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Full Length Article

Pressure transient analysis of commingled multilayer reservoirs: A deconvolution-based approach for individual layer characterization using sparse PLT data

Mina S. Khalaf

Cairo University, Egypt

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ABSTRACT

Layering is common in hydrocarbon reservoirs due to stratified sedimentary deposition, with each layer exhibits distinct characteristics. Multilayer reservoirs are typically classified as systems with interlayer/formation crossflow and systems without interlayer crossflow (commingled systems). This study focuses on the latter, where fluid exchange between layers occurs solely through the wellbore. Accurately interpreting pressure transient data from commingled systems to extract individual layer properties remains a persistent challenge. Most existing interpretation techniques rely on assumed reservoir models (typically homogeneous, isotropic, and infinite in extent) that involve numerous unknown parameters (such as permeabilities and skin factors) and require nonlinear history matching. These assumptions not only introduce subjectivity but can also yield unreliable estimates. The most reliable way to characterize individual layers has traditionally required isolating and testing each layer separately, which is a process that is both technically complex and costly to implement. This study introduces a testing and analysis methodology developed specifically for commingled multilayer reservoirs. The proposed approach utilizes deconvolution to remove rate variation effects from bottomhole pressure data and generate pressure responses equivalent to constant-rate conditions. This transformation enables the recovery of distinct pressure signatures for each individual layer without requiring assumptions. The recovered pressure signals are wellbore storage (WBS) free and extend over the entire test duration, offering a clearer window into the dynamic behavior of each contributing formation. Deconvolution needs complete sandface rate data for each layer, data that are typically unavailable in field practice. To address this, the study develops a simple yet effective power-law model that reconstructs layer-specific rate profiles using only a few discrete production logging tool (PLT) measurements, making the overall approach feasible for real-world applications. The method was implemented and validated on three simulated oil reservoir scenarios, each exhibiting different reservoir/boundary characteristics. The results show that the estimated rate profiles, when input into a stable deconvolution algorithm, produce layer-specific pressure responses. The technique successfully distinguishes the flow behavior and boundary conditions of each individual layer, even under complex conditions such as heterogeneity, no-flow boundaries, dual-porosity formations, and wellbore crossflow during shut-in periods. This advancement enables efficient reservoir description and production optimization, by eliminating the need for mechanical isolation or extended shut-in periods, offering a practical alternative that is aligned with the industry demands for minimum well intervention and cost-effective surveillance.

1. Introduction

Layering in oil and gas reservoirs is a common outcome of sedimentary depositional processes, which naturally create stratified formations with varying properties such as pressure, permeability, porosity, thickness, and areal extent. These variations mean that each layer can respond differently to production and stimulation efforts, making it essential to understand their individual characteristics. Accurate multilayer reservoir characterization is essential for designing effective well completions and stimulation treatments that fit the conditions of individual layers.

E-mail address: minaskhalaf@gmail.com.
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This, in turn, enhances hydrocarbon recovery, improves production efficiency, and supports informed decision-making in reservoir management and field development planning.

Pressure transient analysis in layered reservoirs is generally categorized into two main types: (1) systems with interlayer or formation crossflow, where fluid communication occurs between layers within the reservoir due to non-zero vertical permeability, and (2) systems without formation crossflow, commonly referred to as commingled reservoirs, where layers are hydraulically isolated by impermeable barriers and only interact through the wellbore. In commingled systems, although the formations themselves are disconnected, wellbore crossflow may still take place when fluids move between layers via the wellbore, particularly if both layers are completed in the same well. This type of crossflow becomes problematic when there is a significant pressure differential between layers, leading to fluid transfer from a higher-pressure zone to a lower-pressure one, which can reduce production efficiency. To accurately evaluate and optimize performance in such systems, it is essential to characterize each layer individually and understand its contribution to overall reservoir behavior (Ehlig-Economides and Joseph, 1987; Eissa et al., 2008).

This work focuses on the pressure transient analysis of commingled reservoir systems. In such systems, each layer can be represented by a distinct reservoir and boundary model, with no restrictions on the form of the model. The only constraint is the absence of interlayer communication within the formation, as illustrated in Fig. 1 for a four-layer commingled reservoir. What differentiates the behavior of a commingled system from that of a single-layer reservoir is the presence of flow rate transients in addition to pressure transients. These layer-specific rate variations introduce complexity to the bottomhole pressure response and make interpretation challenging. While the pressure response reflects the behavior of the overall system, the flow rate profile of each layer is governed by that layer's individual properties (Ehlig-Economides and Joseph, 1987; Stewart, 2011). In other words, in commingled systems, the bottomhole pressure is a coupled response resulting from simultaneous contributions of multiple layers, each governed by its own flow regime and boundary conditions. The dynamic interaction through the wellbore results in transient interdependence, especially during periods of production or shut-in, where crossflow may occur in the wellbore even in the absence of formation crossflow. This interaction masks the individual reservoir signals and complicates the identification of layer-specific characteristics such as permeability, skin, and boundary conditions.

The conventional single-layer analysis techniques fall short when applied to commingled multilayer reservoirs, providing only

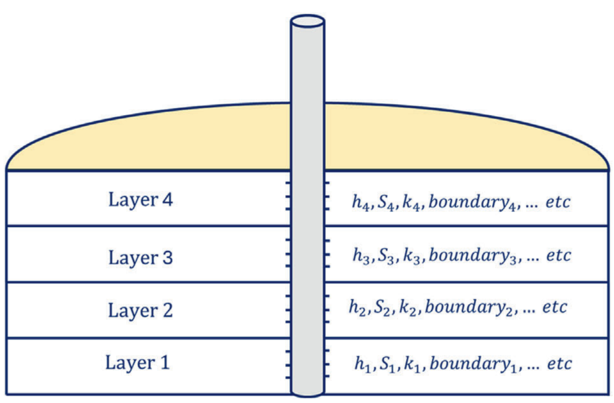

Fig. 1. Schematic illustration of a commingled multilayer reservoir system.

averaged system properties and failing to resolve the underlying complexity (Pan et al., 2010; Stewart, 2011). Efforts to characterize individual layers commingled in a wellbore date back to the early 1960s (Lefkovits et al., 1961; Tempelaar-Lietz, 1961). Nevertheless, the techniques developed since then have remained unable to reliably detail the properties of individual layers. A more advanced analysis methodology is needed to separate and interpret the individual contributions, which is an objective addressed by the technique developed in this study.

This paper is structured as follows: The Introduction provided background on multilayer reservoir systems, outlined the problem, and stated the objectives of the study. The Literature Review summarizes the testing and interpretation methods presented in the literature for analyzing multilayer reservoirs. The Developed Methodology section describes the deconvolution approach introduced in this work for analyzing performance data from commingled reservoirs. The Applications section demonstrates the implementation of the proposed technique on three simulated cases. The Discussion section evaluates the advantages, disadvantages, and limitations of the methodology. Finally, the Summary and Conclusions section summarizes the findings of the study.

2. Literature review

Over the past several decades, research efforts have focused on the pressure transient analysis of multilayer reservoir systems due to their complex and stratified nature. A primary challenge in interpreting well test data from these systems is the uncertainty in defining an accurate reservoir model, particularly regarding outer boundary conditions, interlayer communication, and internal heterogeneity. Additionally, the need to estimate a large number of unknown parameters—such as individual layer permeabilities, skin factors, and initial pressures—makes the problem highly nonlinear and often ill-posed, especially when relying on history matching with limited data. These difficulties are further compounded in commingled systems, where layers are hydraulically isolated in the formation but interact through the wellbore, leading to overlapping pressure and rate transients. In response, the petroleum literature has introduced various analytical and numerical methods aimed at improving interpretation reliability, reducing subjectivity, and enabling more accurate layer-specific characterization. This section provides a comprehensive review of existing methods for pressure transient analysis in multilayer reservoirs, focusing on their assumptions, limitations, and applicability, highlighting the gap that this study addresses.

Lefkovits et al. (1961) probably conducted the first rigorous study on commingled reservoir systems; they developed an analytical model for the layered system and used the model to report observations. The study shows that at late time of flow period, the flow rate from individual layers equals to the ratio of the layer porosity-thickness of that layer to the total system porosity-thickness; early time behavior shows that the flow rate fraction is strongly related to the layer permeability-thickness ratio. Tariq and Ramey Jr. (1978) extended the model by including wellbore storage effect and used Stehfest algorithm (Stehfest, 1970) for numerical inversion of Laplace transform. It is good to mention that Tariq and Ramey Jr (Tariq and Ramey Jr, 1978). along with Sandal et al. (1978) are possibly the first to bring Stehfest algorithm to the petroleum engineering applications. The commonly used commingled testing procedure was introduced by Kuchuk et al. (1986a). This multilayer transient (MLT) test entails performing sequential flow tests. During these tests, continuous and simultaneous pressure and sandface flow rate measurements are obtained just above a different layer at each flow test. And each flow test should be continued until the stabilized flow rate is reached. Moreover, production logging tool (PLT) survey is required at the end of each flow test. Conducting this procedure requires considerable testing duration.

The stabilized rate surveys at the end of the flow tests are used to develop a series of inflow performance relationship (IPR) curves. This is accomplished by applying selective inflow performance (SIP) analysis and the developed curves are used to evaluate the average layer pressures (Stewart et al., 1981). Kuchuk et al. (1986a) used the data acquired from their MLT test to sequentially interpret the system layers for permeabilities and skin factors starting with the lowest layer proceeding to the upper. This is completed through history matching process of the measured data and data generated form a presumed reservoir model (that is composed of homogenous isotropic layers infinite in extent with a fluid of constant viscosity and compressibility). The history matching process is an inverse problem that aims to determine the reservoir properties by minimizing the differences between the measured and model-generated data. However, in the forward problem, the reservoir description is known, and the model is used to generate the reservoir performance data. Kuchuk et al. (1986a) validated their technique using simulated examples. The technique was then extended to field applications by several researchers (Joseph et al., 1988; Kuchuk et al., 1986b; Morris, 1987). Shah et al. (1988) proposed an alternative analysis method to interpret the data from MLT test of Kuchuk et al. (1986a). Rather than following sequential interpretation approach, Shah et al. proposed a technique that embraces simultaneous determination of the system permeabilities and skin factors through history matching process using a presumed reservoir/boundary model. They validated their simultaneous analysis technique against simulated examples. Eissa et al. (2008) presented two field cases of multilayer systems and discussed the standard MLT procedure and the analysis techniques.

Ehlig-Economides and Joseph (1987) developed an analytical solution to a general layered reservoir model that consists of an arbitrary number of layers. The model can account for crossflow between any two adjacent layers. In addition, the authors introduced a two-rate testing procedure and developed an interpretation technique that exploits their analytical model for properties evaluation of the individual layers. The analysis procedure relies on history matching of the measured data with presumed-model data. The analysis methodology utilized the pressure transients and the rate transients (resulting from a change in the surface flow rate) in an inverse problem to solve for layer permeabilities, skins, and vertical permeabilities (for interlayer cross flow). Furthermore, Ehlig-Economides and Joseph (1987) proposed two approaches for flow rate analysis: (1) flow rate transient type curves analysis and (2) flow rate/pressure convolution. The flow rate/pressure convolution alternative is used as precursor of the work developed in this paper. Moreover, they presented an extensive survey of layered reservoirs work published before 1987. Ehlig-Economides (1987) summarized the techniques for testing layered reservoirs that field tests have shown to be successful, and Ehlig-Economides (1993) presented a technique for better selection of the model to be used in the history matching process.

Kuchuk and Wilkinson (1991) developed a general analytic model for pressure transient of stratified formations without interlayer crossflow. The analytical solution considers the effects of surface flow rate changes and wellbore storage. The model accounts for different initial and outer-boundary conditions of individual layers. In addition, the production of individual layers can start at different times. Jackson and Banerjee (2000) developed a workflow that incorporates reservoir simulation and automated history matching for obtaining layer permeabilities and skin factors. They implemented their procedure to simulated and field cases. The procedure was then adopted to analyze field data in western Kazakhstan (Pan et al., 2010; Sullivan, 2007). Kgogo and Gringarten (2011) investigated well test analysis of gas-condensate multilayered reservoirs. Larsen (1989) developed an analytical model to investigate the effect of different outer boundaries. Larsen (1994) discussed the problems and possibilities of combining the pressure data and the rate/PLT data (through history matching) and the implantation of single-layer analysis methods to multilayer systems. Lolon et al. (2008) and Rahman and Mattar (2007) developed solutions to different commingled reservoirs problems. Vieira Bela et al. (2019) developed analytical solution for injectivity tests of commingled reservoirs and they employed their model to estimate the reservoir equivalent permeability from fall-off data.

Li et al. (2022) developed an analytical well-test model for hydraulically fractured wells in stratified reservoirs with variable fracture conductivity. The model improves pressure transient interpretation by capturing conductivity changes across layers. Liang et al. (2024) analyzed the production dynamics of fractured wells in multilayer reservoirs, presenting a model to evaluate well performance and flow characteristics across complex layered systems. Many other researchers directed their efforts to discuss commingled systems testing (Aguilar et al., 2016; Galvao and Guimaraes, 2017; Onwunyili and Onyekonwu, 2013; Poe et al., 2006; Spivey, 2006; Sun et al., 2003). Sui et al. (2008) and Sui and Zhu (2012) proposed a testing procedure and developed an interpretation technique that can exclude transient rate measurements. The approach uses transient temperature and pressure measurements for estimating the permeabilities and skin factors of the layered system. They developed a forward model to generate temperature and pressure data. Then, they employed a nonlinear least-square regression technique in an inverse problem to solve for different layers properties.

These conventional techniques were developed based on the concept of performing the analysis using history matching of continuously measured data with analytically or numerically simulated data. These techniques assume a system model (usually homogenous isotropic radial-flow layers infinite in extent) including the system properties (e.g., permeabilities and skin factors) as unknowns. Moreover, extended flow periods can cause boundary effects, which add more complexity to the analysis (since the assumed model considers the layers to be infinite-acting). As a result, these conventional approaches often lack the robustness needed to reliably characterize individual layers in commingled systems.

The only reliable way to characterize the commingled systems is to isolate and test each layer separately. However, this procedure is costly and requires operational efforts (Pan et al., 2010). Geravand et al. (2020) applied deconvolution to drill stem test (DST) data obtained from separately tested layers in a commingled gas carbonate reservoir. Furthermore, they applied single-layer analysis techniques (which are applicable, as the individual layers are tested separately) to the same data. They showed the superiority of the deconvolution over the conventional single-layer methods. Building on these insights, this work introduces a practical testing procedure and a novel analysis methodology that addresses the limitations in the existing techniques while avoiding the need for physical layer isolation.

3. Developed methodology

This section introduces a practical approach for analyzing multilayer reservoir systems by combining a two-rate testing procedure with a robust deconvolution-based analysis technique.

Unlike conventional methods that often require physical isolation and separate testing of each layer or rely on predefined assumptions about the reservoir model, this approach offers a more practical and assumption-free alternative. The developed method leverages a limited number of production logging tool (PLT) surveys in conjunction with bottomhole pressure data to recover distinct pressure signals for each individual layer in the reservoir. These extracted signals are free from wellbore storage (WBS) effects and are extended over the entire test period, encompassing both drawdown and buildup stages. Each pressure response is unique and captures the specific characteristics of its associated layer, including permeability, heterogeneity, and boundary influences. This layer-specific resolution enhances the understanding of reservoir behavior without the need for intrusive testing or reliance on predefined reservoir models, making it valuable for commingled systems.

This work leverages the deconvolution technique to remove the effects of rate variations (rate transients) from the measured pressure data collected during the test. Deconvolution is a mathematical process that transforms variable-rate pressure data into an equivalent pressure response corresponding to a constant unit-rate drawdown test. This transformation retrieves the reservoir response by eliminating the influence of changing production rates, which often complicate the interpretation of pressure data. As a result, the deconvolved pressure signal captures the true reservoir behavior and extends over the entire test period, combining both drawdown and buildup durations into a single, continuous response. This extended signal not only improves the interpretability of the data but also enhances the ability to identify reservoir characteristics such as flow regimes, boundaries, and heterogeneity with reliable accuracy and confidence. The deconvolution calculations are accomplished by solving the convolution integral (Van Everdingen and Hurst, 1949) for the constant-rate pressure function g (t):

$$\Delta p(t) = p_o - p(t) = \int_0^T q(\tau)\, g(t-\tau) d\tau \tag{1}$$

$$g(t) = \frac{dG}{dt} \tag{2}$$

where, Δp is measured pressure drop, p_o is initial pressure, p(t) is measured pressure, q is measured flow rate, G is constant rate pressure drop function, and g is the derivative of the constant rate pressure drop function. Solving the convolution equation (Equation (1)) is an ill-posed (ill-conditioned) problem, meaning that small errors in the measured pressure and rate data can lead to large errors in the computed constant-rate pressure response. Efforts to develop stable deconvolution methods began as early as the 1950s (Hutchinson and Sikora, 1959). However, the stable deconvolution algorithm was not introduced until the 2000s, when von Schroeter et al. (2004) presented the first stable method. It is worth noting that since the introduction of the first stable deconvolution algorithm to the petroleum literature, deconvolution has made significant progress toward becoming a widely adopted tool in well test data analysis. Beyond its core capability of transforming multi-rate test data into an equivalent constant-rate pressure response, the technique has been extended to layered reservoirs pressure transient data (Geravand et al., 2020), multi-well interference testing (Cumming et al., 2014; Levitan, 2007; Onur et al., 2011; Tung et al., 2016), and thermal response tests (Beier, 2020).

For successful implementation of the deconvolution process, a complete sandface rate profile for each individual layer is required. However, in practice, continuous measurements of layer-specific sandface rates are rarely available due to technical and operational limitations. To address this, the current work introduces a simple and practical model to estimate the rate behavior of individual layers using a limited number of production logging tool (PLT) surveys. It has been observed that the relationships described by Equations (3) and (4) for layer rate transients can be effectively fitted using a power-law equation of the form $y = a\, x^b$, which involves only two parameters and requires just two data points to define the curve. This model provides a straightforward means to reconstruct continuous sandface rate profiles from sparse PLT data, as demonstrated in the application examples presented later in this paper.

$$\text{For the flowing period}: \frac{q_{layer}\ (t)}{q_{total}\ (t)} \times t \text{ Vs. } t \tag{3}$$

$$\text{For the shut}-\text{in period}: \frac{q_{layer}\ (\Delta t) - q_{layer}(\Delta t = 0)}{q_{total}(\Delta t) - q_{total}(\Delta t = 0)} \times \Delta t \text{ Vs. } \Delta t \tag{4}$$

where q_{layer} is layer sandface rate, q_{total} is total sandface rate, t is time during flowing period, and Δt is shut-in time.

The deconvolution calculations employed in this work follow the algorithm presented by von Schroeter et al. (2004) (Appendix A). The deconvolution technique performs the calculations in the real time domain by formulating the deconvolution as a separable, nonlinear total least squares problem, which is effectively minimized using the variable projection algorithm (O'Leary and Rust, 2013a, 2013b). The original objective function is constructed as a minimization problem of error sum of three terms: a pressure matching term, a flow rate matching term, and a smoothing or curvature term to stabilize the solution and suppress the noise amplification. To enhance the flexibility of the algorithm, a modified objective function that allows assignment of weights to the individual pressure and rate data points is developed, enabling more control over the influence of each measurement during the optimization process, and can be described as follows:

$$E = \sum_{i=1}^{m} \left(w_{pi}\, [p_{mi} - p_{Mi}] \right)^2 + \sum_{j=1}^{n} \left(w_{qj} \left[q_{mj} - q_{Mj} \right] \right)^2 + \lambda \| DZ - K \|_2^2 \tag{5}$$

where w_{pi} and w_{qj} are weights of pressure and rate points respectively; p_{mi} and p_{Mi} are pressures of measurements and model respectively; q_{mj} and q_{Mj} are rates of measurements and model respectively; $\lambda \| DZ - K \|_2^2$ is smoothing term. The weights to the pressure and rate data in the optimization process are given as:

$$w_{pi} = \frac{1}{G_P * p_{mi}} \tag{6}$$

$$w_{qi} = \frac{1}{G_q \times R_j \times q_{mj}} \tag{7}$$

where G_q and G_P are constant values introduced to provide flexibility in weighting the flow rate and pressure terms during the optimization process. The values R_j are positive and less than one and are designed to vary with time to reflect confidence in the estimated rate data. Specifically, R_j is set to a minimum value at time points corresponding to the PLT measurements, where the data is considered most reliable, thereby assigning higher weight to these points in the objective function. Between successive PLT measurements, R_j increases linearly and reaches a maximum value of one at the midpoint, reducing the influence of the less certain estimated rates obtained using Equations (3) and (4). This weighting strategy improves the stability and accuracy of the deconvolution results, by emphasizing the measured data and moderating the effect of interpolated estimates.

3.1. Testing procedure

This section outlines the recommended testing procedure for acquiring pressure and rate data from commingled reservoir systems to enable successful implementation of the proposed analysis methodology. A two-rate test sequence, consisting of a production period followed by a shut-in period, is considered ideal for this purpose, as it provides both flow and pressure transients needed for reliable deconvolution. The objective of the testing procedure is to collect a complete bottomhole pressure signal along with at least three production logging tool (PLT) measurements of sandface rate during each transient period (drawdown and buildup) for each contributing layer. An exception can be made for the top layer, whose sandface rate can be inferred by subtracting the sum of the measured rates from the underlying layers from the total sandface rate, which itself can be estimated using Equation (8). This approach reduces the required number of PLT passes without compromising the ability to estimate individual layer contributions.

As transient flow rates exhibit the greatest variation during early times, it is advisable to conduct PLT surveys early in each transient period to improve the accuracy of the rate estimation via Equations (3) and (4). Early PLT data points are particularly valuable for fitting the power-law model, which relies on a limited number of high-quality measurements to reconstruct continuous rate profiles. In two-layer systems, the PLT tool can remain stationary just above the bottom layer to continuously monitor its pressure and rate. For three-layer systems, a dual-spinner PLT (see Fig. 2) can be used, with the lower spinner positioned above the bottom layer and the upper spinner placed above the middle layer. This configuration allows simultaneous rate detection for two layers while maintaining a stationary tool position, reducing tool movement and associated operational complexities.

In systems with more than three layers, a single-spinner or dual-spinner PLT tool may be positioned at a selected depth, such as just above the lowest contributing layer, and then repositioned as needed to conduct rate surveys for the remaining layers. While this requires some tool movement, the approach minimizes the total number of passes while still collecting the necessary data to estimate rate profiles for all layers. To ensure consistent and accurate pressure referencing throughout the test, it is essential to install a pressure gauge below the perforation of the bottommost layer. Additionally, a wireline formation tester (WFT) survey is recommended prior to the test to determine the initial reservoir pressures for all layers. These initial pressure values are required for the formulation of the deconvolution problem.

3.2. Analysis approach

The following steps show how the proposed approach recovers a distinct WBS-free pressure signal for each layer in a commingled reservoir system from the measured data:

1. Estimation of Total Sandface Rate: Begin by estimating the total sandface rate using the van Everdingen and Hurst model (Van Everdingen and Hurst, 1949) (Equation (8)). The total sandface rate q_{sf} in Equation (8) corresponds q_{total} to used in Equations (3) and (4). To apply Equation (8), the wellbore storage coefficient C_s should first be determined. This is

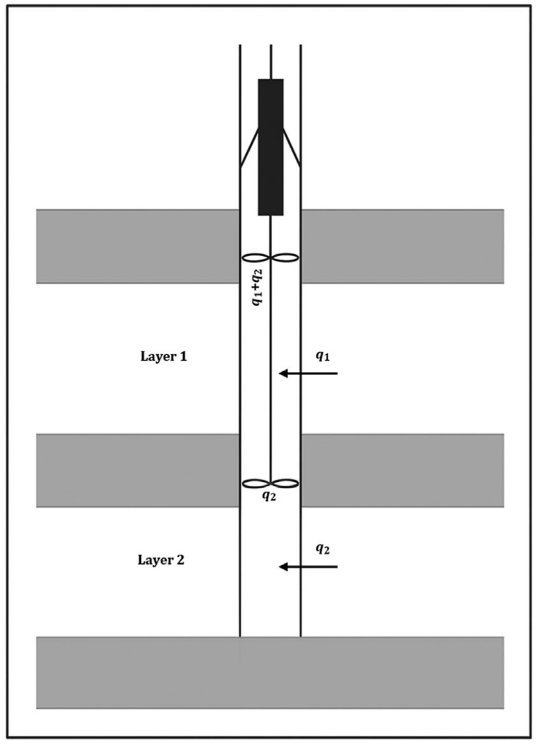

Fig. 2. Dual spinner PLT device (Stewart, 2011).

typically achieved by analyzing the measured bottomhole pressure signal using the standard log-log diagnostic plot, which allows for estimation of C_s based on the early-time pressure behavior.

$$q_{sf} = q_{surface} + \frac{24\,C_s}{B_o}\frac{dp_{wf}}{dt} \tag{8}$$

2. Estimation of Layer Sandface Rates: Use the available PLT surveys to estimate the continuous sandface rate profiles for each individual layer by applying the proposed model described in Equations (3) and (4). These equations utilize a power-law relationship fitted to the limited PLT data to reconstruct the full rate behavior over the test duration.
3. Stepwise Rate Approximation: Convert the reconstructed continuous rate profiles into a stepwise function, as described by Equation (9). In this step, the continuous rate q_{cont} in Equation (9) corresponds to q_{layer} obtained from Equations (3) and (4). This approximation ensures compatibility with the deconvolution algorithm, which operates under the assumption of piecewise constant flow rates.

$$q_{step\ i} = 0.5\left(q_{cont\ i} + q_{cont\ i+1}\right) \tag{9}$$

where, $q_{step\ i}$ are the resulting stepwise rates and q_{cont} are chosen points on the continuous rate function.

4. Deconvolution of Pressure and Rate Data: Apply the deconvolution algorithm to the measured bottomhole pressure data and the estimated stepwise rate profiles for each layer (as obtained in Step 3). This step recovers the distinct pressure response for each individual layer. The resulting pressure signals are free from wellbore storage effects and are extended over the entire test duration, providing a clear representation of the reservoir intrinsic behavior.

The complete procedure is summarized in the flowchart shown in Fig. 3. In Step 3, the continuous layer rate profiles are approximated using 150 rate steps uniformly distributed along the rate axis, which has been found sufficient to accurately represent the transient behavior in stepwise form. In Step 4, deconvolution pressures are estimated at 30 time nodes that are uniformly spaced on a logarithmic scale, consistent with von Schroeter implementation. The deconvolution algorithm assumes that wellbore storage (WBS) effects, characterized by a unit-slope straight line on the log-log diagnostic plot, fully dominate the early-time behavior before the first selected time node. To ensure this assumption does not significantly affect the results, the first time node should be selected small enough (typically 0.0001 h or less). The smoothing or curvature term weight λ, which controls the trade-off between data fidelity and smoothness in the deconvolved pressure signal, is estimated initially using Equation (10), as suggested by von Schroeter et al. (von Schroeter et al., 2004). This initial estimate may then be adjusted subjectively to achieve a smooth and physically meaningful pressure derivative response.

$$\lambda = \frac{\|\Delta p\|_2^2}{m} \tag{10}$$

The novelty of this work lies in its ability to address several limitations associated with the conventional multilayer analysis methods. It is worth mentioning that it is likely the first approach of its kind to incorporate stable deconvolution calculations into multilayer pressure transient analysis. Additionally, it enables the extraction of distinct pressure signals for each layer, as though each were independently produced at a constant rate, thereby recovering its pressure response from the combined system behavior. This capability provides more accurate insight into individual layer properties and flow dynamics.

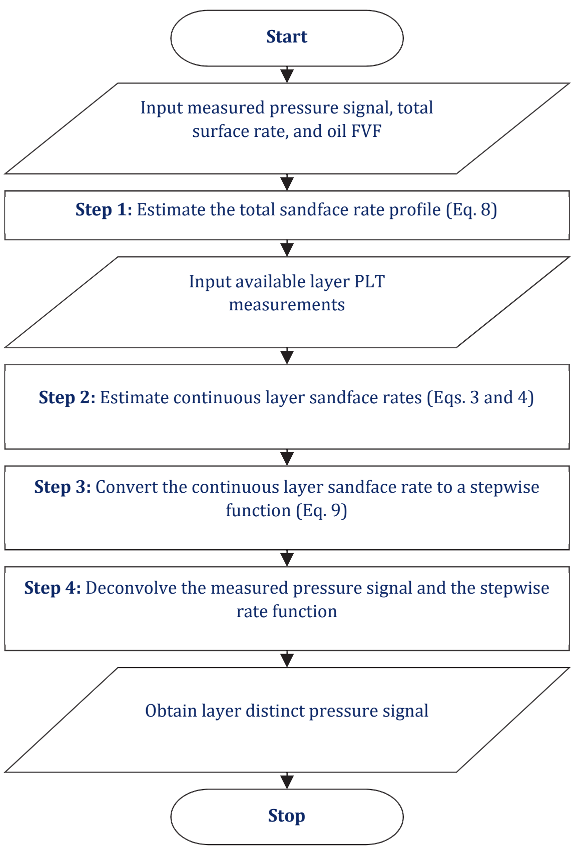

Fig. 3. Flowchart of processing multilayer systems test data.

4. Applications

This section presents three simulated single-phase oil flow cases featuring different layer configurations to validate the applicability of the developed method. The test data are generated using the multilayer convolution algorithm for commingled systems proposed by Stewart (2011). Each case involves a two-rate test sequence, comprising a production period followed by a shut-in period, with the details summarized in Table 1. These cases are designed to include a variety of reservoir and boundary conditions, such as dual-porosity formations, no-flow channels, heterogeneous and homogeneous layers, and systems exhibiting wellbore crossflow during shut-in. For analysis, three production logging tool (PLT) measurements per transient are used to estimate the continuous sandface rate profiles for each layer. These limited measurements are sufficient for accurate rate reconstruction using the power-law model introduced in this study. The PLT surveys are indicated by diamond-shaped markers in the rate profile plots, helping to visualize the points used for curve fitting. The reconstructed rate profiles are then used in the deconvolution calculations to generate distinct pressure signals for each individual layer. The weighting parameters and in Equations (6) and (7) (G_P and G_q) are assigned values of 0.001 and 1, respectively, for all three cases to control the influence of data during optimization. These examples demonstrate the robustness and accuracy of the proposed methodology under various reservoir complexities.

4.1. Two-layer case

This example simulates a commingled two-layer reservoir system subjected to a 25-h test. The test consists of a 20-h production period at a constant surface flow rate of 300 STB/D, followed by a 5-h shut-in period. The system includes two distinct layers: one is homogeneous with a no-flow boundary, while the other is heterogeneous and extends infinitely. The detailed reservoir properties of each layer are provided in Table 2. Figs. 4 and 5 present the sandface transient rate and bottomhole pressure data, respectively. As shown in Fig. 4, the system exhibits wellbore crossflow during the shut-in phase, where one layer continues to produce fluids while the other receives them, indicating interlayer communication through the wellbore despite hydraulic isolation within the formation. This behavior highlights the need for accurate layer-specific analysis to properly characterize reservoir contributions and optimize production strategies.

Table 1
General data of the three cases.

	Unit	Case 1	Case 2	Case 3
Number of layers		2	3	4
Production duration	Hrs.	20	15	30
Shut-in duration	Hrs.	5	9	10
Total test duration	Hrs.	25	24	40
Surface production rate	STB/D	300	400	700
Wellbore storage coefficient, C_s	bbl/psi	0.005	0.0005	0.005
Oil formation volume factor, B_o	bbl/STB	1.5	1.1	1.5
Oil viscosity, μ	cp	2	10	2
Wellbore radius, r_w	ft.	0.3	0.3	0.3

Table 2
System description of the two-layer case.

Layer	h, ft.	k, md	Skin	∅, %	C_t, psi^{-1}	p_i, psi	Reservoir	Boundary
1	100	30	1	10	5×10^{-6}	6000	Homogeneous	No flow channel $L = 200$ ft.
2	70	50	5	12	3×10^{-6}	6000	Dual porosity $\omega = 0.11$; $\lambda = 6E-7$	infinite

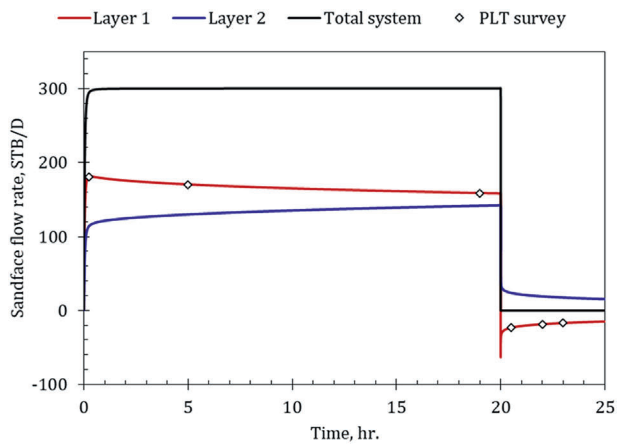

Fig. 4. Rate profile of the two-layer case.

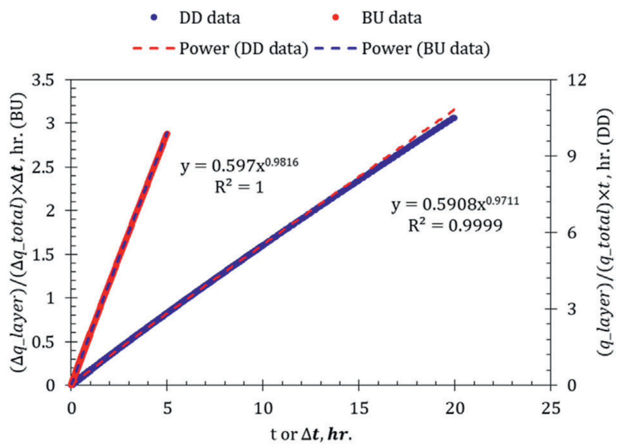

Fig. 6. Fitting the rate data for Layer 1 DD and BU by power equations – the two-layer case.

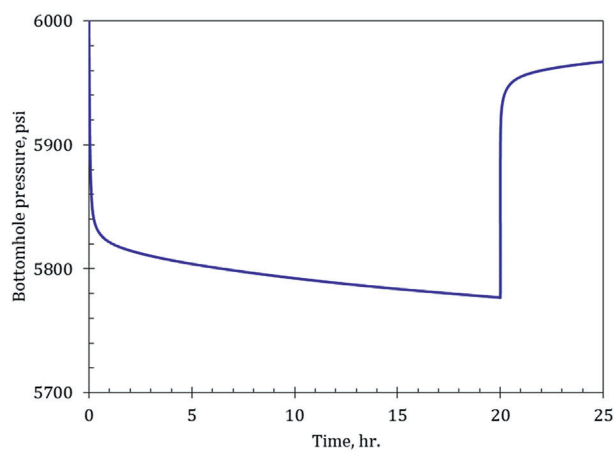

Fig. 5. Pressure data of the two-layer case.

Before demonstrating the application of the proposed analysis approach, the rate transient prediction model (Equations (3) and (4)) is first validated to ensure its reliability in estimating continuous sandface rate profiles from limited production logging tool (PLT) measurements. Fig. 6 presents this validation, where the drawdown (DD) rate data are shown as blue points and fitted using a red dashed line, while the buildup (BU) rate data are represented by red points and fitted with a blue dashed line. These fitting curves are derived from the power-law model introduced in this study, which only requires two or three data points to construct a continuous rate trend. The strong agreement between the fitted curves and the rate data confirms that the power-law model effectively captures the transient rate behavior during both production and shut-in periods. By demonstrating that the model can reconstruct the rate history using sparse PLT data, this step shows the practical feasibility of the methodology in real-world testing scenarios where continuous rate measurements are typically unavailable.

For pressure and rate measurements, the PLT tool is held stationary above the bottom layer throughout the test duration to continuously record pressure and sandface rate data. However, for the purpose of this analysis, only six discrete PLT readings for Layer 1 (three during the drawdown period and three during the buildup period) are used to construct the layer continuous rate profile. These readings are listed in Table 3 and visually represented as diamond-shaped markers on the red curve in Fig. 4. Despite the limited number of data points, they are sufficient for applying the power-law fitting model to estimate the full rate transient. The subsequent steps of the analysis approach, including rate estimation, conversion to stepwise format, and deconvolution, are illustrated in Figs. 7–11 for Layer 1. For Layer 2, the rate profile is derived by subtracting Layer 1 estimated rates from the total sandface rate, and the corresponding analysis results are presented in Figs. 12 and 13.

The first step in the analysis approach is to calculate the total sandface rate profile, represented by the black curve in Fig. 4. This is achieved by plotting the bottomhole pressure data on a log-log scale to evaluate the wellbore storage (WBS) coefficient. Once determined, this coefficient is input into the van Everdingen and Hurst model (Equation (8)) to compute the total sandface rate. In the second step, the PLT-measured flow rates are used to reconstruct the continuous sandface rate profiles for each layer through the power-law model described by Equations (3) and (4). For Layer

Table 3
PLT measurements– the two-layer case.

Time, hr.	Flow rate - Layer 1, BPD
0.25	181
5	170
19	159
20.5	−23
22	−19
23	−17

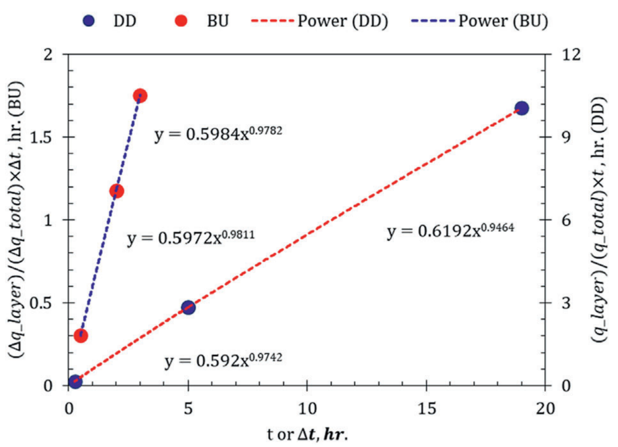

Fig. 7. Model development for estimating Layer 1 rate profile during DD and BU periods – the two-layer case.

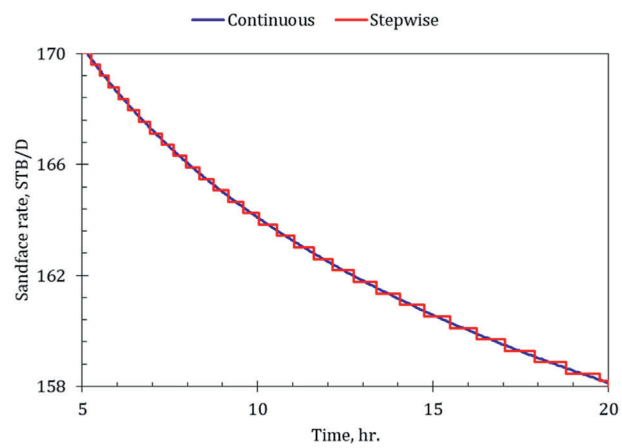

Fig. 10. Converting Layer 1 continuous rate profile to a stepwise function (zoomed on the interval 5:20 h) – the two-layer case.

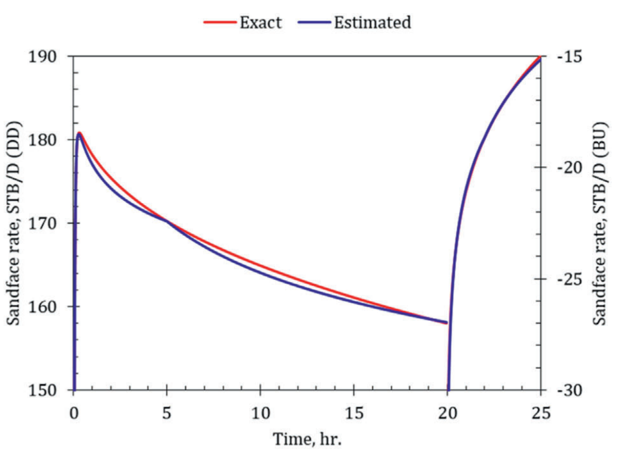

Fig. 8. Model-estimated sandface rate of Layer 1 for the two-layer case. The left y-axis corresponds to drawdown (DD) rates, while the right y-axis corresponds to buildup (BU) rates.

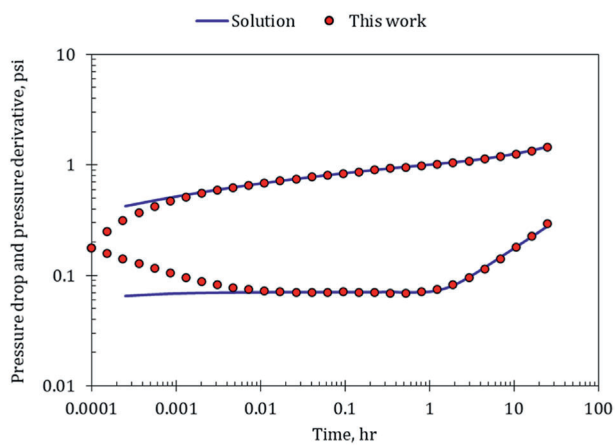

Fig. 11. Deconvolution results of Layer 1 – the two-layer case.

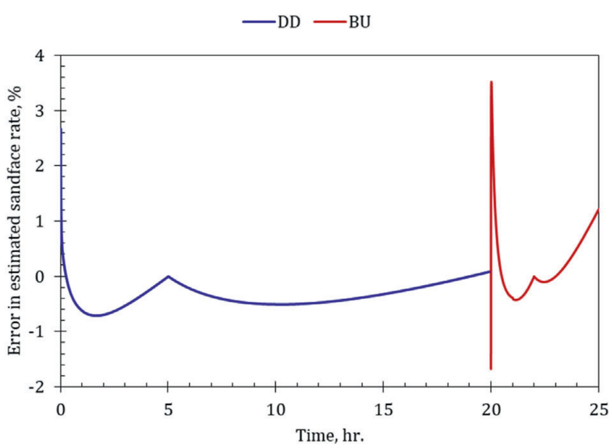

Fig. 9. Error function of model-estimated Layer 1 rate profile– the two-layer case.

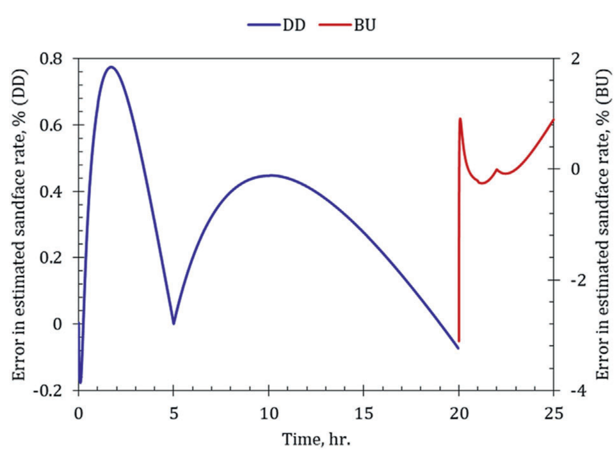

Fig. 12. Error function of Layer 2 rate profile– the two-layer case.

1, this model is illustrated in Fig. 7, where the PLT survey points are shown as blue markers for the drawdown (DD) period and red markers for the buildup (BU) period. The fitted power-law curves appear as red and blue dashed lines for the DD and BU periods, respectively. These fitted models are then used to calculate the full continuous rate profile of Layer 1, shown as the blue curve in Fig. 8. For reference, the exact (true) rate profile is plotted as a red curve in the same figure, matching the red curve shown earlier in Fig. 4. The accuracy of the estimated rate profile is evaluated by calculating the error function, presented in Fig. 9, which demonstrates

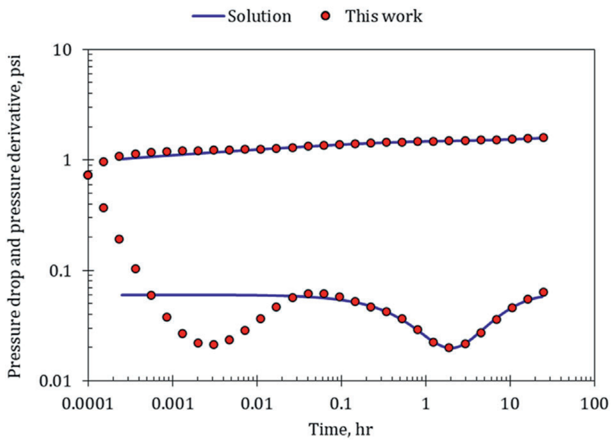

Fig. 13. Deconvolution results of Layer 2 – the two-layer case.

that the power-law model provides a close approximation with minimal deviation from the exact values.

In Step 3, the estimated continuous sandface rate profile of Layer 1 is approximated using a stepwise function to satisfy the assumptions of the deconvolution algorithm. This approximation is shown in Fig. 10, where the blue line represents the original continuous rate profile and the red curve denotes its stepwise counterpart. This rate transformation enables compatibility with von Schroeter algorithm, which assumes stepwise rate input for stable deconvolution. In the final step of the analysis (Step 4), the stepwise rate function and the bottomhole pressure data are input into von Schroeter deconvolution algorithm to extract the distinct pressure response of Layer 1. The results are shown in Fig. 11, where the circles represent the deconvolved pressure signal, and the solid line corresponds to the exact solution. The close alignment shows the method effectiveness in recovering layer-specific pressure behavior.

For the second layer, the sandface rate profile is estimated by subtracting the previously reconstructed Layer 1 rates from the total sandface rate, which was calculated in Step 1 using Equation (8). The accuracy of this estimated rate profile is evaluated in Fig. 12, showing the error level across the test duration. Following the same procedure, the continuous rate profile of Layer 2 is then approximated using a stepwise function and paired with the pressure data for deconvolution. The resulting distinct pressure response for Layer 2 is presented in Fig. 13.

The results indicate that the model-estimated rate profiles for both layers (Figs. 9 and 12) maintain a reasonably low error margin, within approximately ±3.5%, suggesting that the reconstructed rates are suitable for use in deconvolution. The deconvolved pressure responses shown in Figs. 11 and 13 generally align well with the reference solutions. A slight early-time deviation in the pressure derivative curve of Layer 2 (prior to 0.02 h) is observed in Fig. 13, but overall, the recovered pressure signals extend across the entire 25-h test period, capturing the full dynamic behavior of the individual layers.

4.2. Three-layer case

A 24-h test dataset is generated for a three-layer commingled reservoir system consisting of a dual-porosity formation, a layer bounded by a channel no-flow boundary, and a homogeneous layer. The detailed description of the system properties is provided in Table 4, while the corresponding sandface rate and bottomhole pressure data are shown in Figs. 14 and 15, respectively. Production logging tool (PLT) data for the first two layers are listed in Table 5 and are used to reconstruct their sandface rate profiles. The complete pressure and rate datasets are then analyzed using the developed methodology. The deconvolution results for each layer are illustrated in Figs. 16–18. In these figures, the solid red lines represent the exact pressure solutions, while the circles show the deconvolved pressure responses obtained from the analysis. The results show good overall agreement between the deconvolved signals and the reference solutions. A minor exception is observed in Fig. 16, where the deconvolved signal for the dual-porosity layer displays some irregular behavior at early times, which may be attributed to the complex flow dynamics typical of such formations. However, the method demonstrates reasonable accuracy across the layers and provides distinct pressure responses for individual characterization.

4.3. Four-layer case

This case simulates a 40-h test for a commingled reservoir system composed of four layers with varying boundary conditions and rock properties. The system includes a layer of one side with a constant pressure boundary, while the remaining three sides are infinite-acting. Another layer is bounded by two parallel faults (no-flow boundaries), and the other two reservoirs include both homogeneous and heterogeneous infinite-acting formations. The general system description is provided in Table 6, and the simulated sandface rate and bottomhole pressure data are shown in Figs. 19 and 20. Production logging tool (PLT) data for the first three layers are listed in Table 7 and used in the analysis.

Following the steps of the proposed methodology, rate profiles are reconstructed and deconvolved to obtain distinct pressure responses for each layer. The resulting deconvolution outputs are presented in Figs. 21–24. In the figures, the solid line represents the exact pressure solution, while the circles denote the deconvolved signals obtained from the analysis. The results show good consistency with the reference solutions for all layers, though some early-time erratic behavior is noted in the pressure derivative curves.

For the fourth layer, shown in Fig. 24, the initial deconvolution result using the weight parameter $\boldsymbol{G_q} = 1$ (green dashed line) deviates from the true solution for an extended period, up to approximately 0.3 h. However, this deviation is reduced significantly by increasing the rate weight parameter to 10, as illustrated by the improved match in the circles. This adjustment adds an extra log cycle to the interpretable portion of the pressure derivative curve, enhancing the reliability of the result and demonstrating the method flexibility in tuning parameter weights for better early-time behavior.

In this study, an initial value of the rate weight ($G_q = 1$) was used across all cases and yielded satisfactory results, except for the four-layer case (Layer 4), where a higher value improved early-time derivative stability. However, in field applications, exact pressure responses for individual layers are not available for direct comparison. Therefore, the optimal rate weight parameter is typically determined through a sensitivity analysis in which a range of values is tested, and the resulting deconvolved pressure and derivative responses are evaluated for physical plausibility, smoothness, and stability. Diagnostic checks involve confirming the absence of nonphysical oscillations in the derivative, ensuring consistency of late-time behavior with expected boundary conditions, and seeking agreement with independent reservoir information such as permeability-thickness estimates from other sources. This procedure illustrates how, in practice, one may begin with an initial value and adjust as needed to obtain stable and interpretable responses over the widest possible time range.

Table 4
System description of the three-layer case.

Layer	h, ft.	k, md	Skin	∅, %	C_t, psi^{-1}	p_i, psi	Reservoir	Boundary
1	80	20	2	9	1×10^{-6}	5000	Dual porosity $\omega = 0.1$; $\lambda = 2E-6$	Infinite
2	20	100	0	12	5×10^{-6}	5000	Homogeneous	No flow channel $L = 170$ ft.
3	50	10	7	10	3×10^{-6}	5000	Homogeneous	Infinite

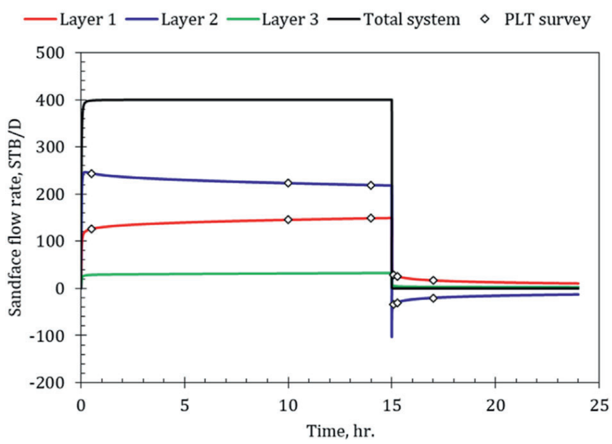

Fig. 14. Rate profile of the three-layer case.

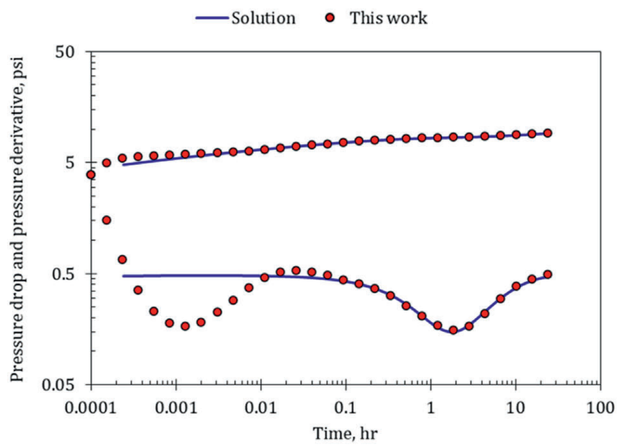

Fig. 16. Deconvolution results of Layer 1 – the three-layer case.

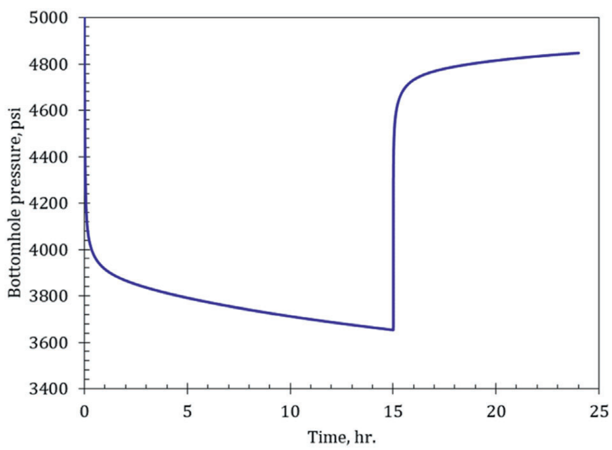

Fig. 15. Pressure data of the three-layer case.

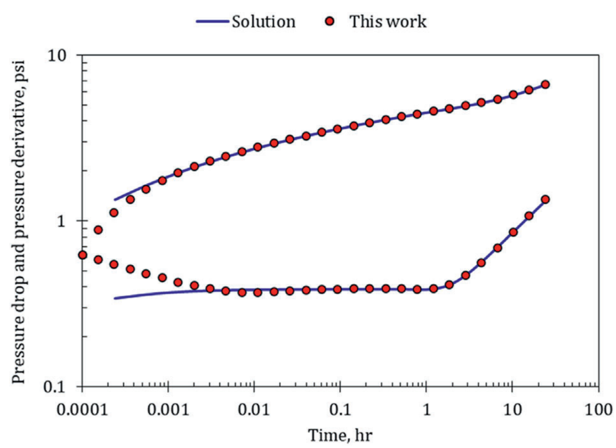

Fig. 17. Deconvolution results of Layer 2 – the three-layer case.

Table 5
PLT measurements – the three-layer case.

Time, hr.	Flow rate – Layer 1, BPD	Flow rate – Layer 2, BPD
0.5	126	244
10	145	223
14	149	219
15.1	29	−35
15.25	26	−30
17	17	−20

Although the reconstructed pressure responses generally follow the exact pressure solutions, some discrepancies were observed (e.g., Figs. 21–24). These deviations are primarily attributed to uncertainties in reconstructing layer rate profiles from sparse PLT data. Early-time mismatches may also arise from wellbore storage effects and from assumptions inherent in the von Schroeter algorithm, which enforces a unit-slope straight line before the first selected time point. In practice, such errors can be mitigated by improving the accuracy of layer rate estimation through more frequent PLT surveys, especially at early times, by tuning/optimizing the smoothing parameter (λ) in the deconvolution, and by applying weighting schemes that prioritize the most reliable rate measurements.

5. Discussion

This advancement in the pressure transient analysis of commingled systems can enhance reservoir description, support production optimization, reduce testing costs, and improve decisions in field development planning. The methodology aligns closely with current industry needs for efficient, low-intervention

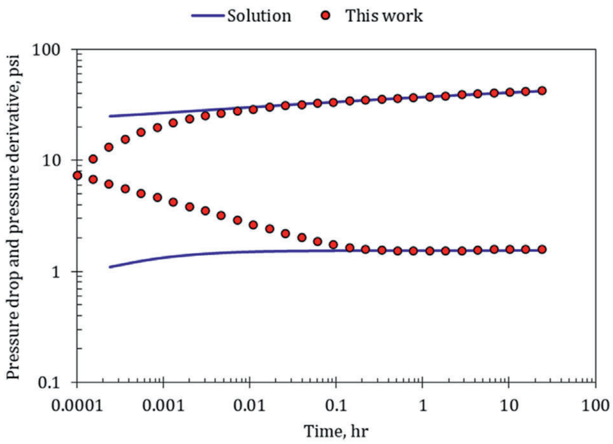

Fig. 18. Deconvolution results of Layer 3 – the three-layer case.

reservoir characterization techniques. In commingled multilayer systems, traditional layer-by-layer testing is often impractical due to operational constraints, costs, and deferred production. The proposed approach offers an alternative by extracting individual layer responses from standard well test pressure data combined with minimal PLT input, eliminating the need for extended shut-ins or mechanical isolation. This makes the method highly applicable where operational simplicity and cost-effectiveness are essential. Furthermore, downhole sensing technologies such as permanent downhole gauges (PDGs) (SLB, 2025), fiber-optic flowmeters (Kragas et al., 2003), and distributed acoustic sensing (DAS) systems (Sharma et al., 2020) now enable continuous, real-time monitoring of pressure, temperature, and multiphase flow rates at the sandface. These technologies reduce reliance on conventional PLT surveys and make high-frequency, layer-specific rate data increasingly accessible in field operations. By leveraging these capabilities, the presented approach becomes more deployable in real-world settings, aligning with modern industry trends toward non-intrusive reservoir surveillance and efficient production optimization.

The proposed deconvolution-based approach demonstrated strong performance across a range of multilayer reservoir configurations. By integrating rate estimation from sparse PLT surveys and applying stable deconvolution, the method was able to generate distinct, wellbore-storage-free pressure signals for each layer, even in the presence of heterogeneity and boundary effects. The results show consistent alignment with true reservoir responses in simulated two-, three-, and four-layer cases. Although minor early-time deviations were observed in some pressure derivative curves, these were generally minimal and could be mitigated through parameter adjustments. Importantly, the method avoids the need for physical layer isolation and subjective model assumptions, offering a practical and scalable solution for field implementation. It also provides extended pressure signals over the entire test duration (drawdown and buildup periods), enhancing the interpretability of flow regimes and boundary effects. These capabilities make the approach a valuable tool for improving production and injection management in commingled systems, with potential applicability to reservoirs with limited interlayer crossflow, as their behavior is identical to commingled systems at the early times (Ehlig-Economides and Joseph, 1987).

The continuous sandface rate is approximated to a stepwise function in Step 3 to address the assumption made in the von Schroeter deconvolution algorithm, which requires flow rate inputs to follow a stepwise format. Additionally, the algorithm assumes that the early-time pressures in the deconvolved signal are dominated by wellbore storage, represented by a unit-slope straight line on a log-log plot. To minimize the influence of this assumption, the first time-node for evaluating the deconvolved pressure derivative can be chosen sufficiently early. In the

Table 6
System description of the four-layer case.

Layer	h, ft.	k, md	Skin	∅, %	C_t, psi^{-1}	p_i, psi	Reservoir	Boundary
1	150	10	10	21	3×10^{-6}	4000	Homogenous	Constant Pressure $L = 150$ ft.
2	40	100	2	24	5×10^{-6}	4000	Homogenous	No flow channel $L = 200$ ft.
3	85	18	3	20	2×10^{-6}	4000	Homogenous	Infinite
4	80	20	0	22	1×10^{-6}	4000	Dual porosity $\omega = 0.075$; $\lambda = 9.5E-7$	Infinite

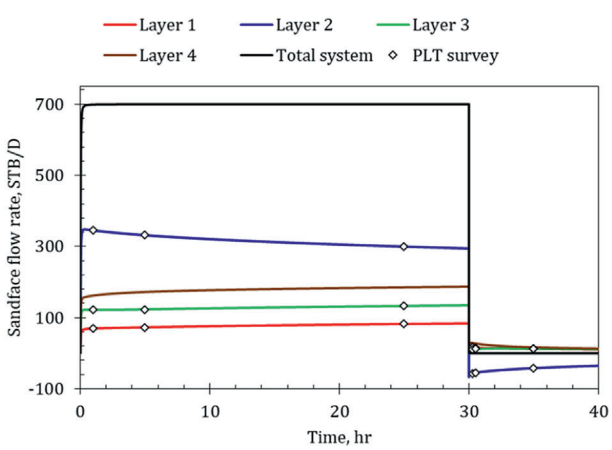

Fig. 19. Rate profile of the four-layer case.

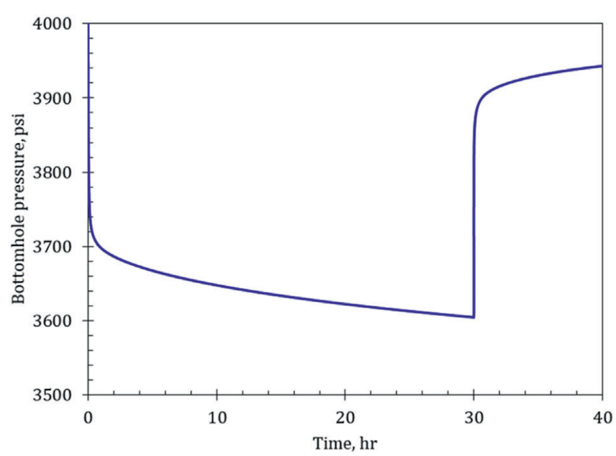

Fig. 20. Pressure data of the four-layer case.

Table 7
PLT measurements– the four-layer case.

Time, hr.	Flow rate – Layer 1, BPD	Flow rate – Layer 2, BPD	Flow rate – Layer 3, BPD
1	69	345	122
5	73	332	123
25	82	300	133
30.25	16	−57	12
30.5	15	−55	12
35	12	−42	13

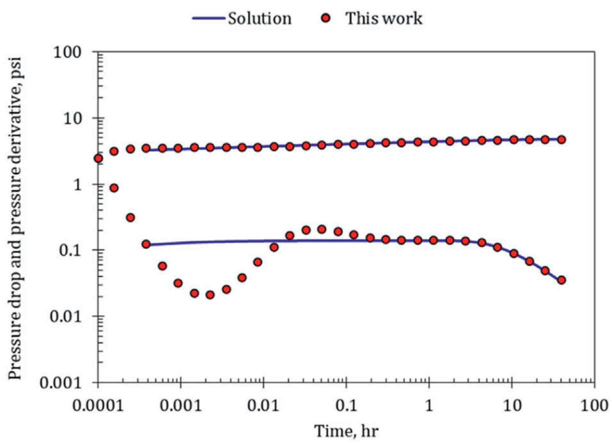

Fig. 21. Deconvolution results of Layer 1 – the four-layer case.

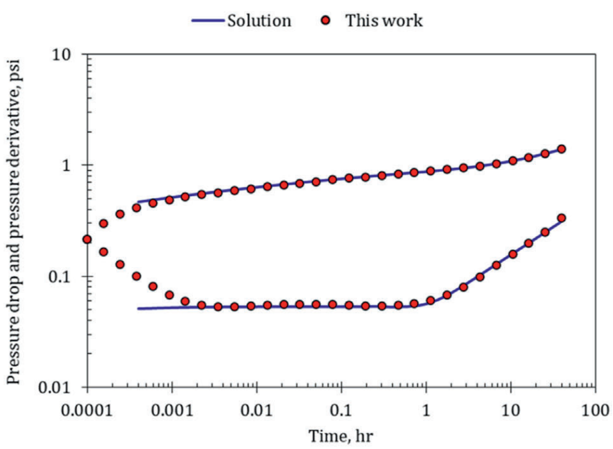

Fig. 22. Deconvolution results of Layer 2 – the four-layer case.

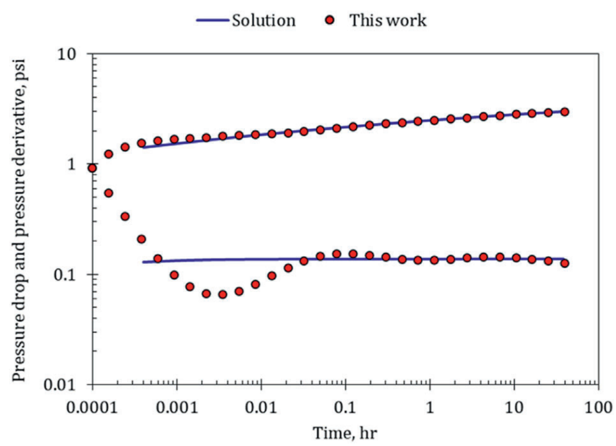

Fig. 23. Deconvolution results of Layer 3 – the four-layer case.

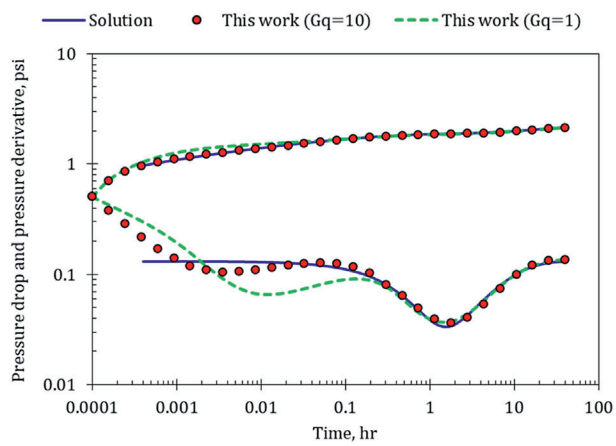

Fig. 24. Deconvolution results of Layer 4 – the four-layer case.

examples presented here, a first time-node of 0.0001 h is used to reduce potential distortion in the recovered pressure signal.

Although von Schroeter deconvolution approach is utilized in this work, other stable deconvolution algorithms can also be applied to perform the deconvolution calculations in Step 4 of the proposed approach Examples include the methods developed by Ilk et al. (2006), Levitan (2005), Nomura and Horne (2009), Onur and Kuchuk (2012), von Schroeter et al. (2004). These algorithms have been subjected to investigation by several researchers (Chen and Jones, 2012; Levitan and Wilson, 2012; Onur et al., 2008; Pimonov et al., 2010; Wu et al., 2009).

As time progresses, the magnitude of change in transient flow rates diminishes, and the rates tend to stabilize. In other words, the most significant rate variations occur during the early stages of each transient period. Therefore, it is recommended to conduct PLT surveys early in each transient to improve the accuracy of the rate estimates generated using the model proposed in Equations (3) and (4). It has been found that using three PLT surveys per transient provides sufficiently accurate estimations of the layer rate profile while capturing main variations. It is also important to note that with advancements in downhole monitoring technologies, gauges are now available that can provide continuous rate measurements throughout the entire test duration (Cahill and Dinsdale, 2025; Enyekwe and Ajienka, 2014; Sharma et al., 2020; Webster et al., 2006). The pressure/rate datasets used in this study were generated including both wellbore storage and skin effects (Tables 1, 2, 4 and 6). For validation purposes, the deconvolution results are compared against the reference reservoir responses/solutions that exclude wellbore storage but retain skin, since the deconvolution objective is to recover WBS-free pressure signals.

Although the methodology is demonstrated here with vertical wells, its principles are not restricted to that geometry. In deviated or horizontal completions, the same workflow can, in principle, be applied if layer-specific rate allocation and bottomhole pressure measurements are available. In practice, these requirements are challenging to meet, since flow partitioning and layer identification are less straightforward. With continued advances in permanent downhole sensors and distributed measurements, the extension of this approach to non-vertical completions is becoming feasible, though further validation remains necessary.

The main advantages of the methodology are its ability to deliver accurate, assumption-free, and cost-effective characterization of commingled multilayer reservoirs. It removes rate variation and wellbore storage effects from pressure transient data to generate extended, distinct pressure responses for each layer. These responses reliably capture permeability, heterogeneity, flow regimes, and boundary conditions without the need for mechanical isolation or predefined reservoir models, making it practical for efficient reservoir description and production optimization. Despite the advantages of the proposed workflow, it is subject to limitations. This study workflow is based on the convolution integral (Equation (1)), which is valid for linear systems (e.g., single-phase oil or water flow). In single-phase oil reservoirs, linearity is inherent, while for single-phase gas reservoirs the assumption can still hold after pseudo-pressure linearization (Khalaf et al., 2021). However, its applicability becomes limited in multiphase systems such as gas-condensate reservoirs, where strong nonlinearities arise due to phase behavior. The method also assumes no interlayer crossflow and requires at least a few PLT measurements in each transient period. In addition, early-time deviations may occur from wellbore storage effects. These constraints should be considered when applying this study workflow, and extension to more complex flow scenarios remains an area for future work.

6. Conclusions

This study introduces a novel analysis methodology for commingled multilayer reservoir systems, addressing limitations in conventional pressure transient analysis techniques. The proposed methodology offers several technical advantages for analyzing commingled multilayer reservoirs. By integrating sparse production logging tool (PLT) data with a simple power-law model, it reconstructs continuous layer-specific sandface rate profiles without requiring extensive or continuous measurements. These reconstructed rates are then used in a stable deconvolution framework (e.g., von Schroeter algorithm) to remove rate variation and wellbore storage effects from bottomhole pressure data, producing extended, wellbore-storage-free pressure responses unique to each layer. This allows accurate identification of permeability, heterogeneity, flow regimes, and boundary effects while avoiding reliance on subjective reservoir model assumptions or costly mechanical isolation of layers. The method is computationally robust, adaptable through weighting schemes to handle uncertainties in estimated rates, and validated across two-, three-, and four-layer simulated cases, demonstrating consistent accuracy even under heterogeneity, dual-porosity, no-flow boundaries, and wellbore crossflow. Collectively, these features make it a practical, assumption-free, and cost-effective approach for efficient reservoir characterization and production optimization in complex multilayer systems.

While primarily designed for commingled systems, the proposed framework also shows potential for application to systems with limited formation crossflow during early times. Future work will also focus on extending the proposed methodology to field applications, which will provide practical validation and demonstrate its value for multilayer reservoir characterization.

CRediT authorship contribution statement

Mina S. Khalaf: Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization.

Declaration of competing interest

The author declares that he has no known competing financial interests or personal relationships that could have influenced the work reported in this paper.

Nomenclature

B_o	Oil formation volume factor, RB/STB
C_s	Wellbore storage coefficient, bbl/psi
C_t	Total compressibility, psi^{-1}
G_q and G_P	Parameters allow flexibility while optimization
g (t)	Derivative of constant rate pressure drop, psi/hr
G (t)	Constant rate pressure drop function, psi
h	Thickness, ft
k	Permeability, md
m	Number of pressure points
n	Number of rate points
p_D	Dimensionless pressure
p_i	Initial pressure, psi
p_{mi}	Measured pressures, psi
p_{Mi}	Model estimated pressures, psi
p_{wf}	Bottomhole pressure, psi
$q_{cont\ i}$	Continuous rate points, BPD
q_{layer}	Layer sandface rates, BPD
q_{mj}	Measured rates, BPD
q_{Mj}	Model estimated rates, BPD
q_{sf}	Sandface flow rate, BPD
$q_{step\ i}$	Stepwise rate, BPD
$q_{surface}$	Surface flow rate, BPD
q_{total}	Total sandface rates, BPD
q	Flow rate, BPD
r_w	Wellbore radius, ft
R_j	Rate scaling parameter
w_{pi}	Weights to pressure points
w_{qj}	Weights to rate points
Δp	Pressure drop data, psi
Δt	Shut-in time, hr
λ	Weight to curvature term
μ	Viscosity, cp
$\varnothing$	Porosity

Abbreviations

BPD	Barrel per day
BU	Buildup
DD	Drawdown
FVF	Formation volume factor
IPR	Inflow performance relationship
MLT	Multilayer transient
PLT	Production logging tool
SIP	Selective inflow performance
WBS	Wellbore storage

Appendix A. von Schroeter Deconvolution Algorithm

This section provides details on the von Schroeter algorithm to support the implementation of the deconvolution procedure in Step 4 of the proposed analysis approach. The objective function introduced by von Schroeter et al. (von Schroeter et al., 2004) is defined as follows:

$$\text{Error function} = \|p_0 Y_m - p - C(z)y\|_2^2 + \upsilon\|y - q\|_2^2 + \lambda\|DZ - K\|_2^2 \tag{A-1}$$

In the formulation, the first term $\|p_0 Y_m - p - C(z)y\|_2^2$ corresponds to the pressure error term $\sum_{i=1}^{m}([p_{mi} - p_{Mi}])^2$ in Equation (5). The second term $\|y - q\|_2^2$ accounts for rate errors and correspond to $\sum_{j=1}^{n}\left(\left[q_{mj} - q_{Mj}\right]\right)^2$ in Equation (5). The third term $\|DZ - K\|_2^2$ is a curvature penalty term. p_0 is the initial pressure, Y_m is a vector of ones and has a length equal to the number of pressure measurements, p is the vector of measured pressures, y is the vector of optimized flow rates, and q is a vector of measured flow rates. $C(z)$ is the design matrix of pressure response (given by Eq. A-9 in the appendix), while D is a constant matrix and K is a constant vector (defined in Equations A-10 to A-12). The parameters υ and λ are weights applied to the flow rate match and curvature terms, respectively.

von Schroeter et al. (2004) applied three main constraints within the objective function to ensure stable and physically meaningful deconvolution results. The first term in Equation A-1 imposes constraints on pressure calculations, minimizing the mismatch between the measured and reconstructed pressures. The second term applies constraints on the rate calculations to ensure consistency between the measured and reconstructed flow rates. The third term introduces a curvature penalty on the resulting well test derivative, promoting smoothness in the deconvolved pressure signal and helping to suppress oscillations. In addition to these explicit constraints, the algorithm also includes an implicit constraint on the reservoir response function to ensure its non-negativity. This is accomplished using the following encoding of the reservoir response function g(t):

$$Z(\sigma) = ln(t\,g(t)), \sigma = ln\,(t) \tag{A-2}$$

where, z is the constant rate well test derivative, and σ represents the selected time nodes where the derivative is evaluated. The time nodes (σ) are selected to be uniformly spaced on a logarithmic scale, such that:

$$-\infty = \sigma_0 < \sigma_1 < \cdots\cdots\cdots < \sigma_n = ln\,(T) \tag{A-3}$$

where, T is the total test duration and n is the number of the selected time nodes. Substituting Equation A-2 into the convolution integral (Equation (1)) leads to:

$$\Delta p(t) = \int_{-\infty}^{ln\,(t)} e^{Z(\sigma)}\, q(t - e^{\sigma})d\sigma \tag{A-4}$$

The well test derivative function z(σ) is approximated using a linear interpolation scheme:

$$Z = \alpha_k + \beta_k \sigma, \text{and } \sigma_{k-1} < \sigma < \sigma_k \tag{A-5}$$

where, α_k and β_k are the intersection and slope of the line segment over the time interval $\sigma_{k-1} < \sigma < \sigma_k$. The data before the first time node ($-\infty = \sigma_0 < \sigma \le \sigma_1$) is assumed to follow a unit-slope straight line ($\beta_1 = 1$). The flow rate $y(t)$ is interpolated as a stepwise function:

$$y(t) = \sum_j y_j\, \theta_j(t) \tag{A-6}$$

$$\theta_j(t) = \begin{cases} 1t & , \in I_j \\ 0 & , t \notin I_j \end{cases} \tag{A-7}$$

Substituting Equations A-5 and A-6 in Equation A-4 and evaluating at $t = t_i$, give a system of equations that can be written in matrix form as follows:

$$\Delta p = C(Z)y \tag{A-8}$$

$$C_{ij}(Z) = \int_{-\infty}^{ln\,(T)} \theta_j(t - e^{\sigma})\, e^{Z(\sigma)}\, d\sigma \tag{A-9}$$

where, T is the full test duration. Equation A-9 gives the elements of design matrix C(Z) for the model pressure calculations.

In the curvature term, the matrix D is defined as given by Equations A-10 and A-11. The elements of the first row are given by Eq. A-10, while Eq. A-11 generates the rest of the D matrix. The D matrix has dimensions of (n-1) x n, where n is the number of the selected time nodes.

$$D_{1j} = \begin{cases} -(\sigma_2 - \sigma_1)^{-1}, & j = 1 \\ (\sigma_2 - \sigma_1)^{-1} & , j = 2 \\ 0 & , j = 3, \ldots, n \end{cases} \tag{A-10}$$

$$D_{ij} = \begin{cases} (\sigma_i - \sigma_{i-1})^{-1}, \; j = i - 1 \\ -\dfrac{\sigma_{i+1} - \sigma_{i-1}}{(\sigma_{i+1} - \sigma_i)(\sigma_i - \sigma_{i-1})}, \; j = i \\ (\sigma_{i+1} - \sigma_i)^{-1}, \; j = i + 1 \\ 0, \text{ otherwise} \end{cases} \tag{A-11}$$

The K vector in the third term of the objective function has the dimensions of (n-1) and is given by

$$K = (1, 0, 0, \ldots, 0) \tag{A-12}$$

The first element of the K vector enforces a unit-slope condition on the initial segment of the deconvolved pressure derivative, representing the wellbore storage (WBS)-dominated regime commonly observed at early times in pressure transient data. However, in the context of the proposed analysis approach, where the objective is to recover WBS-free pressure signals for individual layers, this constraint becomes limiting. To eliminate WBS influence and capture the reservoir-specific behavior, the first row of the D matrix (Eq. A-10) and the corresponding first element of the vector K (Eq. A-12) are removed.

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