

Probability estimation method for fatigue crack growth rate based on $a \sim t$ data from service

Probability
model of crack
growth rate

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Abstract

Purpose – The study aims to provide a basis for the effective use of safety-related information data and a quantitative assessment way for the occurrence probability of the safety risk such as the fatigue fracture of the key components.

Design/methodology/approach – The fatigue crack growth rate is of dispersion, which is often used to accurately describe with probability density. In view of the external dispersion caused by the load, a simple and applicable probability expression of fatigue crack growth rate is adopted based on the fatigue growth theory. Considering the isolation among the pairs of crack length a and crack formation time t ($a \sim t$ data) obtained from same kind of structural parts, a statistical analysis approach of t distribution is proposed, which divides the crack length in several segments. Furthermore, according to the compatibility criterion of crack growth, that is, there is statistical development correspondence among $a \sim t$ data, the probability model of crack growth rate is established.

Findings – The results show that the crack growth rate in the stable growth stage can be approximately expressed by the crack growth control curve $da/dt = Q \bullet a$, and the probability density of the crack growth parameter Q represents the external dispersion; t follows two-parameter Weibull distribution in certain values.

Originality/value – The probability density $f(Q)$ can be estimated by using the probability model of crack growth rate, and a calculation example shows that the estimation method is effective and practical.

Keywords Crack growth, $a \sim t$ data, Growth rate, Probability estimation

Paper type Research paper

1. Introduction

Metal components are widely used in various mechanical-related industries, and fatigue fracture of key components is one of the most serious safety risks. As a key parameter, crack growth rate can well describe the evolution law of the fatigue fracture of key components. Therefore, as a prerequisite for safe use, quantitative estimation of crack growth rate becomes an important content to predict and evaluate the damage tolerance of metal components. However, the process of fatigue crack growth is usually dispersive, which means that the distribution of crack growth rate may not be properly described by the mean crack growth rate.

The dispersion of crack growth rate can be divided into two types according to different sources: one is inherent dispersion coming from metal components, which is mainly due to the material metallurgy and manufacturing processes; and the other is external dispersion caused by external load and environment (Zhong, Jin, Hong, & Tao, 2000; James, 2001).

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With the development of material metallurgy and manufacturing technology, inherent dispersion is effectively controlled. Especially when the samples are used to simulate the actual components in the laboratory, the effect of the dispersion of the samples on the random crack growth can be ignored so that the effect of external dispersion on crack growth rate can be well investigated. For the external dispersion, the loading process is generally regarded as a steady-state stochastic process (Yang & Yao, 1995). For example, rails, wheels, bogies and other components while running a railway rolling stock are subjected to steady, random external loads.

In practical engineering, the inherent dispersion and the external dispersion must exist at the same time, which will both affect the crack propagation. Obviously, when the inherent dispersion is small, the load dispersion is the main influence factor of the random crack growth. Though the distribution law of crack growth rate can be obtained by testing a large number of specimens, the cost is too high to bear especially for large-scale metal components. As a result, it is much difficult to evaluate the fatigue crack growth rate with probability.

Yang and Yao (1995) reviewed the random crack growth models considering the two kinds of dispersions, respectively, and suggest that the initiation and propagation of crack because of no exact physical description may be regarded as a “black box” for phenomenological study. The average growth rate model based on the Paris formula was used by Dui, Liu, Wang, and Dong (2020) to describe the average crack growth under the random load spectrum, which was treated as the “equivalent constant amplitude spectrum” so as to reflect all the complicated load sequence effects and other effects, and high-precision prediction of crack growth rate under the random load spectrum has been achieved. Many scholars pay more attention to the durability problem caused by the inherent dispersion and focus on the methods of determining the relatively small crack growth rate (Mannig & Yang, 1984; He, 1994; Zuo, He, & Li, 2021; Chen, Bao, Zhang, & Fei, 2004; Barter, Athinotiis & Clark, 1997). In other literature (Zhang, Chen, & Huang, 2003; Jin, Zhong, Hong, & Xiong, 2000; Wang, Fang, Li, & Liu, 2021), it is emphasized that the distribution of crack growth life is deduced by using the existing data or the incomplete data of fatigue crack growth tests, so that the reliability analysis of crack growth can be carried out. Although much research has been done on fatigue crack growth, the proposed models or methods are mainly based on the analysis and modeling of correlation data.

In the industrial field, many times the same type of metal components are used in large quantities. For example, the 840D wheel was once massively used in the railway freight car, and its material metallurgy, the processing manufacture and so on were identical. In practice, the common mode failure often occurs, such as the fatigue crack propagation of the same type of metal components with one mechanism at the same positions. Recently, with the development and wide application of big data information systems, it has become more and more practical to collect a large amount of $a \sim t$ data (where a is the crack length and t is the time to crack formation) from the components with common mode failure in service, presenting plenty of data about the crack information to digest and benefit from. Although these data are isolated, they definitely provide a new path for probability assessment of fatigue crack growth rate. Chen, Xi, and Wang (2005) proposed an estimation method for average crack growth rates by statistical analysis, using in situ, isolated data on the $a \sim t$ data related to the wheel spoke hole cracks.

In this paper, based on the isolated $a \sim t$ data of metal components in service, a statistical analysis method of data distribution is proposed, and a model as well as an estimation method of fatigue crack growth probability is presented and verified by a practical example. It provides a basis for the effective use of safety-related information data and a quantitative assessment way for the occurrence probability of the safety risk such as the fatigue fracture of the key components in the railway locomotives and vehicles.

2. Statistical method for analyzing t distribution with $a \sim t$ data

2.1 Distribution of time to crack formation

A large number of studies have shown that the time to form a given length of fatigue crack obeys a three-parameter Weibull distribution. For the fatigue of metal components, it is entirely possible that there is a certain length of crack in the initial period of service ($t \approx 0$). Therefore, it can be considered in engineering that the position parameter of Weibull is equal to 0, that is, the time to crack formation t is suitable for Weibull distribution by two parameters. There are:

$$F_{a_i}(t) = 1 - \exp \left[- \left(\frac{t}{\beta_i} \right)^{\alpha_i} \right] \quad (1)$$

In the formula,

a —crack length;

a_i —given crack length;

$F_{a_i}(t)$ —cumulative distribution function of t under given a_i ;

t —time to crack formation;

α_i —Weibull shape parameter under given a_i , closely related to the mechanism of crack propagation;

β_i —Weibull scale parameter under given a_i , mainly related to fatigue load spectrum.

For a large number of $a \sim t$ data of the same type components in service, based on the common-mode failure mechanism, the α_i under different a_i can be taken as constant α . Then, [Formula \(1\)](#) may be reworded as follows:

$$F_{a_i}(t) = 1 - \exp \left[- \left(\frac{t}{\beta_i} \right)^{\alpha} \right] \quad (2)$$

In the formula,

α —Weibull shape parameter under arbitrary crack length.

2.2 $a \sim t$ data collation

For the fatigue crack under service conditions, we may get many isolated $a \sim t$ data. Let n_1 be the group amount of $a \sim t$ data, and the data be arranged in the order of crack length from small to large. Hence, the $a \sim t$ data can be recorded as:

$$(a \sim t)_i = (a_i, t_i), i = 1, 2, \dots, n_1, a_i < a_{i+1} \quad (3)$$

In the formula,

n_1 —the group amount of total $a \sim t$ data;

$(a_i, t_i), (a \sim t)_i$ — $a \sim t$ data of No. i ;

a_i —crack length in the $(a \sim t)_i$ data;

t_i —time to crack formation in the $(a \sim t)_i$ data.

In order to get the t distribution under different crack lengths, it is necessary to divide the total $a \sim t$ data into different crack length segments. Three principles have been taken into consideration on the crack length segments:

- (1) The volume of the crack length segments should not be too small, so as to avoid large error in fitting the probability density of the crack growth parameter. Generally, it should be greater than or equal to 5.
- (2) In order to well fit the probability density of t , the sample number of each crack segment should be as much as possible.
- (3) The range of each crack length segment should not be too large. In this way, the average crack length can be used to represent the given crack length.

Let n_2 be the volume of the crack length segments, and the crack length in each segment be regarded as a fixed value, that is, an average value, then the probability density function and the cumulative distribution function of t in a given segment are as follows:

$$\begin{cases} f_{\bar{a}_j}(t) \\ F_{\bar{a}_j}(t) \end{cases}, j = 1, 2, \dots, n_2 \quad (4)$$

In the formula,

- $f_{\bar{a}_j}(t)$ —probability density function of t under given crack length \bar{a}_j ;
- $F_{\bar{a}_j}(t)$ —cumulative distribution function of t under given crack length \bar{a}_j ;
- \bar{a}_j —average crack length of No. j segment;
- n_2 —the volume of the crack length segments, $n_2 \geq 5$.

At this point, [Formula \(2\)](#) should be rewritten accordingly as follows:

$$F_{\bar{a}_j}(t) = 1 - \exp\left[-\left(\frac{t}{\beta_j}\right)^\alpha\right] \quad (5)$$

2.3 Weibull parameters fitting under different crack lengths

First, based on all the $(a \sim t)_i$ data in the total amount of $a \sim t$ data, a two-parameter Weibull fitting is carried out according to [Formula \(2\)](#) by using common statistical analysis software, such as SPSS software. As a result, the Weibull shape parameter α value for an arbitrary crack length is achieved.

Then, for each crack length segment, using statistical analysis software, set the Weibull shape parameter as the obtained α value, and two-parameter Weibull fitting is performed according to [Formula \(5\)](#), so that the Weibull scale parameters β_j values of n_2 under given \bar{a}_j is obtained. Consequently, the probability distribution $f_{\bar{a}_j}(t)$ for each given \bar{a}_j is obtained.

In order to guarantee the validity of the parameter fitting, it is necessary to check the correlation coefficient given by the fitting. In general, a correlation coefficient greater than 0.9 can meet the engineering needs.

3. Fatigue crack growth model based on load dispersion

3.1 Function relationship between a and t

At the stable stage of crack propagation, the crack growth rate da/dt can be described by the Paris formula in engineering:

$$da/dt = C(Y\Delta\sigma\sqrt{\pi a})^m \quad (6)$$

In the formula,

- da/dt —crack growth rate;
- $\Delta\sigma$ —stress range;

C 、 m —constants relating to the material;

Y —shape factor.

When the external load exists dispersion, the fatigue stress range is not a constant value, [Formula \(6\)](#) can be rewritten as:

$$da/dt = Q \bullet a^b \tag{7}$$

In the formula,

Q —crack growth parameter, relating to the load spectrum;

b —constant relating to the material.

Comparing [Formula \(6\)](#) and [Formula \(7\)](#) shows that the dispersion of external load is entirely reflected by the dispersion of Q . While ignoring the inherent dispersion of material shows that b can be taken as a constant value, and can be treated as $b = 1$ when the crack length a is relatively very small to the size of the component. There are:

$$da/dt = Q \bullet a \tag{8}$$

$$\ln a = Q(t - t_0) \tag{9}$$

In the formula,

t_0 —time to crack formation at $a = 1$ mm.

[Formula \(7\)](#) and [Formula \(8\)](#) are consistent with the formulas for crack growth in structural durability and probabilistic damage tolerance analysis ([Yang & Yao, 1995](#); [Dui et al., 2020](#); [Mannig & Yang, 1984](#); [He, 1994](#); [Zuo et al., 2021](#); [Chen et al., 2004](#)). The crack growth parameter Q represents the dispersion of da/dt . Again, [Formula \(9\)](#) shows that $(\ln a) \sim t$ is of a linear relationship, and its slope is the crack growth parameter Q .

3.2 Probability model of crack growth rate

It is obvious that for the $(a \sim t)_i$ data of n_i , it is difficult to establish a direct correspondence among these isolated data points, so Q value cannot be simply deduced by fitting [Formula \(9\)](#). However, for the $a \sim t$ data with a large amount, we can confirm that the crack growth has a statistical development correspondence, that is, $f_{\bar{a}_{j+1}}(t)$ is derived from the crack propagation of $f_{\bar{a}_j}(t)$ under the action of a certain load spectrum, as shown in [Figure 1](#).

As shown in [Figure 1](#), the crack propagation is controlled by the crack growth curve $da/dt = Q_{jk} \bullet a$ and obeys the compatibility criterion. The so-called compatibility criterion of crack growth means that the cumulative distributions of t at different crack lengths are equal under a given reliability R_k . That is:

$$R_k = 1 - F_{\bar{a}_j}(t_{jk}) = 1 - F_{\bar{a}_{j+1}}(t_{(j+1)k}), k = 1, 2, \dots, n_3 \tag{10}$$

In the formula,

R_k —reliability for $t < t_k$, often set to $R_k = 0.05, 0.1, 0.15, \dots, 0.95$;

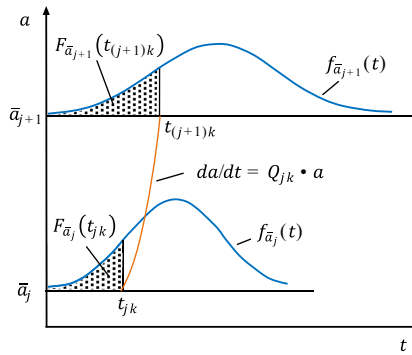
t_{jk} —time to crack formation under given R_k 、 \bar{a}_j ;

n_3 —number of set values of reliability R_k ;

$F_{\bar{a}_j}(t_k)$ —cumulative distribution of t under given R_k 、 \bar{a}_j ;

$F_{\bar{a}_{j+1}}(t_{k+1})$ —cumulative distribution of t under given R_k 、 \bar{a}_{j+1} .

Figure 1.
Probability model of
crack growth rate



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3.3 Estimation of crack growth parameter

Under each given R_k values, t_{jk} values of n_2 can be calculated from [Formula \(10\)](#) and the $f_{\bar{a}_j}(t)$ given by statistical analysis. Furthermore, $n_2 \times n_3$ groups of (\bar{a}_j, t_{jk}) values can be computed. According to [Formula \(9\)](#) and the least square method, we know that:

$$Q_k = \frac{\sum (t_{jk} - t_{lk}) (\ln \bar{a}_{jk} - \ln \bar{a}_{lk})}{\sum (t_{jk} - t_{lk})^2}, j > l; l = 1, 2, \dots, j-1; j = 2, 3, \dots, n_2 \quad (11)$$

In the formula,

Q_{jk} —crack growth parameter under given R_k , \bar{a}_j ;

\bar{Q}_k —average crack growth parameter under given R_k .

Up to this point, n_3 groups of \bar{Q}_k values have been gained.

Many load spectrums are regarded as steady-state stochastic processes in engineering, for example, the load spectrum while trains are running on railway lines. Therefore, it is reasonable to take the crack growth parameter as a normal distribution, that is, $Q \sim N(\mu, \sigma^2)$. Where:

μ —mathematical expectation parameter of normal distribution;

σ^2 —deviation parameter of Normal distribution.

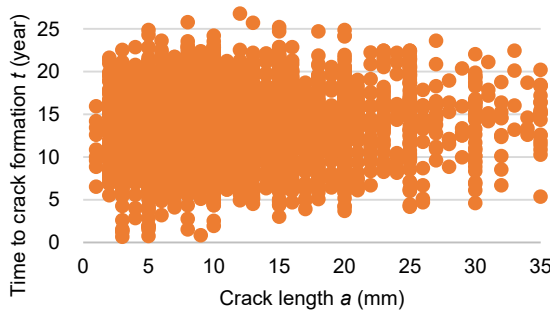
As a result, probability estimation of crack growth parameter can be performed after determining normal distribution parameters by fitting a series of \bar{Q}_k values. At the same time, the normal distribution check is necessary to carry out in order to ensure the validity of fitting.

4. Case demonstration and verification

4.1 The $a \sim t$ data from service

A kind of freight car wheel has been widely used on railway, and a large number of wheels have cracked due to fatigue at the spoke hole location. It is shown that when the crack length $a \leq 35$ mm, the crack is in the stable propagation stage ([Chen, Huang, Gao, Xi, & Liu, 2007](#)).

A total of 3,949 pairs of $a \sim t$ data with $a \leq 35$ mm were collected as shown in [Figure 2](#). The Weibull shape parameter $\alpha = 3.285$ for arbitrary crack length is obtained by two-parameter



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Figure 2. $a \sim t$ data ($n_1 = 3949$)

Weibull fitting for time to crack formation according to Formula (2), as shown in Figure 3. In addition, Figure 3 indicates that t well fits the two-parameter Weibull distribution, and the fitting has a high correlation coefficient R^2 .

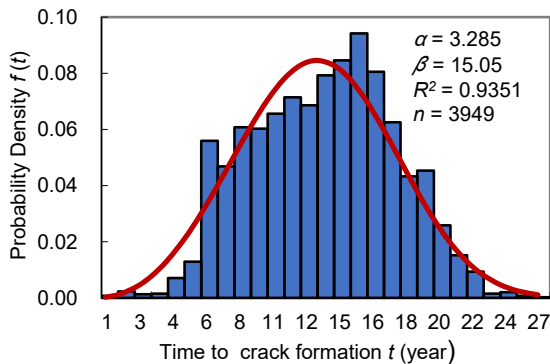
4.2 Determination of probability density of t under various a

According to the total $a \sim t$ data, the 3,949 points are divided into $n_2 = 12$ crack length segments. Two-parameter Weibull fitting was performed by applying Formula (5) with $\alpha = 3.285$. Figures 4–7 display probability densities of t under typical crack lengths, and Table 1 gives the Weibull scale parameter β_j values under the 12 average crack length \bar{a}_j . These results mean that twelve $f_{\bar{a}_j}(t)$ as well as twelve $F_{\bar{a}_j}(t)$ are obtained.

4.3 Distribution function of Q

Given $R_k = 0.05, 0.1, 0.15, \dots, 0.95$, that is, $n_3 = 19$, first 12×13 groups of (\bar{a}_j, t_{jk}) values can be derived from the data given in Table 1, and then 19 \bar{Q}_k values are obtained by calculating according to Formula (12), as shown in Table 2.

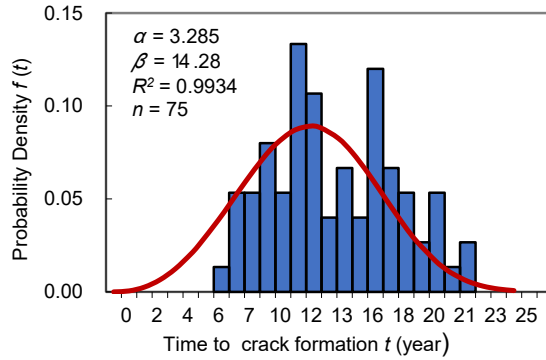
The normal distribution parameters are estimated based on the 19 \bar{Q}_k values, and the Kolmogorov-Smirnov test is performed. It reveals that Q accords with normal distribution, that is, $Q \sim N(1.23, 0.435^2)$, as shown in Figure 8.



Source(s): Authors own work

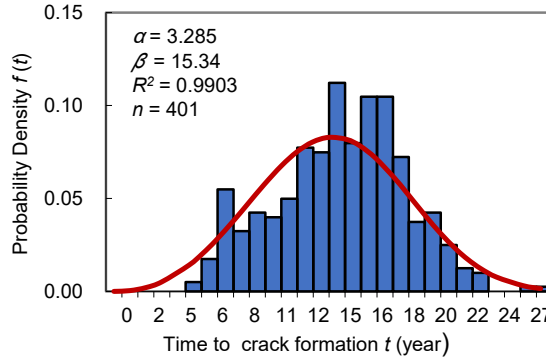
Figure 3. t density $f_a(t)$ under $a \leq 35$ mm

Figure 4.
 $f_{\bar{a}_1}(t)$ under
 $\bar{a}_1 = 1.88$ mm



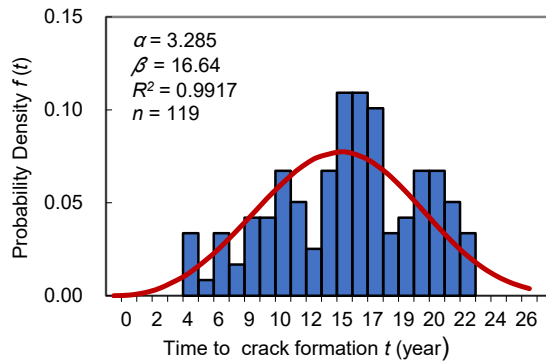
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Figure 5.
 $f_{\bar{a}_5}(t)$ under
 $\bar{a}_5 = 12.63$ mm

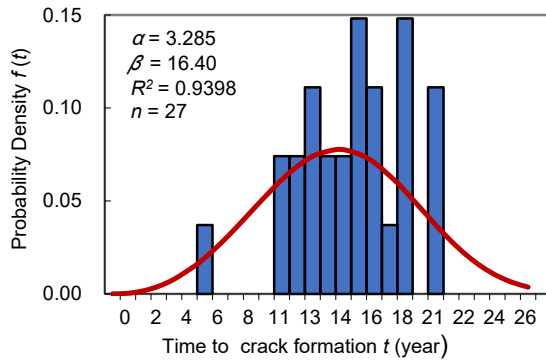


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Figure 6.
 $f_{\bar{a}_9}(t)$ under
 $\bar{a}_9 = 24.97$ mm



Source(s): Authors own work



Source(s): Authors own work

Figure 7. $f_{\bar{a}_{12}}(t)$ under $\bar{a}_{12} = 34.56$ mm

j	Range of segment/mm	\bar{a}_j /mm	Parameters of Weibull distribution			Quantity of data points
			α	β_j	R^2	
1	(0, 3)	1.88	3.285	14.28	0.9934	75
2	[3, 6)	4.40		14.17	0.9973	776
3	[6, 9)	7.03		14.83	0.9970	740
4	[9, 12)	9.99		14.60	0.9964	802
5	[12, 15)	12.63		15.34	0.9903	401
6	[15, 18)	15.36		15.09	0.9950	501
7	[18, 21)	19.42		15.11	0.9968	332
8	[21, 24)	21.97		15.98	0.9584	93
9	[24, 27)	24.97		16.46	0.9917	119
10	[27, 30)	27.72		15.70	0.9401	29
11	[30, 33)	30.46		15.73	0.9713	54
12	[33, 35]	34.56		16.40	0.9398	27

Source(s): Authors own work

Table 1. Weibull distributions of t under different crack lengths

k	1	2	3	4	5	6	7	8	9	10
R_k	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
\bar{Q}_k	0.7095	0.7687	0.8153	0.8572	0.897	0.9364	0.9763	1.0175	1.0611	1.1078
k	11	12	13	14	15	16	17	18	19	
R_k	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	
\bar{Q}_k	1.1588	1.2156	1.2804	1.3561	1.4478	1.5642	1.7227	1.9657	2.4472	

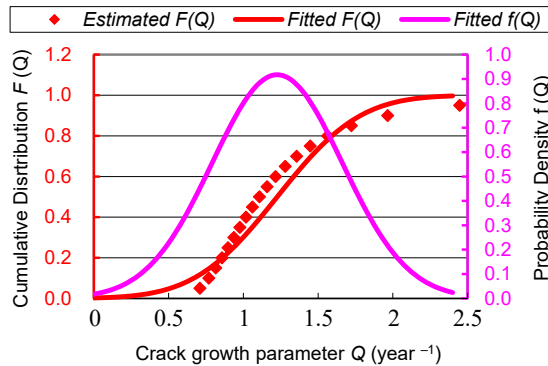
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Table 2. Crack growth parameter values \bar{Q}_k and their cumulative distribution values $F(\bar{Q}_k)$

5. Conclusions

- (1) The crack growth rate can be approximately expressed by $da/dt = Q \bullet a$, and the probability density of crack growth parameter Q represents the external dispersion for the fatigue cracking mode of the same kind of components.
- (2) A large amount of $a \sim t$ data from service are isolated each other, but there is a corresponding relationship of statistical development, which can be reflected by

Figure 8.
Crack growth
parameter Q and its
distributions



Source(s): Authors own work

dividing the crack length range into several segments and statistically analyzing the distribution of time to crack formation.

- (3) According to the compatibility criterion of crack propagation, the probability model of crack growth under different reliability is established, and the probability density of Q can be estimated based on the $a \sim t$ data from service.
- (4) A case study is carried out to demonstrate the method application, and it verifies that the estimation method is effective and practical.

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