An improved fatigue life prediction model based on loading sequence

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Abstract

Purpose – In view of the difficulty in determining the key parameters *d* in the Corten-Dolan model, based on the introduction of small loads, damage degrees and stress states to the Corten-Dolan model and the existing improved model, the sequential effects of the adjacent two-stage load were further considered.

Design/methodology/approach – Two improved Corten-Dolan models were established on the basis of modifying the parameter *d* by two different methods, namely, increasing stress ratio coefficient as well as considering the effects of loading sequence and damage degree as independent influencing factors respectively. According to the test data of the welded joints of common materials (standard 45 steel), alloy materials (standard 16Mn steel) and Q235B steel, the validity and feasibility of the above two improved models for fatigue life prediction were verified.

Findings – Results show that, compared with the traditional Miner model and the existing Corten-Dolan improved model, the two improved models have higher prediction accuracy in the fatigue life prediction of welding materials whether under two-stage load or multi-stage load.

Originality/value – Because the mathematical expressions of the models are relatively simple and need no multi-layer iterative calculation, it is convenient to predict the fatigue life of welded structure in practical engineering.

Keywords Loading sequence, Corten-Dolan model, Fatigue life, Accumulated damage, Parameter correction, Welding material

Paper type Research paper

1. Introduction

With the leap-forward development of heavy-haul freight and high-speed passenger transport of railways in China, welded load-bearing parts have been widely used in various new types of rolling stock structures. However, fatigue cracking is one of the main failure modes of these welded load-bearing parts. As fatigue life prediction is related to the safety of the whole welded structure, more reasonable fatigue life prediction technology is urgently needed in the domestic railway vehicle industry to meet the actual needs of engineering practice. It is of great practical significance to study the life prediction of welded load-bearing structures (Liu *et al.*, 2017; Kim *et al.*, 2013), and many experts and scholars have done a lot of research work on this issue with quite a few results achieved (Zhou & Li, 2010; Yue *et al.*, 2016; Wang, Liu, & Shang, 2013).

For some key welded parts of practically complex rolling stock structures, such as welded joints of train truck frame, a significant feature of its load-bearing state is to withstand multi-stage small loads. In fatigue damage assessment, the effect of small loads,

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instead of being ignored, must be taken into consideration; otherwise, the prediction results will be too conservative, which is not conducive to the safe use of welded structures. Corten-Dolan model can describe the effect of small loads on fatigue damage in a proper way, and can also deal with the interaction effect between loads. Though this model has been applied for fatigue accumulated damage assessment and life prediction in practical engineering (Meng, Wang, & Xue, 2014; Huang *et al.*, 2011; Shang & Yao, 1998, 1999), the definition of its key parameter *d,* which is directly related to the prediction accuracy of fatigue life, is still controversial (Gao *et al.*, 2015). Therefore, how to define the parameter *d* in Corten-Dolan model is the focus of the study, and it is necessary to carry out further analysis and discussion.

The earliest parameter *d* is considered to remain unchanged, and can be determined by the fatigue test of two-stage load (Marciniak, Rozumek, & Macha, 2008). Subsequently, it is found that the parameter *d* cannot be regarded as a constant. Although the parameter can be determined by fatigue test, the values obtained from different fatigue load tests differ a lot (Chen, 1996). With the influence of stress state and real-time damage on fatigue accumulated damage considered, Gao (2016) has redefined the function of parameter *d*. Good results have also been obtained in numerical verification. The prediction accuracy of the model, though getting enhanced significantly, still needs further improvement.

In the actual service environment of rolling stock, the stress states of welded parts of the vehicle structure are very complex. For the fatigue accumulated damage under complex multi-stage loads, the loading sequence and the interaction between loads should also be fully considered. Although the key parameter *d* in Corten-Dolan model and the existing improved model can reflect the interaction between loads, it fails to consider the influence of loading sequence on fatigue damage accumulation, which will to a large degree affect the prediction results that are highly dispersed and often can hardly meet the actual engineering requirements.

Based on small loads, damage degrees and stress states considered, this paper goes further to include the relations of loading sequence to improve the Corten-Dolan model, and it verifies the validity and feasibility of Corten-Dolan model according to the test data of the welded joints of common materials, alloy materials and Q235B steel.

2. Corten-Dolan model and the existing improved model

According to the Corten-Dolan model, under constant-amplitude loading, after n stress cycles, the accumulated fatigue damage D_d can be expressed as,

$$D_d = prn^{a'} \tag{1}$$

Where, P is the number of nucleus flaw under stress; r is the damage coefficient; a' is a constant related to the material.

According to Formula (1), the critical fatigue damage is,

$$D_c = p_{\max} r_{\max} N_{f\max}^{a'_{\max}} \tag{2}$$

Where, $P_{\max}$, $r_{\max}$ and $a'_{\max}$ are respectively the number of nucleus flaw, the damage coefficient and the corresponding material-related constant under the maximum stress; $N_{f\max}$ is the fatigue life under the maximum stress.

Under variable amplitude loading, the fatigue accumulated damage equation can be established according to the total damage amount, that is, the sum of damage in each section should be equal to the sum of damage under constant-amplitude loading alone. Under the action of multi-stage variable amplitude loads, the accumulated damage D_d corresponding to failure is consistent with the critical fatigue damage D_c, then D_d can be expressed as follows:

$$D_d = \sum\nolimits_{i=1}^{k} p_i r_i n_i^{a'_i} = p_{\max} r_{\max} N_{f\max}^{a'_{\max}} \tag{3}$$

Where, i is the number of stages of the stress cycle.

In fatigue failure analysis, the fatigue life prediction formula of Corten-Dolan model under multi-stage loads and multi-stage stress cycles can be expressed as follows:

$$N_f = \frac{N_{f\max}}{\sum_{i=1}^{k} a_i \cdot \left\{ \sigma_i / \sigma_{\max} \right\}^{d}} \tag{4}$$

Where, N_f is the fatigue life predicted by the model under the action of multi-stage stress cycles; a_i is the percentage of the i-stage stress cycles to the total cycles; σ_i is i-stage stress; $\sigma_{\max}$ is the maximum stress in multi-stage stress.

In Formula (4), parameter d is related to material properties and stress level. Gao (2016) defines parameter d as a function including cycle times ratio and stress ratio, which is as follows:

$$d = \exp\left\{ \left\{ n_i / N_{fi} \right\}^{\sigma_i/\sigma_{\max}} \right\} + \gamma \tag{5}$$

Where, n_i is the fatigue life corresponding to the actual stress cycle; N_{fi} is the fatigue life of the i-stage stress σ_i under the action of single cycle.

Formula (5) introduces a parameter γ, which is a constant related to material and can be determined by the test data in combination with the fatigue failure criterion. With Formula (5) substituted into Formula (4), the existing improved model of Corten-Dolan model can be expressed as follows:

$$N_f = \frac{N_{f\max}}{\sum_{i}^{k} a_i \left\{ \sigma_i / \sigma_{\max} \right\}^{\exp\left\{ \left\{ n_i / N_{\mathrm{f}} \right\}^{\sigma_i/\sigma_{\max}} \right\} + \gamma}} \tag{6}$$

According to the parameter d defined by Formula (5), the existing improved model of the corresponding Corten-Dolan model can also be written into another expression form, which is,

$$\frac{\sum n_i}{N_f} = \sum\nolimits_{i=1}^{k} \frac{n_i}{N_{f\max}} \cdot \left\{ \sigma_i / \sigma_{\max} \right\}^{\exp\left\{ \left\{ n_i / N_{\mathrm{f}} \right\}^{\sigma_i/} \right\} + \gamma} \tag{7}$$

Compared with Miner's rule and Manson-Halford model, the existing improved model of Corten-Dolan model reflects the interaction between loads by the ratio of stress of various stages to the maximum stress while considering the contribution of small loads. Thus the prediction results of fatigue life have higher reliability, but the calculation of the only parameter d is yet to be further discussed.

3. An improved Corten-Dolan model based on loading sequence

The existing improved model of Corten-Dolan model does not consider the effect of real-time loading sequence of loads of various stages, and is derived based on the linear Miner accumulated damage theory where Miner accumulated damage is independent of the loading sequence. However, numerous experimental data show that the accumulated fatigue damage is related to the loading sequence. For simple two-stage test loading, the accumulated fatigue damage during failure is often not equal to 1. When the loading sequence is low first and then high, the accumulated fatigue damage $\Sigma n_i/N_{fi}$ is more than 1 and the crack initiation time is delayed. When the loading sequence is first high and then low, the accumulated fatigue damage $\Sigma n_i/N_{fi}$ is less than 1, and the high stress causes the crack to form ahead of time, while the low stress causes the crack to expand. For multi-stage loading, the same conclusion can be drawn.

According to the above analysis, when the definition of key parameter d is to be modified, besides the influence of small loads, real-time damage and stress states on fatigue accumulated damage, the influence of loading sequence also needs to be considered, so as to further improve the existing improved model of Corten-Dolan model.

Since it is often multi-stage variable amplitude loads that act in practical engineering, it is not appropriate to consider loads of all stages before loading. Therefore, only the sequential effects of the adjacent two-stage loads are taken into consideration. The parameter d can be modified with the following two methods.

Method 1: The stress ratio coefficient is directly increased on the basis of Formula (5) to correct d, then,

$$d_1^* = d * \sigma_{i+1}/\sigma_i \tag{8}$$

Where, $d*$ 1 is the calculation parameter when method 1 is adopted to improve the prediction model.

With Formula (8) substituted into Formula (6), the improved fatigue life prediction model 1 (prediction model 1 for short) after correction by method 1 can be obtained as,

$$N_f = \frac{N_{f\max}}{\sum_{i=1}^{k} a_i \cdot \left\{\sigma_i/\sigma_{\max}\right\}^{\frac{\sigma_{i+1}}{\sigma_i}\left\{\exp\left\{\left\{n_i/N_{\mathrm{f}}\right\}^{\sigma_i/\sigma_{\max}}\right\}+\gamma\right\}}} \tag{9}$$

Method 2: As the contribution of parameter γ, a constant related to materials, to the key parameter d is fixed, the effects of loading sequence and damage degree are taken as independent influencing factors respectively to modify d, then,

$$d_2^* = \exp\left\{\left\{n_i/N_{fi}\right\}^{\sigma_1/\sigma_{\max}}\right\} + \exp\left\{\left\{n_i/N_{fi}\right\}^{\sigma_{i+1}/\sigma_i}\right\} \tag{10}$$

Where, $d*$ *2* is the calculation parameter when method 2 is adopted to improve the prediction model.

With Formula (10) substituted into Formula (6), the improved fatigue life prediction model 2 (prediction model 2 for short) after correction by method 2 can be obtained as,

$$N_f = \frac{N_{f\max}}{\sum_{i=1}^{k} a_i \cdot \left\{\sigma_i/\sigma_{\max}\right\}^{\left\{\exp\left\{\left\{n_i/N_{\mathrm{fi}}\right\}^{\sigma_i/\sigma_{\max}}\right\}+\exp\left\{\left\{n_i/N_{\mathrm{fi}}\right\}^{\sigma_{i+1}/\sigma_i}\right\}+\gamma\right\}}} \tag{11}$$

After the key parameter d is modified by the above two models, when the adjacent loads ($\sigma_{i+1}/\sigma_i > 1$) adopt the loading sequence of first low and then high, the predicted life is increased. When the adjacent loads ($\sigma_{i+1}/\sigma_i < 1$) adopt the loading sequence of first high and then low, the predicted life is reduced, which is consistent with the analysis of the relationship between fatigue accumulated damage and loading sequence.

The inversion identification of the undetermined parameter γ in Formula (11) can be carried out using test measurement data with two prediction models by means of inversion optimization. In the inversion identification, parameter γ can be easily identified by the one-dimensional search method where objective function is constructed according to the least square principle based on the test measurement data and the analysis results of two prediction models.

4. Numerical examples

In order to verify the fatigue prediction ability of the improved Corten-Dolan model based on loading sequence, the test data of materials such as standard 45 steel, standard 16Mn steel and hot-rolled 16Mn steel given in Gao (2016) are adopted to carry out the fatigue life prediction. By comparing the predicted value of the model with the test data, the prediction accuracy of the improved model is verified, and the prediction results of different models are compared.

Example 1. Fatigue life prediction of welding materials under two-stage load.

The two-stage loads of standard 45 steel are 331.46 MPa and 284.40 MPa respectively, and the fatigue life of welded joints under their separate action is 5.0×10^4 and 5.0×10^5 times respectively. The calculation results of parameter d obtained by different methods and the comparison of fatigue life prediction results in different models under various loading modes (stress levels corresponding to different loading sequences and different stress cycles) are shown in Tables 1 and 2 respectively.

The two-stage loads of standard 16Mn steel are 562.9 MPa and 392.3 MPa respectively, and the fatigue life of welded joints under their separate action is 3.968×10^3 and 7.8723×10^4 times respectively. The calculation results of parameter d obtained by different methods and the comparison of fatigue life prediction results in different models under various loading modes are shown in Tables 3 and 4 respectively.

The two-stage loads of hot-rolled 16Mn steel are 394 MPa and 345 MPa respectively, and the fatigue life of welded joints under their separate action is 9.35×10^4 and 4.022×10^5 times respectively. The calculation results of parameter d obtained by different methods and the comparison of fatigue life prediction results in different models under various loading modes are shown in Tables 5 and 6 respectively.

It can be seen from the above calculation results that under two-stage load, the Corten-Dolan improved model based on the loading sequence in this paper can accurately predict the fatigue life of welded joints for standard 45 steel, standard 16Mn steel and hot-rolled 16Mn materials. Compared with the linear Miner's rule and the improved method in GAO Huiying (2016), there is significant improvement in the prediction results of the two prediction models in this paper.

Example 2. Fatigue life prediction of welded joints under multi-stage load

According to the test data of Q235B steel under variable amplitude loading in Gao (2016), the fatigue life of its welded joints is predicted by using different prediction models with five-stage load as an example, and the prediction comparison results are shown in Table 7.

Table 1.
Comparison of calculation results of parameter d of standard 45 steel under different loading modes and different solutions

Loading mode	Method in GAO Huiying (2016)	Method 1	Method 2
1	16.5951	14.1786	14.5063
2	15.9515	13.6264	13.8632
3	15.6952	13.4065	13.6069
4	15.4023	13.1552	13.3140
5	15.5698	18.0642	13.9702
6	15.9502	18.5076	14.3506
7	16.3983	19.0297	14.7986

It can be seen from the calculation results in Table 7 that the fatigue life of Q235B welded joints can also be predicted by the improved Corten-Dolan model under five-stage variable amplitude loads, and the prediction accuracy is significantly improved compared with the traditional Miner's rule and the models in references.

Table 2. Fatigue life of standard 45 steel predicted by different models under two-stage load

Loading mode	σ_1/MPa	σ_2/MPa	n_1/10^5	n_2/10^5	Predicted by different models $\left(\frac{\sum n_p}{N_f}\right)$ Miner model	The prediction model in GAO Huiying (2016)	Prediction model 1	Prediction model 2
1	331.46	284.4	0.005	4.237	0.8574	0.6777	0.9765	0.9292
2	331.46	284.4	1.250	2.504	0.7508	0.6854	0.8716	0.8495
3	331.46	284.4	2.500	1.683	0.8366	0.8043	0.9320	0.9190
4	331.46	284.4	3.750	0.645	0.8790	0.8720	0.9221	0.9180
5	284.40	331.46	1.250	0.379	1.0080	0.9884	0.9153	1.0524
6	284.40	331.46	2.500	0.389	1.2780	1.2128	1.0719	1.3334
7	284.40	331.46	3.750	0.434	1.6180	1.4769	1.2750	1.6459

Table 3. Comparison of calculation results of parameter *d* of standard 16Mn steel under different loading modes and different solutions

Loading mode	Method in GAO Huiying (2016)	Method 1	Method 2
1	8.4358	8.1187	8.1743
2	8.1138	7.8943	7.8523
3	8.0463	7.8473	7.7848
4	7.8616	7.7185	7.6001
5	7.3657	7.3729	7.1041

Table 4. Fatigue life of standard 16Mn steel predicted by different models under two-stage load

Loading mode	σ_1/MPa	σ_2/MPa	$n_1/10^4$	n_2/10^4	Predicted by different models $\left(\frac{\sum n_p}{N_f}\right)$ Miner model	The prediction model in GAO Huiying (2016)	Prediction model 1	Prediction model 2
1	562.9	392.3	0.0002	7.360	0.9357	0.8825	0.9895	0.9698
2	562.9	392.3	0.020	5.940	0.8052	0.8500	0.9159	0.9292
3	562.9	392.3	0.100	5.630	0.9674	1.0286	1.0864	1.1055
4	562.9	392.3	0.170	4.760	1.0332	1.1303	1.1675	1.1998
5	562.9	392.3	0.245	2.290	0.9084	1.0213	1.0202	1.0613

Table 5. Comparison of calculation results of parameter *d* of hot-rolled 16Mn steel under different loading modes and different solutions

Loading mode	Method in GAO Huiying (2016)	Method 1	Method 2
1	12.6491	9.7488	8.7324
2	12.4991	9.6175	8.8618
3	12.3860	9.5184	9.1513
4	12.1863	9.3435	9.2760
5	11.8817	9.0768	9.4493
6	12.2705	12.2823	10.3712
7	12.3350	12.3560	10.3225
8	12.4869	12.5295	9.8361

Table 6.
Fatigue life of hot-rolled 16Mn steel predicted by different models under two-stage load

					Predicted by different models $\left(\frac{\sum n_p}{N_f}\right)$			
Loading mode	σ_1/ MPa	σ_2/ MPa	$n_1/10^5$	n_2/ 10^5	Miner model	The prediction model in GAO Huiying (2016)	Prediction model 1	Prediction model 2
1	394	345	0.0935	2.695	0.7701	0.6373	0.8897	0.8671
2	394	345	0.197	2.361	0.7977	0.6908	0.9147	0.8962
3	394	345	0.394	2.095	0.9422	0.8538	1.0544	1.0389
4	394	345	0.4675	1.593	0.8961	0.8377	0.9926	0.9822
5	394	345	0.561	0.740	0.7840	0.7633	0.8371	0.8332
6	345	394	1.810	0.828	1.3363	1.2657	1.2651	1.3990
7	345	394	1.971	0.809	1.3560	1.2757	1.2745	1.4196
8	345	394	2.333	0.595	1.2171	1.1123	1.1097	1.2793

Table 7.
Fatigue life of Q235B welded joints predicted by different models under multi-stage load

					The fatigue life predicted by different models $N_f/10^4$			
Loading mode	Stress σ_i/ MPa	Number of stress actions $N_i/10^4$	Fatigue life under single-stage stress action $N_{fi}/10^4$	Test life N_E/ 10^4	Miner model	The prediction model in GAO Huiying (2016)	Prediction model 1	Prediction model 2
1	22.40	25.03	116.64	158.05	63.49	110.43	195.08	214.89
	23.30	53.07	95.17					
	25.10	44.35	64.81					
	26.00	21.56	54.03					
	30.90	14.04	22.15					
2	32.76	40.00	88.85	173.03	64.25	93.40	139.24	163.93
	34.29	40.00	73.87					
	35.81	40.00	61.98					
	37.33	40.00	52.39					
	38.86	13.03	44.53					
3	32.00	40.00	97.70	252.87	61.33	75.43	90.73	96.69
	33.52	40.00	80.98					
	35.05	40.00	67.60					
	36.57	40.00	56.93					
	38.10	92.87	48.24					

5. Conclusions

(1) An improved Corten-Dolan model based on loading sequence is established, with sufficient consideration for the influence of small loads, damage degree, stress states and loading sequence and can effectively predict the fatigue life.

(2) According to the test data verification results of the welding joints of common materials, alloy materials and Q235B steel, compared with the traditional Miner model, the two improved models in this paper have good feasibility, whether under two-stage load or multi-stage load, and the prediction accuracy is significantly improved; the prediction results are more reliable.

(3) The improved model in this paper is relatively simple. There is no need for multi-layer iterative calculation, and model parameters, small in number, are easy to determine. So it is convenient to predict the fatigue life of welded structures in engineering.

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