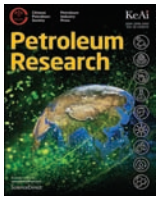
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Full Length Article

Sensitivity analysis of effective parameters in water saturation estimation by using Monte Carlo simulation



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ABSTRACT

Water saturation is one of the primary and influential parameters in evaluating the reservoir quality of hydrocarbon formations and is typically calculated by the Archie equation. This equation employs porosity, cementation factor, saturation exponent, tortuosity factor, formation water resistivity and true reservoir resistivity for water saturation calculation. It should be noted that uncertainty in these values can directly affect water saturation calculation, so the correct water saturation calculation in hydrocarbon reservoirs is essential, and any mistakes in its calculation leads to optimistic or pessimistic consideration of interval of interest. In this study, the Monte Carlo simulation method based on well-logging data were used to investigate the uncertainty of effective parameters in water saturation in the framework of Archie's equation. In this regard, the water saturation is calculated in three wells for the Sarvak reservoir in one of southwestern Iran's giant oilfields. The Monte Carlo simulation was implemented using MATLAB software for each parameter causing uncertainty in the water saturation model, and the distribution function is obtained for each parameter. Water saturation is as a distribution for each depth by investigating the uncertainty of each parameter affecting the Archie equation and is calculated in optimistic, probable and pessimistic cases. The results show that using well-logging data, the mentioned method can calculate the uncertainty of water saturation.

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1. Introduction

In the early life of a petroleum reservoir, volumetric equations are used to calculate the Oil In Place volume (*OOIP*). Typically, porosity, water saturation and net pay are the basic parameters required in volumetric equations. These parameters are not obtained directly but are inferred and extracted from well data and well-logging measurements (Kuchuk et al., 2009; Pasternack, 2009). Therefore, all these parameters have non-negligible uncertainty. In general, uncertainty in petrophysical results such as water saturation (*Sw*), permeability (*K*), porosity (Ø), and contact surfaces (fluid contacts) is not considered for reasons; like underestimation of uncertainty in measurements, such as input parameters, statistical uncertainty in nuclear logs, the effect of heterogeneity, systematic errors that require basic corrections, tool calibration problems, thin bed effects, invasion effects and vertical resolution effects. Also, uncertainty may result from lithological and structural hypotheses, interpretation models selected for porosity and water saturation, shaly sand equations and cutoff. Thus, to determine the uncertainty of petrophysical evaluation, one must first go back to the basic principles and identify these uncertainties and regardless of these uncertainties, calculating hydrocarbon in the reservoir may be accompanied by irreparable errors (Cronquist, 2001; Pasternack, 2009), especially for petroleum reservoirs in the Middle East, with giant reservoirs.

By definition, the percentage of the reservoir porosity, that is occupied with water is called water saturation. This percentage indicates that several percent of pores have water and several percent have hydrocarbon. Therefore, one of the essential parameters for calculating storage volume in hydrocarbon reservoirs is water saturation (Amabeoku et al., 2008; Gupta and Kamal,

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2023; Kadkhodaie, 2018). Typically, water saturation is calculated using the information obtained from well-logging in hydrocarbon reservoirs using the presented equations. The most common method for calculating water saturation was by Archie in 1942, which was derived from the combination of two experimental formulas based on measurements of core data. In clean reservoirs that do not have shale, the amount of shale is negligible; it is used to calculate water saturation (Eq. (1)) (Archie, 1942):

$$S_w = \sqrt[n]{\frac{F.R_w}{R_t}} \tag{1}$$

According to the research conducted by Archie, the amount of water present in a porous environment is directly dependent on the formation resistivity factor (F), formation water resistivity (Rw), the saturation exponent (n), and it has an inverse relationship with the true formation resistivity (Rt). The value of the formation resistivity factor is calculated from Eq. (2) (Archie, 1942):

$$F = \frac{R_o}{R_w} \tag{2}$$

Ro is this equation's rock resistivity value of 100% water saturation, and Rw is the formation's water resistivity.

In the first step, each core plug must be saturated with different salinities at different stages to calculate the formation resistivity factor; In the second step, the resistivity value of the rock (Ro) should be calculated at each stage. Then, the Ro vs. Rw diagrams were drawn. The formation resistivity factor (F) is obtained from the slope of the line drawn to the points of this diagram. This test requires a device that simultaneously measures the resistivity of several core plug samples. In the absence of core samples to calculate the formation resistivity factor, we can use experimental formula (Eq. (3)) (Archie, 1942; Kadkhodaie, 2018; Xu et al., 2015):

$$F = \frac{a}{\varphi^m} \tag{3}$$

By investigating the true resistivity of hydrocarbon reservoirs (Rt), the formation water resistivity (Rw) and porosity ($Ø$), the following fundamental equation can be presented for water saturation calculation. Therefore, the final form of Archie's equation in clean sandstone formations without shale is presented as follows (Eq. (4)) (Alimoradi et al., 2011; Archie, 1942; Gomaa et al., 2022; Kadkhodaie, 2018; Moradzadeh et al., 2013):

$$S_w = \sqrt[n]{\frac{a.R_w}{\varphi^m.R_t}} = \left(\frac{a.R_w}{\varphi^m.Rt}\right)^{1/n} \tag{4}$$

Other water saturation equations are based on the Archie equation and the physical properties of rocks. According to the obtained results, researchers have reported different values for the parameters of these equations. The values m, n, and a are known as Archie coefficients. These coefficients are unknown constants obtained by petrophysical studies and analysis of laboratory cores in different formations (Kadkhodaie, 2018; Tiab and Donaldson, 2004). Nevertheless, when not enough data is available, their value is considered to be 2, 2, and 1, respectively (Kadkhodaie, 2018).

As mentioned before, the main elements of the Archie equation (Archie, 1942) to calculate water saturation are various parameters such as formation water resistivity (Rw), true formation resistivity (Rt), cementation factor (m), tortuosity factor (a), saturation exponent (n) and porosity ($Ø$) (Ikwuakor, 2019). These parameters involved in Archie's equation (Eq. (4)) have uncertainty leading to uncertainty in Sw calculation, and in a petroleum reservoir, the calculated water saturation accuracy depends on these parameters' accuracy (Kuchuk et al., 2009).

Typically, the porosity parameter is obtained from well-log data and well-logging measurements. Porosity uncertainty is likely to be low in the vicinity of the well, but it can increase significantly throughout the formation (Adams, 2005; Kuchuk et al., 2009; Verga et al., 2002). The primary source of uncertainty in determining porosity can have various reasons, including errors in well-logging measurement, model selection, and the uncertainty of the parameters that are the input of the interpretive model (Cronquist, 2001; Kuchuk et al., 2009).

Conventionally, the true formation resistivity (Rt) is measured directly from well-log data; however, calculating its uncertainty may be difficult. For this reason, dependent on the environment of the borehole, the depth that drilling mud penetrates, and the thickness of the layer, it is possible that significant corrections are applied to calculate the true formation resistivity (Cronquist, 2001; Kuchuk et al., 2009).

Of the other most significant and complex variable to calculate is formation water resistivity (Rw) whose uncertainty may have a high probability. In petroleum formations, when the water saturation of the formation is reduced and it is near irreducible water saturation, it is more or less impossible to Gain a water sample with high accuracy. In other words, in petroleum formations, there is usually no free water flow in the initial conditions of the reservoir. If present, the most real value for formation water resistivity is the value that is obtained in water areas under the oil areas for water samples, which would be a better method to calculate (Cronquist, 2001; Kuchuk et al., 2009).

As mentioned earlier, Archie's electrical parameters are a, m, and n. Usually, the mentioned parameters are not calculated directly from the formation, depending on lithology, the geometry of pores, and the distribution of fluid in pores (wettability). Instead, they are calculated from laboratory core samples simulated in reservoir conditions. The values of core samples may not be accepted in areas with much permeability due to incomplete recovery of the core and in areas with insignificant permeability due to measurement problems in the laboratory. In addition, obtaining clean and uncontaminated cores is tough, and the return of cores contaminated to the initial conditions of the reservoir is currently considered a significant industry challenge. Also, upgrading the data of core samples to applications of the reservoir can be challenging and debatable. Particularly for reservoirs that have heterogeneity (Kuchuk et al., 2009).

The parameters m and n are typically achieved from the laboratory plugs of the core samples based on electrical analysis. The expression m for heterogeneous microscopic environments, especially carbonates, is derived from intergranular and vuggy ingredients (Kuchuk et al., 2009; Ramakrishnan et al., 2001). The expression n indicates the effect of electrical resistivity when nonconductor fluid, such as oil, replaces pore-filling water (Kadkhodaie, 2018). In most cases, cementation factor changes and saturation exponent can hardly be evaluated, and uncertainty in m and n spreads in water saturation uncertainty (Eyvazzadeh et al., 2003; Kuchuk et al., 2009). Eyvazzadeh et al. (2005, 2007) show how to obtain water saturation value errors based on resistivity log analysis. Uncertainty is generally neglected in petrophysical interpretations, and Archie's equation parameters are associated with high uncertainty. Therefore, decisions based on definite or deterministic values may lead to inappropriate and incorrect engineering decisions (Eyvazzadeh et al., 2007; Kuchuk et al., 2009).

The current study presents an approach to determine the uncertainty of Archie's equation parameters using Monte Carlo simulation in MATLAB software and calculates water saturation

with 10, 50 and 90 percent probabilities. The results of water saturation calculations will be useful in estimating the Oil In Place volume, technical planning and the investment risk for the production and recovery of hydrocarbon.

2. Methodology

While introducing the Monte Carlo simulation to determine the uncertainty in the water saturation calculation, we review some of its basic equations to determine the water saturation parameter.

Many research and engineering analyses in various fields have been presented and performed based on stochastic methods, mainly Monte Carlo simulation. The interesting name of this method and the extraordinary developments in software and hardware equipment in the computer world have encouraged researchers to discover and solve complex problems using this method. Unlike other methods, Monte Carlo simulation methods, instead of relying on a realistic view, discuss how the process is performed over time, about the occurrence of possible events in future times, based on probability distribution. These methods operate based on a sequence of random numbers in the time or space domain and actionable input data of possible events (Ghiass, 2014; Ghiass et al., 2001; Kroese et al., 2011; Robert, 2009).

In other words, Monte Carlo simulation methods are applied aspects of stochastic simulation according to the computer world expansions and developments. These methods deal with the calculations of the computer and iterative algorithms. Where it is not easy to perform experiments and theoretical analyses and it is associated with exorbitant costs, Monte Carlo simulation has been successfully used for the uncertainty of complex systems, including fluids turbulence, heterogeneous environments, structures of cellular, and inputs of systems. Also, it has been significantly used in mathematical calculations in fields such as chemistry, physics, space research, hydrocarbon exploration, costs forecasting, and other fields (Ghiass, 2014; Ghiass et al., 2001; Imanzadeh et al., 2019).

Monte Carlo simulation is based on the specific probability distribution of the input parameters. It also calculates the overall uncertainty in the prediction concerning each parameter uncertainty (Kuchuk et al., 2009; Robert and Casella, 2004). By definition, the Monte Carlo simulation is a computational algorithm that uses statistics and probabilities based on repetitive random sampling for its result calculation. In other words, it is an algorithm that relies on repeated random samples (Imanzadeh et al., 2019; Robert, 2009). The initiator of this method is a model in the mathematical rules framework, where the dependent variable is affected by the independent variables (Zhang, 2003). For example, the water saturation can be the dependent variable; the porosity, true formation resistivity and the Archie's parameters such as cementation factor can be independent variables (Fig. 1).

Different independent variables might have different statistical distributions, or they might have different parameters even though they are the same kind of distribution. For example, two normal distributions can have various means and standard deviations. After the mathematical model is built, numerous random numbers are generated for each independent variable based on their respective statistical distributions. To generate random numbers for the independent variables, we need probability density functions for them. Thus, those probability density functions have to be determined before the Monte Carlo method can be applied. Each independent variable is plugged into the mathematical model, and a dependent variable is calculated. Thus, many values of the dependent variable are obtained using those of the independent variables (Fig. 1) (Zhang, 2003).

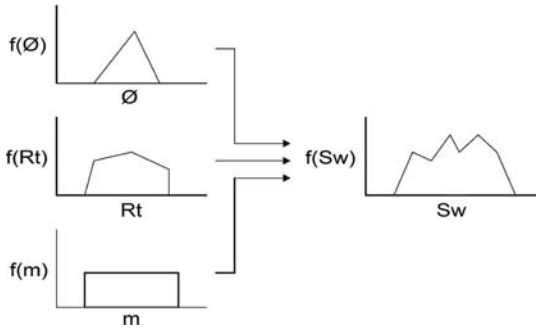


Fig. 1. Monte Carlo simulation schematic for SW.

The basic procedure for performing calculations through the Monte Carlo simulation is similar to the example presented in Fig. 2. Fig. 2 shows the Monte Carlo simulation flow chart. First, a random number is determined, and then the probability of an event occurring is compared with the value of the generated random number. If the generated random number satisfies the probability criterion, a process or set of processes occurs in the next part. This procedure can be repeated several times, producing a measurable output for each iteration. In the final part, the set of experiments or output results are subjected to statistical processing, and a quantitative, understandable, and interpretable amount of the results is announced (Fig. 2) (Ghiass, 2014; Robert, 2009).

As mentioned, the core of any random simulation method with Monte Carlo simulation is based on the continuous use of random

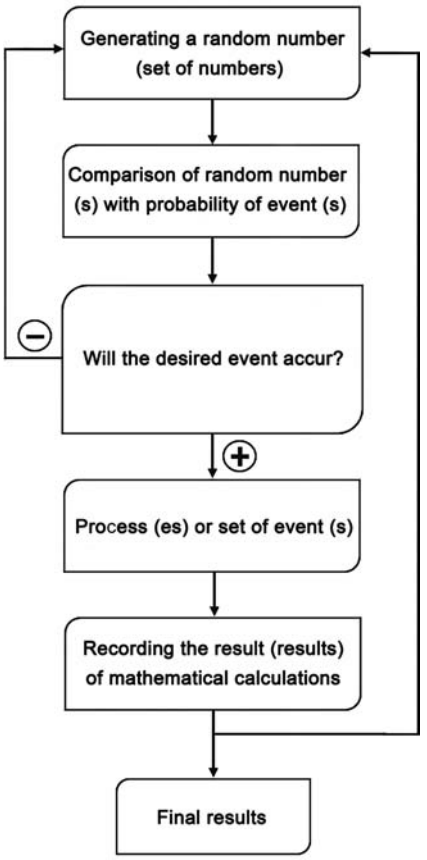


Fig. 2. Diagram of calculation process according to random numbers in a Monte Carlo simulation (Ghiass, 2014).

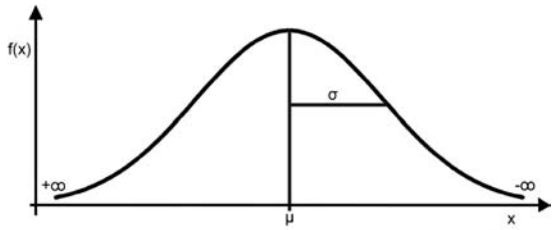


Fig. 3. Probability distribution function (pdf) for arbitrary parameter x with mean μ and standard deviation σ (Kadkhodaie, 2018).

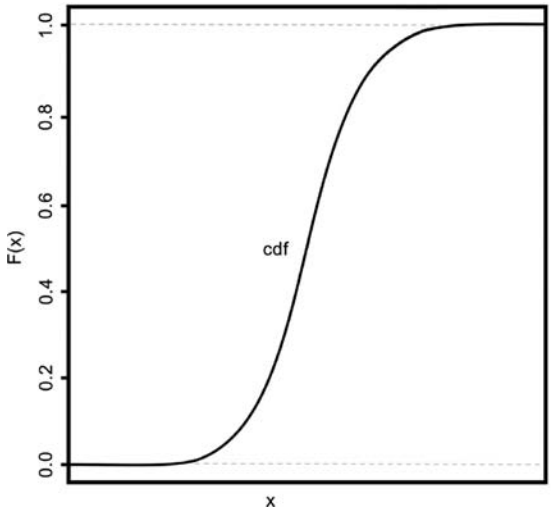


Fig. 4. Cumulative distribution function (cdf) for parameter x (Kadkhodaie, 2018).

numbers According to the mathematical definition, a number determined entirely by chance and without considering specific rules is random. In computers, random numbers are usually obtained based on machine rules. In the sense that in each execution based on the use of particular mathematical equations that generate numbers, a real number in the range between zero and one is produced. Such numbers are called random numbers, and their production program is called a random number generator (Ghiass, 2014; Ghiass et al., 2001; Hammersley and Handscomb, 1964; Kolos and Whitlock, 2008; Kroese et al., 2011; Thomopoulos, 2013).

A probability distribution function for a random numerical value (x) in a specified domain, continuously or discretely, is the value of the probability of its occurrence in the form of $f(x)$, where x indicates the presence of effective parameters in the distribution function (Fig. 3) (Ghiass, 2014; Kadkhodaie, 2018; Kroese et al., 2011). The probability distribution function (*pdf*) is also called the normal distribution function or the gaussian distribution function and is one of the most common functions defined as follows (Eq. (5)) (Kadkhodaie, 2018):

$$f(x) = \frac{1}{\sqrt{2\sigma\pi}} e^{-0.5\left(\frac{x-\mu}{\sigma}\right)^2} \tag{5}$$

The expression μ represents the x parameter mean, which in this figure coincides with the peak of the curve. The expression σ represents the standard deviation of the x parameter and shows the opening value of both sides of the curve compared to the mean. The more compact the *pdf*, the less the standard deviation or dispersion of the data, and the lower the *pdf* compression and the lower the slope of the domain, the more data has dispersion than the mean. If the data's standard deviation is equal to zero, the *pdf* will correspond to the mean point as a straight line (Kadkhodaie, 2018).

In addition, the integral of the probability distribution function is also used as a cumulative distribution function (*cdf*) to check and evaluate the performance of the Monte Carlo algorithm. This function is an elongated *S* shape, and its values change between zero and one. The corresponding values can be obtained by choosing any random number between zero and one. In other words, equivalent answers will be obtained for every probability between zero and one (Fig. 4). The value of the cumulative distribution function is defined as an integral over the entire domain as follows (Eq. (6)) (Kadkhodaie, 2018):

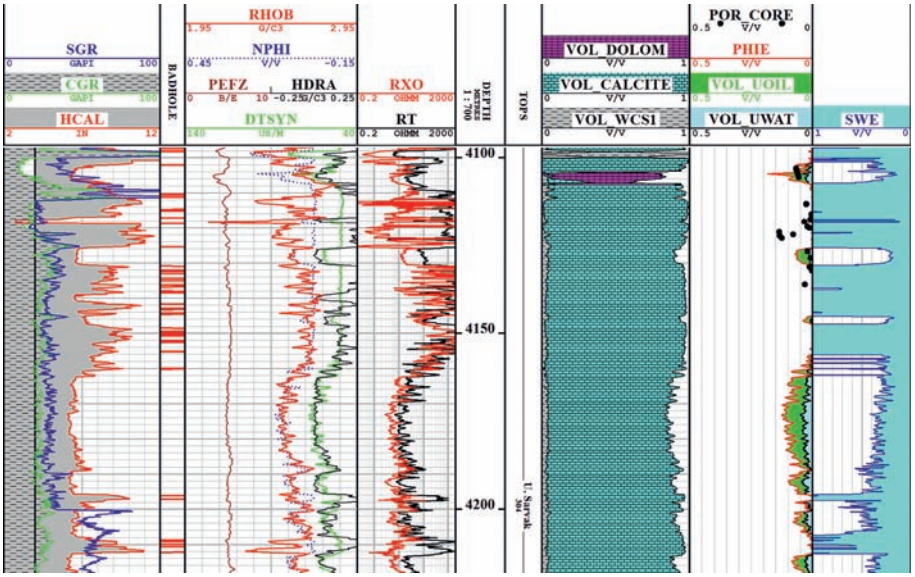


Fig. 5. Plot showing the results of the petrophysical evaluation (stratigraphic column) using the multimin analysis for well A from the Upper Sarvak zone of the Sarvak formation.

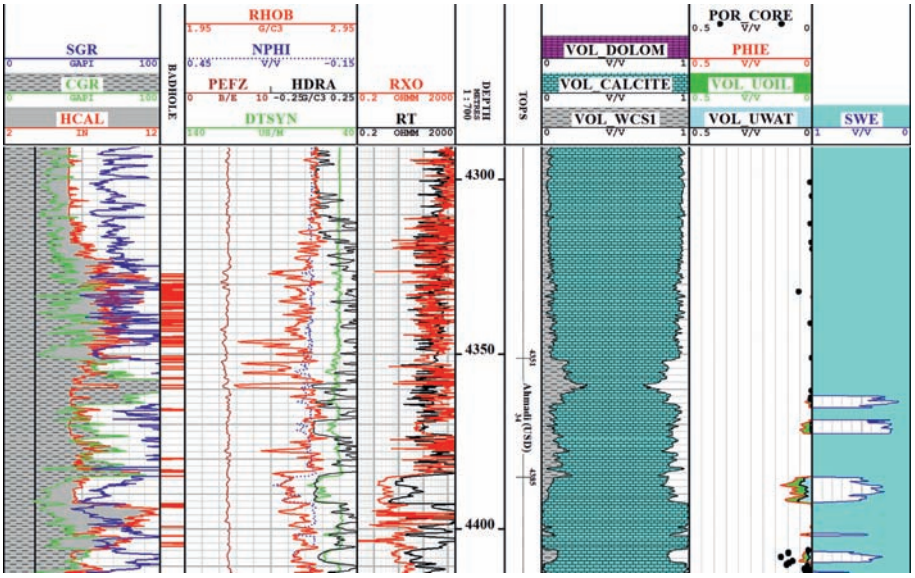


Fig. 6. Plot showing the results of the petrophysical evaluation (stratigraphic column) using the multimin analysis for well A from the Ahmadi zone of the Sarvak formation.

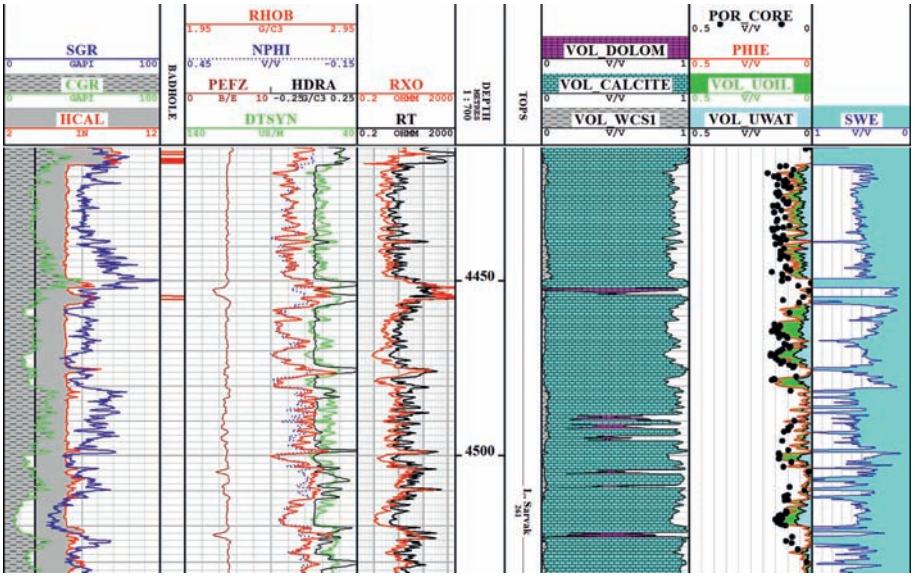


Fig. 7. Plot showing the results of the petrophysical evaluation (stratigraphic column) using the multimin analysis for well A from the Lower Sarvak zone of the Sarvak formation.

$$\int_{-\infty}^{+\infty} f(x)dx = 1 \tag{6}$$

The main purpose of the current study is to determine the uncertainty of the effective parameters in Archie's equation using Monte Carlo simulation. It investigates how the uncertainty in the effective parameters of the Archie equation affects water saturation calculation and propagates in it. Finally, it calculates water saturation with probabilities of 10, 50 and 90%. With this, a more concrete understanding of hydrocarbon reserve estimation will be available, and it is possible to estimate the volume of Oil In Place with different possibilities. For this purpose, a case study was run to investigate the uncertainty and calculate water saturation using well-logging data, such as neutron, density, sonic, gamma and specific resistivity logs, together with the data from the initial petrophysical evaluation including *SWT*, *SWE*, *PHIT* and *PHIE* data. The data of this study were selected from three wells drilled in Sarvak Reservoir in one of southwestern Iran's oilfields. Sarvak formation with the main sequence of carbonate lithology with some shale and thin layers of dolomite is the main interval studied in this project.

3. Results and discussion

Since lithology has an essential effect on the petrophysical parameters of reservoirs, therefore, in the current research to study the lithology of the formation in three wells in the target reservoir area, the cores were studied, and well logs were

Table 1
Maximum, minimum, standard deviation and mean values of the porosity (Ø) in three studied wells.

Well No.	Min (Ø)	Max (Ø)	Mean (Ø)	Std. D (Ø)
Well A	0.000001	0.181666	0.028149	0.038035
Well B	0.000001	0.177536	0.040694	0.046401
Well C	0.000001	0.218132	0.030801	0.03786

simultaneously interpreted; the Stratigraphic column with the petrophysical Interpretation results of the interval studied was drawn to visualize and understand more concretely the actual situation of the reservoir structure and visually compare the modeling calculation results with reality (Figs. 5–7). In the main

Table 2
Maximum, minimum, standard deviation and mean values of the true formation resistivity (Rt) in three studied wells.

Well No.	Min (*Rt*)	Max (*Rt*)	Mean (*Rt*)	Std. D (*Rt*)
Well A	0.579	100000	2066.562	10872.36
Well B	3.255	40000	478.8519	1407.777
Well C	1.705	18316.72	445.344	1004.389

Table 3
Maximum, minimum, standard deviation and mean values of the cementation factor (m) in three studied wells.

Well No.	Min (*m*)	Max (*m*)	Mean (*m*)	Std. D (*m*)
Well A	1.745102	2.12538	1.806664	0.077217
Well B	1.745102	2.221505	1.835197	0.107575
Well C	1.745102	2.17609	1.812884	0.077833

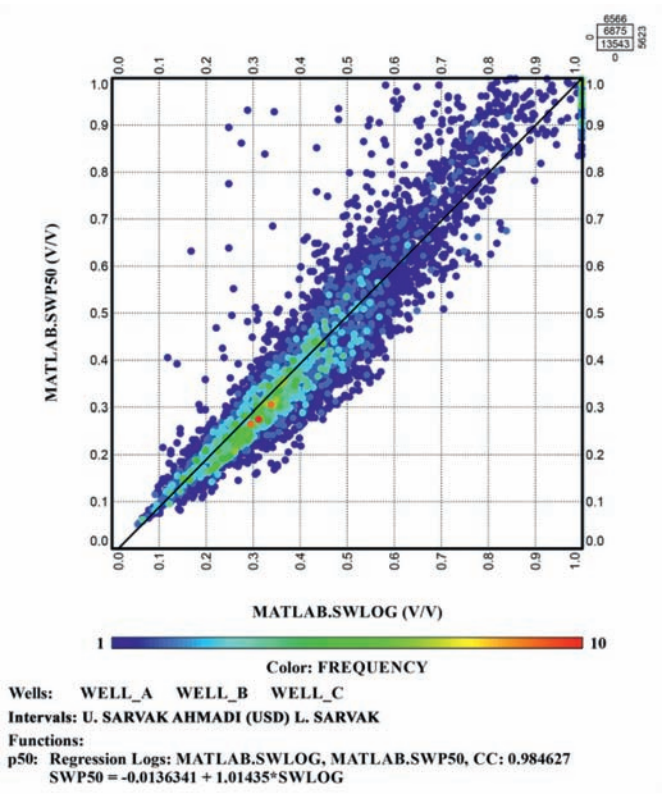


Fig. 9. Cross plot between the water saturation of the petrophysical evaluation and the probable case (p50).

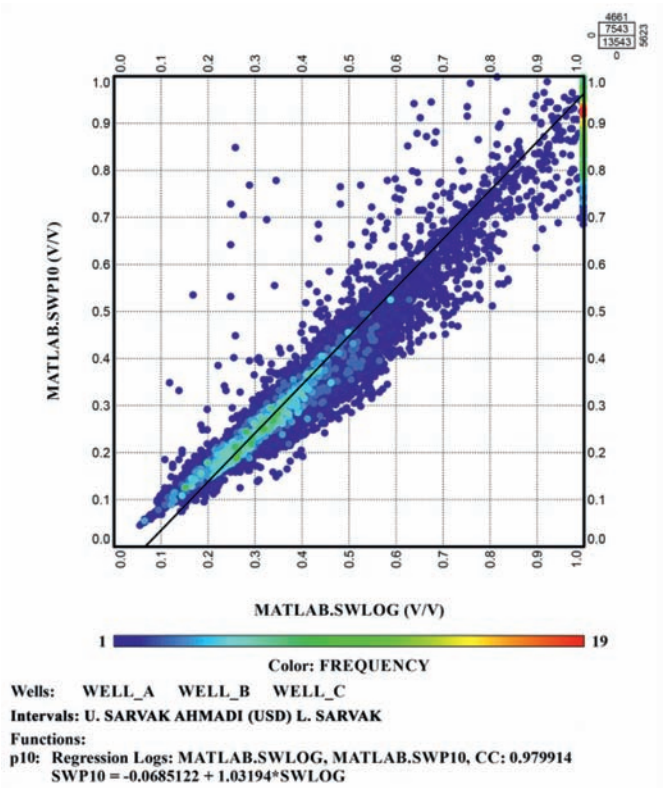


Fig. 8. Cross plot between the water saturation of the petrophysical evaluation and the optimistic case (p10).

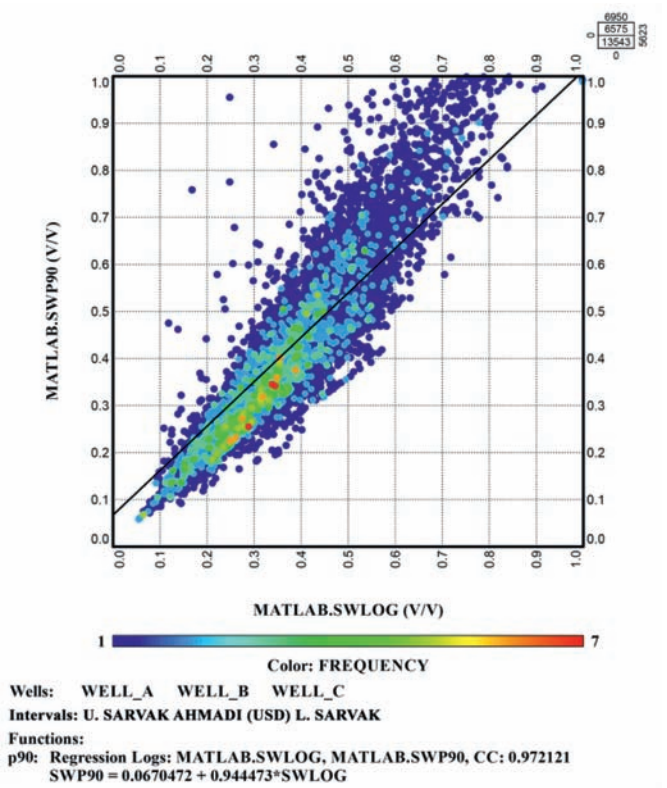


Fig. 10. Cross plot between the water saturation of the petrophysical evaluation and the pessimistic case (p90).

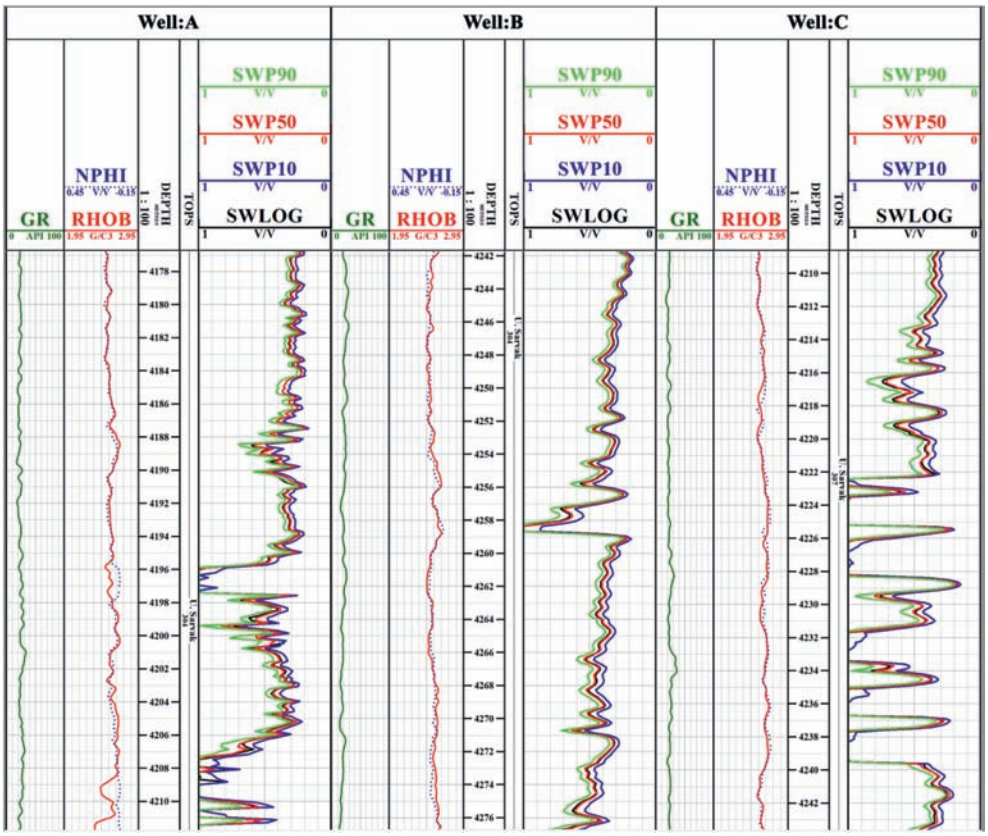


Fig. 11. Water saturation log comparison in optimistic, probable and pessimistic cases with petrophysical evaluation in the Upper Sarvak zone.

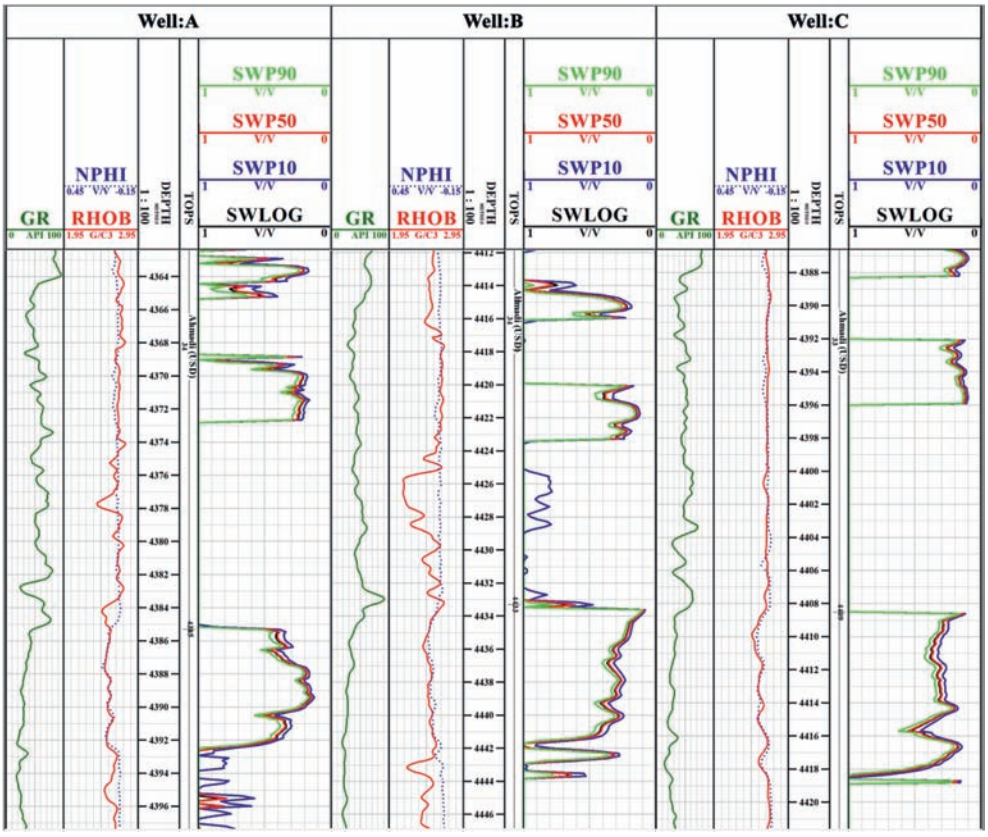


Fig. 12. Water saturation log comparison in optimistic, probable and pessimistic cases with petrophysical evaluation in the Ahmadi zone.

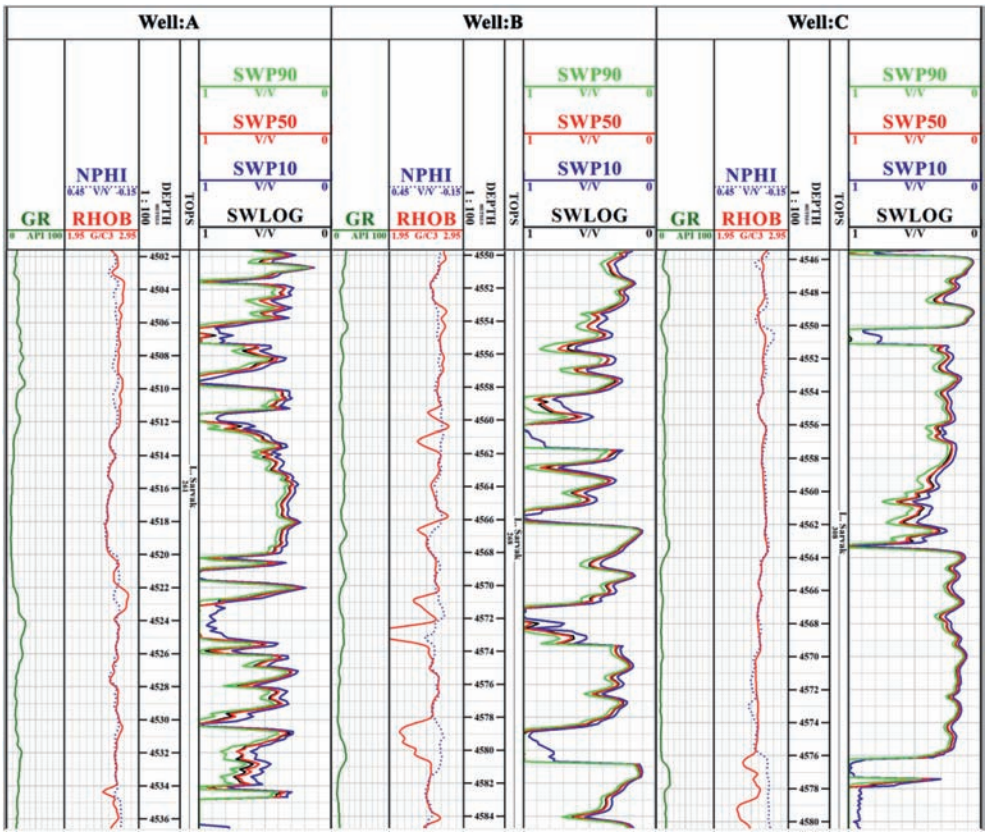


Fig. 13. Water saturation log comparison in optimistic, probable and pessimistic cases with petrophysical evaluation in the Lower Sarvak zone.

Table 4
The average values of effective water saturation calculated in different states in the Sarvak formation.

Well No.	Effective water saturation (log) average (%)	Optimistic (p10) water saturation average (%)	Probable (p50) water saturation average (%)	Pessimistic (p90) water saturation average (%)
Well A	75.82	69.04	74.20	77.37
Well B	66.64	62.76	66.79	70.01
Well C	65.84	62.72	66.35	69.54

reservoir interval, the plots shown on left tracks are gamma ray (CGR, SGR), Caliper, bad-hole flag areas, neutron porosity (NPHI), sonic (DT), density (RHOB), photo-electric (PEF) and density standoff correction log along with resistivity curves (Rxo, Rt); also the reservoir lithology obtained from the petrophysical interpretation such as (calcite (VCAL), dolomite (VDOL), and Shale (VWCS)), together with petrophysical parameters such as porosity (PHIT) and the porosity resulting from the core (black dots)), the volume of porosity occupied by irreducible water (VUWA), the volume of effective porosity is assigned to hydrocarbon (VUO) and water saturation (SWE) are shown on the right tracks of the plot as well.

The petrophysical evaluation results in the studied interval show that the volume of hydrocarbon saturation changes with changes in water saturation. For example, when the water saturation decreases, the hydrocarbon saturation volume increases. In this way, in-depth where water saturation is associated with high uncertainty, the calculation of hydrocarbon saturation is also associated with uncertainty. Therefore, the uncertainty of water saturation should be considered in Oil In Place calculations (Figs. 5–7).

For this purpose, investigates the uncertainty in water saturation according to Archie's equation. Therefore, random numbers generated by the Monte Carlo simulation have been used for parameters such as $\emptyset$, *Rt*, and *m*, depending on their values. Moreover, other parameters of Archie's equation, including *a*, *n*, and *Rw*, are assumed as constant values. In this way, the tortuosity factor is set to one ($a = 1$ as this is of the boundary conditions and generally in the industry, this parameter is used equally to 1(. In this sense, it is assumed that the flow in the porous medium moves directly. The saturation exponent is generally considered equal to 2. In this research, because the carbonate formation is simple and homogeneous, the saturation exponent is considered as 2. According to the geological and field studies in the oil field, *Rw* is set to 0.015.

When the Monte Carlo simulation is used for the Archie equation, it samples the *Sw* based on the random numbers generated for each parameter of the Archie equation. Therefore, according to the statistical characteristics of *Sw*, including mean, standard deviation and frequency diagram for *Sw* can be obtained. Then, *Sw* can be represented with probabilities of 10%, 50%, and 90% for each depth point. Since to do this research, it is necessary to follow the path of

programming language. Therefore, MATLAB software was used in this research to implement the Monte Carlo simulation.

To calculate the water saturation using the Monte Carlo simulation should be obtained a distribution function for each parameter that creates uncertainty in the water saturation model. Then, a Monte Carlo model can be run for water saturation calculation. The model is built using suitable input and output data in MATLAB software environment. Archie coefficients, porosity and formation water resistivity are the essential parameters that create uncertainty in water saturation calculation. In this research, the tortuosity factor (a), formation water resistivity (Rw), and saturation exponent (n) are assumed to be constant. Moreover, the uncertainty on three parameters of cementation factor (m), porosity and true formation resistivity, which are variable, have been investigated.

Therefore, to check the uncertainty of each parameter involved in water saturation calculation, the normal distribution function of that parameter should be obtained. This normal distribution function for each parameter is obtained based on that parameter's mean and standard deviation at each depth. In this research, the standard deviation of the mean values of each parameter is defined as 0.04. Therefore, the data's normal distribution is obtained with the parameters of the mean index and the variables' standard deviation. Then using this distribution, the cumulative distribution curve is obtained for the data. In the following, a number is chosen randomly and read its equivalent Sw from the cumulative distribution function. That is, crossing this point to the Y-axis, the corresponding quantity value is read from the X-axis. This process is repeated several times (in this study, the number of realizations is 1000). This way, we will have 1000 different realizations for each of the parameters involved in Archie's equation, especially the porosity and cementation factor. Therefore, the desired function generates 1000 random numbers for two parameters of porosity and cementation factor at each depth using the mean and standard deviation and its cumulative distribution curve. Also, according to the coding in the MATLAB environment, optimistic ($p10$), probable ($p50$), and pessimistic ($p90$) values are calculated between the random data generated for each parameter (porosity and cementation factor) at each depth. In order to obtain $p10$, $p50$, and $p90$ values at each depth, the cumulative curve of each parameter was used.

In this regard, a total of 1000 realizations were calculated for water saturation (Sw) at each depth based on Archie's equation. For this purpose, the data from three wells were used (Tables 1–3). Tables 1–3 outline the three wells' maximum, minimum, standard deviation and mean values of the porosity, true formation resistivity and cementation factor.

According to Archie's equation, there are four values of water saturation in the entire interval and for each depth including petrophysical evaluation water saturation, optimistic water saturation ($p10$), probable water saturation ($p50$), and pessimistic water saturation ($p90$), which can be represented as a well log. Three logs of optimistic, probable, and pessimistic water saturation for all three wells were plotted to compare with the obtained from the petrophysical evaluation. On the horizontal axis the water saturation from the petrophysical evaluation and on the vertical axis the water saturation calculated in the optimistic ($p10$), probable ($p50$) and pessimistic ($p90$) cases using the Monte Carlo simulation method (Figs. 8–10). According to the discussed figures, the correlation coefficient (CC) value between these two parameters has been obtained as 97.99, 98.46 and 97.21% respectively (Figs. 8–10). Obviously, according to the values of the correlation coefficient in the three cross plots, the amount of water saturation in the probable case ($p50$), in which the uncertainty of two parameters of porosity and cementation factor is taken into account, shows the highest correlation with the water saturation calculated from the petrophysical evaluation. Because $p50$ is the average value around which all results are distributed and has a higher probability of occurrence. $p90$ and $p10$ are the highest and lowest calculated values for water saturation, respectively, both of which have a lower probability of occurrence than $p50$.

The comparison of the four mentioned water saturation values, in the form of the log, for each zone of the Sarvak reservoir, shows that the maximum correlation is the water saturation log from the petrophysical evaluation with the probable case derived from the Monte Carlo simulation, which indicates the high resolution and accuracy of the implemented model results (Figs. 11–13).

Finally, the mean values of the four discussed water saturation cases are outlined in Table 4 such that the sensitivity analysis for this parameter can also be analyzed numerically. According to Table 4, by examining the Sw values in this table, the results obtained from the mean water saturation in the probable case are more similar to the mean water saturation values of the petrophysical evaluation.

The method described for the examples shown in this study can be used for any reservoir information, and determine its uncertainty by the Monte Carlo simulation. Even on the calculation of the Oil In Place volume, Monte Carlo can be run and the uncertainty defined on parameters such as fluids contact surface (fluid contacts), formation volume factor, reservoir upper and lower surfaces, porosity and water saturation, and their effect on the Oil In Place volume checked. It means that the cumulative function obtained for the Oil In Place volume is a result of the uncertainty of the mentioned parameters. In this way, three values with probabilities of 10%, 50% and 90% for the volume of Oil In Place will be achieved, which correspond to the pessimistic, probable, and optimistic reserve volume, respectively. It should be noted that in the calculation of water saturation, $p10$ is considered as an optimistic value. Because it obtains less water saturation, but in the calculation of the Oil In Place volume, the value of $p10$ is considered as a pessimistic storage volume and is accepted as a storage volume. However, due to the possibility of high accuracy of the $p50$ number, it is usually relied on in technical and investment planning. In the framework of Archie's equation, MATLAB codes used to generate random numbers for its parameters in order to quantify the uncertainty of water saturation of the present research data are shown in Appendix 1.

4. Conclusion

In the current research, an efficient method based on the Monte Carlo simulation using well-logging data such as porosity logs (density, neutron, sonic), special resistivity logs, gamma logs together with the data from initial petrophysical evaluation including SWT, SWE, $PHIT$ and $PHIE$ logs was used to calculate the water saturation of the reservoir rock in one of southwestern Iran's oilfields. The Monte Carlo method is a computational algorithm that uses statistics and probabilities based on repetitive random sampling for its result calculation. The initiator of this method is a model in the mathematical rules framework, where the dependent variable such as water saturation is affected by the independent variables such as porosity, true formation resistivity, formation water resistivity and the Archie's parameters such as cementation factor, tortuosity factor and saturation exponent. The results of this study showed that the determination of uncertainty by the Monte Carlo Simulation method can be considered as an effective approach in analyzing petrophysical properties. By examining the

uncertainty of the two parameters of cementation factor and porosity, the uncertainty of water saturation was also determined. Based on this, 1000 different realizations were generated for the water saturation parameter (*Sw*) at each depth, according to Archie's equation, and it was calculated with 10%, 50% and 90% probabilities. In this way, there are four water saturation logs within each interval including water saturation resulting from petrophysical evaluation, optimistic water saturation (*p*10), probable water saturation (*p*50) and pessimistic water saturation (*p*90). Satisfactory results were achieved from the comparison of the four mentioned water saturations with different methods. Accordingly, the amount of water saturation in the probable state (*p*50), in which the uncertainty of two parameters of porosity and cementation factor is taken into account, shows the highest correlation of 98.46 with water saturation resulting from the petrophysical evaluation. The correlation coefficient (*CC*) between optimistic water saturation (*p10*) and pessimistic water saturation (*p90*) with the water saturation obtained from the petrophysical evaluation was 97.99 and 97.21, respectively. Also, four water saturation logs comparison for each zone of the Sarvak reservoir shows that the highest correlation between the water saturation log of petrophysical evaluation and the probable case (*p50*) of Monte Carlo simulation has been obtained. Also, comparing the mean values of water saturation in different probabilities with the mean values of water saturation obtained from petrophysical evaluation, shows that the best fit and consistency to the log data is the probable case value (*p50*). With uncertainty-based Monte Carlo methods, it is not only possible to estimate the amount of water saturation in different probabilities, but also allows petroleum engineers to provide acceptable results of reservoir storage volume with different water saturation probabilities. The proposed approach can be used to any other reservoir data such as Oil In Place calculation, drilling data, and production data.

CRediT authorship contribution statement

Rahman Norouzi: Writing – original draft, Software, Formal analysis, Data curation. **Hossein Mossadegh:** Writing – review & editing, Supervision, Formal analysis, Conceptualization. **Ali Kadkhodaie:** Writing – review & editing, Supervision, Software. **Vali Mehdipour:** Writing – original draft, Software, Methodology, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix 1. MATLAB codes used to generate random numbers for parameters of the Archie equation to quantify the uncertainty of water saturation.

```
function [x,p] = make_random(min,max,meann,stdd,n,casee)

stdd=0.04*meann;
x=normrnd(meann,stdd,[1,n]);  %random numbers
p=prctile(x,[10,50,90]);   %P10, P50, P90

pd=makedist('normal',meann,stdd);
y1=pdf(pd,x);
y2=cdf(pd,x);

end
```

```
clear
clc

%%
%Input Data
well_num=input('Input well Numer (A, B, C): ');
sheetname=['Well', num2str(well_num)];
if well_num==A
    cellrange=['A2:C', '3934'];
    min_porosity=0.000001;
    max_porosity=0.181666;
    std_porosity=0.038035;
    min_resistivity=0.579;
    max_resistivity=100000;
    std_resistivity=10872.36;
    min_cementation=1.745102;
    max_cementation=2.12538;
    mean_cementation=1.806664;
    std_cementation=0.077217;
elseif well_num==B
    cellrange=['A2:C', '3931'];
    min_porosity=0.000001;
    max_porosity=0.177536;
    std_porosity=0.046401;
    min_resistivity=3.255;
    max_resistivity=40000;
    std_resistivity=1407.777;
    min_cementation=1.745102;
    max_cementation=2.221505;
    mean_cementation=1.835197;
    std_cementation=0.107575;
elseif well_num==C
    cellrange=['A2:C', '5681'];
    min_porosity=0.000001;
    max_porosity=0.218132;
    std_porosity=0.03786;
    min_resistivity=1.705;
    max_resistivity=18316.72;
    std_resistivity=1004.389;
    min_cementation=1.745102;
    max_cementation=2.17609;
    mean_cementation=1.812884;
    std_cementation=0.77833;
end


d=xlsread('Raw Data.xlsx', sheetname,cellrange);
depth=d(:,1)';
a=1;
Rw=0.015;
n_random=1000;  %number of ran_depthom values

mean_porosity=d(:,2);  %average
mean_resistivity=d(:,3);  %average
%%

n_depth=length(depth);

p_porosity=zeros(n_depth,4);   %includes P10,P50,an_depth P90 in each depth
p_resistivity=zeros(n_depth,4);
p_cementation=zeros(n_depth,4);
p_saturation=zeros(n_depth,4);

porosity=zeros(n_depth,n_random);  %all random numbers for porosity
resistivity=zeros(n_depth,n_random);
cementation=zeros(n_depth,n_random);
saturation=zeros(n_depth,n_random);

for i=1:n_depth
    %min, max, mean,   std
```

```
    [porosity(i,:), p_porosity(i,:)] = make_random(min_porosity, max_porosity, mean_porosity(i), std_porosity, n_random,'porosity');
    [resistivity(i,:), p_resistivity(i,:)] = make_random(min_resistivity, max_resistivity, mean_resistivity(i), std_resistivity, n_random,'resistivity');
    [cementation(i,:), p_cementation(i,:)] = make_random(min_cementation, max_cementation, mean_cementation, std_cementation, n_random,'cementation');
    %     saturation(i,:)=sqrt(a*Rw./(porosity(i,:).^cementation(i,:).*resistivity(i,:)));
%     Rw=0.02*rand(1,1)+0.014;
    saturation(i,:)=sqrt(a*Rw./(porosity(i,:).^cementation(i,:).*mean_resistivity(i)));
    saturation(saturation>1)=1;
    p_saturation(i,:)=prctile(saturation(i,:),[10,50,90]);


    %     %this section is for plotting CDF and PDF
    %     x=saturation(i,:);
    %     pd=makedist('normal',mean(x),std(x));
    %     y1=pdf(pd,x);
    %     y2=cdf(pd,x);
    %     figure(2*i-1);
    %     plot(x,y1,'.');
    %     title('Saturation PDF');
    %     plot(x,y2,'.');
    %     figure(2*i+6);
    %     title('Saturation CDF');

end
```

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