$\begin{aligned} \widehat{a}_i' &= \mu_a[(\beta_i + 1)N_i x_i' - \zeta_1 \widehat{a}_i] \\ \widehat{b}_i' &= \mu_b[(\beta_i + 1)N_i x_i' - \zeta_2 \widehat{b}_i] \\ \widehat{c}_i' &= \mu_c[(\beta_i + 1)N_i x_i^{2'} - \zeta_3 \widehat{c}_i] \end{aligned} \tag{11}$$

Where $\mu_a, \mu_b, \mu_c, \zeta_1, \zeta_2, \zeta_3$ are positive constants, $\zeta_1, \zeta_2, \zeta_3$ are used as compensation factors to correct estimation errors.

Lemma (about tracking performance and closed-loop signal boundedness): For the multi-train dynamic system under adjacent communication topology, if the above control laws and parameter adaptive laws are selected, only when the linear weighting error N_i and parameter estimation error $\overline{g}_i$ finally converge to compact set $\boldsymbol{\Omega}_{Ni}$ and $\boldsymbol{\Omega}_{gi}$, can the multi-vehicle closed-loop signal be bounded and eventually consistent. A compact set is defined as,

$$\begin{aligned} \boldsymbol{\Omega}_{Ni} &:= \left\{ N_i \in R, \left| N_i \leq \sqrt{2V_i(0) + \frac{2\theta_i}{\mu_1}} \right| \right\} \zeta_1 \\ \boldsymbol{\Omega}_{gi} &:= \left\{ \overline{g}_i \in R, \left| \overline{g}_i \leq \sqrt{2\mu_g V_i(0) + \frac{2\mu_g \theta_i}{\mu_1}} \right| \right\} \end{aligned} \tag{12}$$

3.4 Proof of stability

In order to verify the stability and effectiveness of the controller designed in this paper, the Lyapunov function is used for stability proof (Zhang & Wang, 2021). Combined with the cascade error and adaptive parameters, the Lyapunov function shown below is designed

$$\begin{aligned} V_{c1} &= \sum_{i=1}^{n} V_i \\ V_i &= \frac{1}{2} N_i^2 + \frac{1}{2\mu_a} \overline{a}_i^2 + \frac{1}{2\mu_b} \overline{b}_i^2 + \frac{1}{2\mu_c} \overline{c}_i^2 \end{aligned} \tag{13}$$

The derivative of V_i can be obtained

$$\begin{aligned}V_i' &= -k_1 N_{\mathrm{i}}^2 + \frac{1}{\mu_a}\overline{a}_i\overline{a}_i' + \frac{1}{\mu_b}\overline{b}_i\overline{b}_i' + \frac{1}{\mu_c}\overline{c}_i\overline{c}_i' - (\beta_{\mathrm{i}}+1)\mathrm{N}_i\left[\overline{a}_i + \overline{b}_i x_i' + \overline{c}_i x_i^{2'} + k_2\mathrm{sgn}(S_i)d_i\right]\\ &= -k_1 N_{\mathrm{i}}^2 - (\beta_{\mathrm{i}}+1)\mathrm{N}_i[k_2\mathrm{sgn}(S_i)d_i a_i - a_i] - \zeta_1\overline{a}_i\widehat{a}_i - \zeta_2\overline{b}_i\widehat{b}_i - \zeta_3\overline{c}_i\widehat{c}_i\\ &\le -k_1 N_{\mathrm{i}}^2 + (\beta_{\mathrm{i}}+1)D_i^{+}[|N_{\mathrm{i}}| - N_i\mathrm{sgn}(S_i)] - \zeta_1\overline{a}_i\widehat{a}_i - \zeta_2\overline{b}_i\widehat{b}_i - \zeta_3\overline{c}_i\widehat{c}_i\end{aligned} \tag{14}$$

According to Young's inequality $-\overline{q}\,\widehat{q} = -\overline{q}(\overline{q}+q) \le -\frac{1}{2}\overline{q}^2 + \frac{1}{2}q^2, q \in \{a_i, b_i, c_i\}$, it can be obtained according to the above lemma

$$(\beta_{\mathrm{i}}+1)D_i^{+}[|N_{\mathrm{i}}| - N_i\mathrm{sgn}(S_i)] \le k_3(\beta_{\mathrm{i}}+1)D_i^{+}/\lambda_i \tag{15}$$

then

$$\begin{aligned}V_i &\le -k_1 N_{\mathrm{i}}^2 - \frac{1}{2}\left(\zeta_1\widetilde{a}_i^2 + \zeta_2\widetilde{b}_i^2 + \zeta_3\widetilde{c}_i^2\right) + \frac{1}{2}\left(\zeta_3 a_i^2 + \zeta_1 b_i^2 + \zeta_2 c_i^2\right) + \frac{k_3(\beta_{\mathrm{i}}+1)D_i^{+}}{\lambda_i}\\ &\le -\mu_1 V_i + \theta_i\end{aligned} \tag{16}$$

where μ_1 and θ_i are defined as follows

$$\begin{aligned}\mu_1 &:= \frac{\min\{2\mathrm{k}_1, \zeta_1, \zeta_2, \zeta_3\}}{\max\{1, 1/\mu_a, 1/\mu_b, 1/\mu_c\}}\\ \theta_i &:= \frac{1}{2}\left(\zeta_1 a_i^2 + \zeta_2 b_i^2 + \zeta_3 c_i^2\right) + \frac{k_3(\beta_{\mathrm{i}}+1)a_i^{+}}{\lambda}\end{aligned} \tag{17}$$

According to the above Lemma, V_i is finally bounded. At the same time, when it tends to infinity, $V_i \le V_i(0) + \frac{\theta_i}{\mu_1}$, and $V_i(0)$ is the initial value of V_i, according to the definition of V_i, the linear cascade error N_i and parameter estimation error $\overline{g}_i$ can converge to the following compact set, where,

$$\begin{aligned}&\frac{1}{2}N_{\mathrm{i}}^2 \le V_i(0) + \frac{\theta_i}{\mu_1} \Rightarrow |N_i| \le \sqrt{2V_i(0) + \frac{2\theta_i}{\mu_1}},\\ &\frac{1}{2\mu_g}\overline{g}_i^2 \le V_i(0) + \frac{\theta_i}{\mu_1} \Rightarrow |\overline{g}_i| \le \sqrt{2\mu_g V_i(0) + \frac{2\mu_g\theta_i}{\mu_1}}.\end{aligned} \tag{18}$$

Theoretical proof is completed.

Through the Lyapunov function constructed above, the feasibility of this control algorithm is mathematically proved, and the stability of the designed distributed control law is verified on a theoretical basis, so the controller can achieve the goal of multi-train cooperative control state.

4. Simulation comparison and analysis

In order to verify the effectiveness of the proposed algorithm in cooperative formation control, CRH380AL EMU is the simulation control object. Its specific parameters are shown in Table 1.

Under the mobile occlusion system, the coordinated tracking operation of five high-speed trains is selected. In order to fit the actual operating characteristics of high-speed trains, Figure 3 shows the expected operation curve of train 1. The entire run time is 4,400 s, the upper limit of speed in the operation process is 97 m/s, the acceleration in the start-up stage is set as 0.16 $\mathrm{m/s^2}$, the temporary speed limit is 75 m/s, and the braking deceleration is 0.12 $\mathrm{m/s^2}$. This paper tests the cooperative control effect of the designed control algorithm, it is assumed that the trains depart synchronously at a given initial spacing. Figure 4 shows the external disturbance during the whole operation process.

The goal of collaborative control in this paper is that each train will eventually maintain a consistent running speed and a stable expected position interval in a complex operating environment. The initial speed of the train is set as 0 m/s, and the relative initial position interval of the five trains is set to 10,000 m. Meanwhile, a small amplitude of initial position error is added in the process of simulation execution. The search range of parameters is determined based on prior knowledge, and then the optimal value is obtained by traversing the parameter set in trial-and-error method. We can get these parameters as $k_1 = 0.75$,

Parameter	Numerical value
Power configuration	6M2T
Train quality (t)	365
Maximum operating speed (m/s)	111
Current collecting voltage (Kv)	25
Train rotation coefficient γ	0.11
Basic resistance parameter	$w_0 = 0.16 + 0.0053v + 0.00018v^2$

Table 1. Parameters of CRH380A EMU

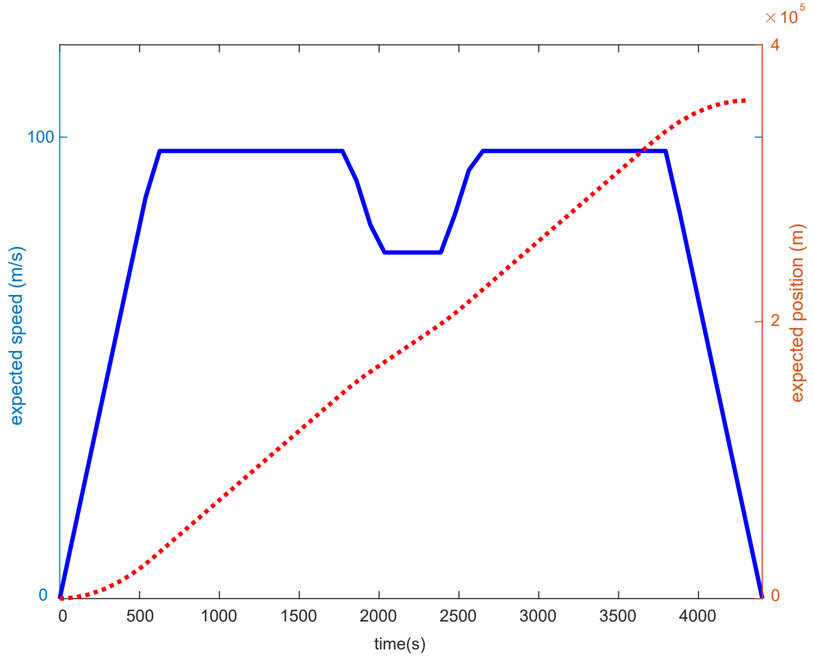

Figure 3. Reference speed curve and displacement curve during train operation

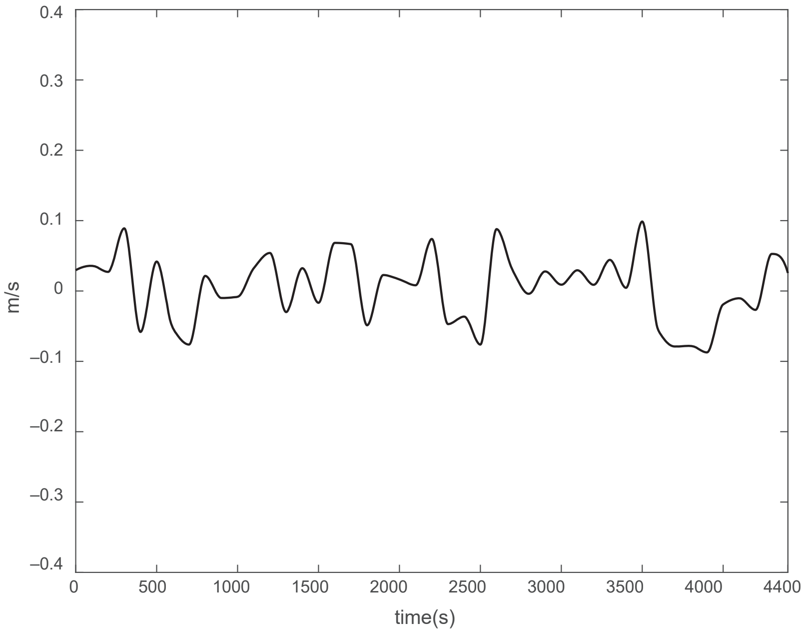

Figure 4.
External disturbance during train operation

$k_2 = 0.5$, $k_3 = 0.2875$. The initial values of basic resistance coefficients $\widehat{a}_i, \widehat{b}_i, \widehat{c}_i$ are 0.16, 0.0053 and 0.00018, respectively; the calculus operators are $\lambda = 15$, $\alpha = 0.9$. Correction coefficients are set as $\mu_a = \mu_b = \mu_c = 10^{-3}$, and compensation factors are set as $\zeta_1 = \zeta_2 = \zeta_3 = 0.1$. And "V-X" represents the speed and position curves of the train.

Through the simulation experiment of CRH380AL EMU in its tracking operation process, it is assumed that the first train obtains the expected speed curve, the subsequent trains obtains the expected interval and expected speed in real-time through multi-agent interactive topology, then it realizes cooperative control through distributed control law and adopts the calculation method to adjust each train online. Figure 5 shows the displacement-velocity tracking operation curve realized by using the distributed control law of traditional PID control algorithm, although the algorithm keeps the basic consistent operation trend of each train, in the cruising stage, the speed error is controlled within the range of (−0.4, 0.4) m/s, and the range fluctuation is more severe, which is difficult to meet the accuracy requirements of passenger comfort and smooth operation of high-speed trains. When the unimproved sliding mode controller model is selected and the resistance coefficient is selected to give a fixed empirical value, the control effect obtained by simulation is shown in Figure 6, and the subsequent train can basically track the reference trajectory of train 1, and the train running state basically tends to be consistent, but the actual speed error fluctuates in the range of (−0.3, 0.3) m/s in the cruising stage of maintaining the highest speed, due to the problem of jitter in the traditional sliding mode control, so that the train speed fluctuates back and forth at 97m/s, without a decreasing trend. Figure 7 shows the speed curve obtained by using the fractional order adaptive sliding mode control algorithm, from the overall tracking operation, each train can stably track the calculated reference curve, and maintain a stable tracking interval between each train. According to the local magnification figure, when the train is in the cruise mode, the actual speed error is controlled in the range of (−0.1, 0.1) m/s, and the speed error gradually tends to 0 during the cruising process, compared with the previous two

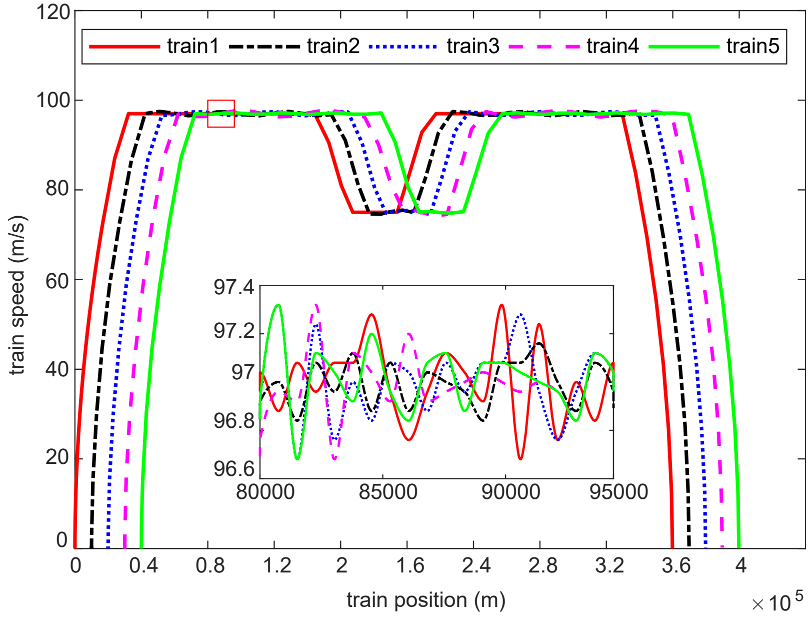

Figure 5.
V-X operation curve generated based on PID control algorithm

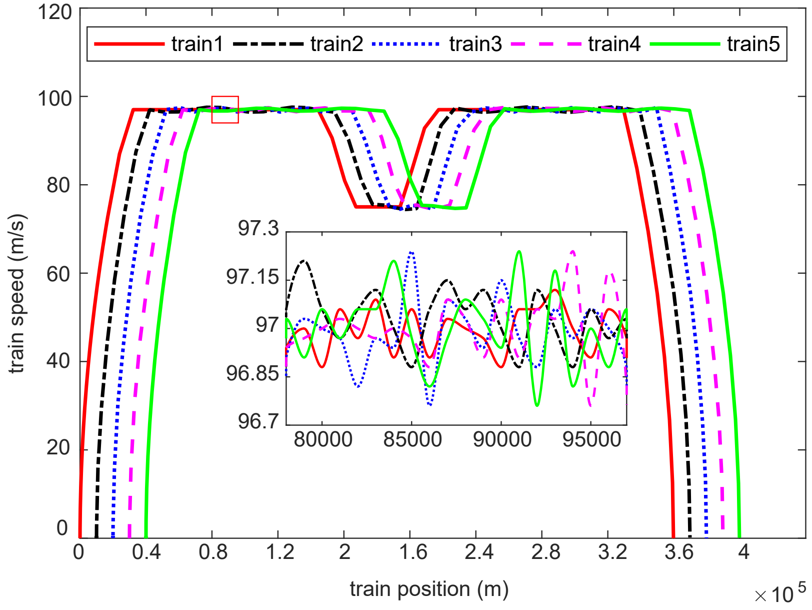

Figure 6.
V-X operation curve based on ordinary sliding mode control algorithm

algorithms, the speed error range is further reduced, and the multi-train collaborative control effect is improved, and the jitter amplitude is better suppressed. Therefore, the proposed algorithm meets the requirements for high-speed train tracking operation speed tracking accuracy.

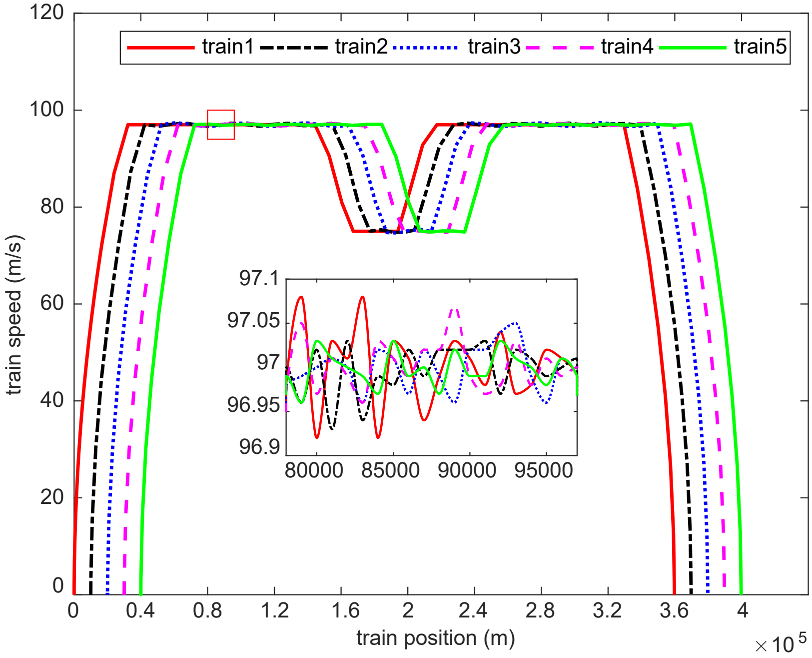

Figure 7.
V-X operation curve generated based on fractional order adaptive sliding mode control algorithm

Figures 8–10 show the train position error under the modes of PID control, ordinary sliding mode control and fractional-order adaptive sliding mode control, respectively. By comparing and observing the following three figures, it can be seen that the three algorithms meet the safety requirements for the train tracking spacing, the error fluctuation range is within the warning line, and there is no collision risk. The PID control algorithm in Figure 8 responds quickly, but a position error of approximately 10 m is generated at the initial stage of the train, and in the subsequent 4400s operating time, the error amplitude is large and fluctuates frequently, the system is prone to overshoot, and the overall position error is controlled within (−10, 10) m. The ordinary sliding mode control algorithm in Figure 9 is robust and has a certain inhibitory effect on interference, so that the initial position error is reduced to less than 8m, and the number of error fluctuations is reduced, and the position error is controlled in the range of (−0.5, 0.5) m, but there is still the problem of system inaccuracy caused by jitter. Under the condition of setting the initial position error, we can see from Figure 8 that the PID control algorithm responds quickly but the error amplitude is large in the operation process, and the fluctuation is due to the inherent overshoot phenomenon of the PID control algorithm; The ordinary sliding mode control algorithm has a strong robustness, and the system response speed is fast, but there is the problem of excessive chattering in Figure 9, which is caused by the discontinuous switching phenomenon inherent in sliding mode control. When the fractional-order adaptive sliding mode controller designed in this paper is adopted, the error of the position distance of each train is basically the zero-crossing point of the horizontal line, and the tracking error is basically 0. It can realize tracking without static error and has good tracking accuracy. Each train is basically kept at the desired tracking position, and the fluctuation trend is similar according to the locally enlarged small window in Figure 10, it shows that under this algorithm, the trains can realize safe and efficient cooperative operation. And there are some short fluctuations due to discontinuous switching at switching

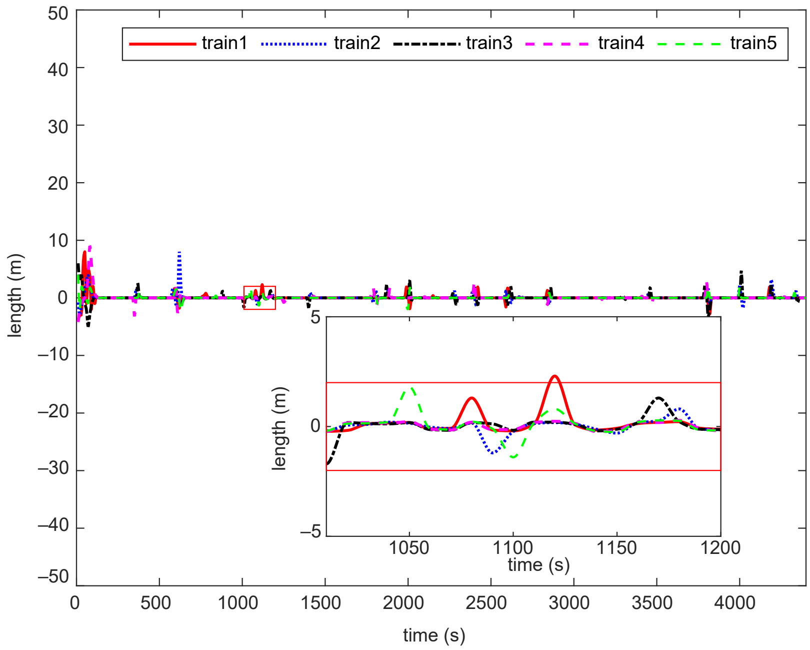

Figure 8.
Train position error based on PID control algorithm

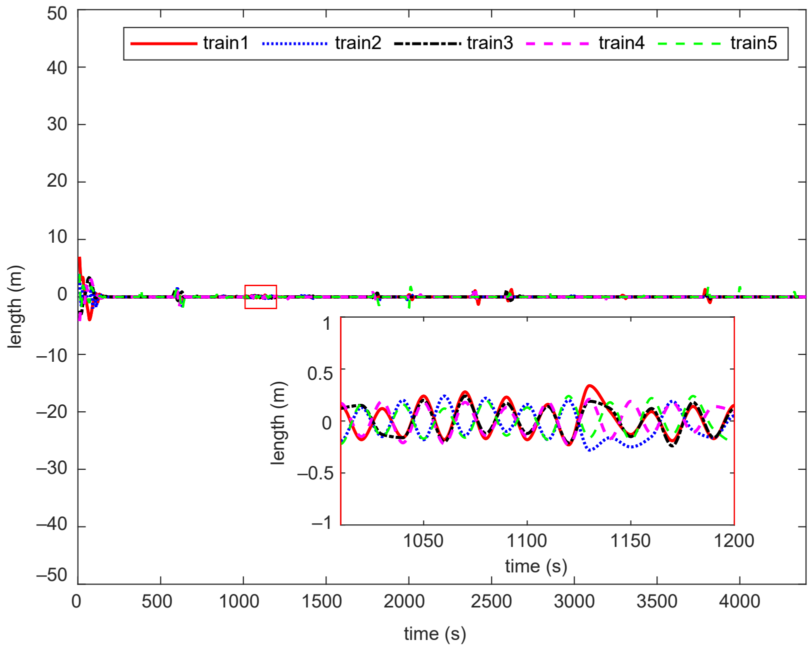

Figure 9.
Train position error based on ordinary sliding mode control algorithm

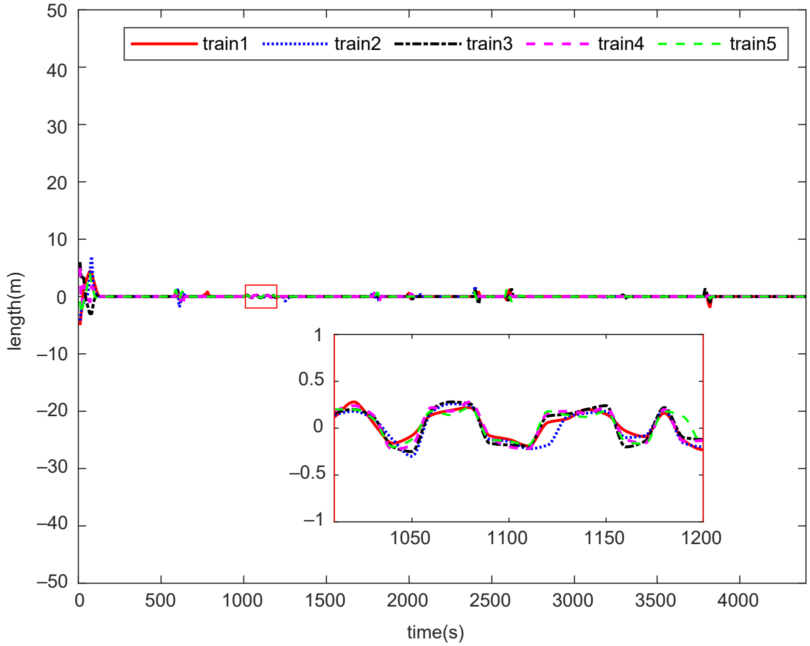

Figure 10.
Train position error based on fractional order adaptive sliding mode control algorithm

points such as traction and cruise braking, which are an inevitable problem in the transition of train operating conditions and it does not affect the stability of the control system.

5. Conclusion

(1) Based on the multi-agent algorithm, the network topology of train information transmission is established through algebraic graph and matrix theory, and the train operation control algorithm is designed by a sliding mode controller.

(2) Based on the fractional-order adaptive sliding mode control, the cascading error of the integrated weight design is used as the input of the control law variable to realize the control, so as to ensure the cooperative predetermined performance.

(3) The distributed control law of train tracking operation and the strict mathematical stability proof of the corresponding closed-loop system are given to realize the stable tracking operation between multiple high-speed trains. Compared with the simulation results of the traditional PID controller and ordinary sliding mode control algorithm, it is proved that the control algorithm proposed in this paper has higher robustness and control accuracy. When the model have parameter uncertainty or strong external disturbance, the control algorithm proposed ensures the accuracy of the train in tracking the reference curve and realize a safe cooperative operation of the train queue efficiently.

(4) The coordinated operation scenario of the train selected in this paper is the tracking operation of a large area, and the trains are synchronized to start and stop synchronously. In future research, it is necessary to be closer to the actual scenario,

and consider the collaborative control algorithm between short stations and the state of unsynchronized start and departure of trains.

(5) In order to simplify the calculation, the single particle model of the train used in this paper does not consider the coupling force between the carriages between trains, and at the same time does not study the complexity of the external environment faced by the train operation in detail, but combines various external disturbances and additional resistance into the simulation, which reduces the model accuracy to a certain extent, and can be studied in more detail to establish a more accurate model for the above shortcomings in the future, so as to further improve the control accuracy.

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Corresponding author
Huadian Liang can be contacted at: 1446049372@qq.com