复合材料层合悬臂板的非线性动力学研究

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摘要 以飞机机翼的颤振为实际工程背景,考虑高阶横向剪切效应、几何大变形和横向阻尼的影响,基于 Reddy 的高阶剪切变形理论和 von Karman 的大变形理论,利用 Hamilton 原理对纤维增强复合材料层合悬臂板的非线性动力学问题进行了研究。建立了复合材料悬臂板在面内激励和横向外激励联合作用下悬臂板广义位移形式的偏微分运动控制方程。利用 Galerkin 方法,选取二阶模态对复合材料层合悬臂板偏微分形式运动控制方程进行二阶模态截断,得到了具有三次非线性项、参数激励项和横向激励项的常微分形式二自由度非线性动力学方程。在考虑主参数共振和 1:2 内共振的情况下,用多尺度法获得了复合材料层合悬臂板四维直角坐标形式的平均方程。在平均方程的基础上,利用数值方法分析面内激励和横向激励幅值对系统非线性动力学特性的影响,得到了 1:2 内共振时复合材料层合悬臂板动力学方程的平面相图、波形图、三维相图和频谱图。结果表明,随着外激励的变化,系统会出现单倍周期运动、多倍周期运动、概周期运动和混沌运动。

关键词 复合材料;悬臂板;非线性振动;混沌

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Nonlinear Dynamics of a Composite Laminated Cantilever Plate

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Abstract The nonlinear dynamics of composite laminated cantilever plates subjected to in-plane and transversal excitations is investigated in this paper. With the wing flutter of a flying aircraft as the background, based on the Reddy′s high-order shear deformation theory and von Kármán type equations for the geometric nonlinearity, the nonlinear governing partial differential equations of motion are derived for the composite laminated cantilever plate subjected to in-plane and transverse excitations by using the Hamilton′s principle with considerations of the effects of higher-order transverse shear deformation, nonlinear geometry deformation and transversal damping. The Galerkin approach is utilized to transform the nonlinear partial differential governing equations of motion for the composite laminated cantilever plate to a two-degree-of-freedom nonlinear system. The principal parametric resonance, the 1/2 subharmonic resonance and the 1:2 internal resonance are considered. The method of multiple scales is employed to obtain the four-dimensional averaged equations of the composite laminated cantilever plate subjected to in-plane and transverse excitations. The dynamical behaviors of the cantilever plate under combined in-plane and transversal excitations are investigated with numerical simulations of the averaged equations. The two-dimensional phase portrait, waveform and three-dimensional phase portrait are obtained as results of the nonlinear dynamic behaviors of the composite laminated cantilever plate. The results of numerical simulations demonstrate that there exist one-periodic, multi-periodic and chaotic motions of the composite laminated cantilever plate.

Keywords composite material; cantilever plate; nonlinear dynamics; chaotic motion

0 引言

复合材料是指由两种或两种以上不同物质以不同方式组合而成的材料,它可以发挥各种材料的优点,克服单一材料的缺陷,并且可以通过改变纤维取向和铺层顺序对其性质

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加以调整。复合材料层合板结构具有重量轻、强度高、耐疲劳、可设计性好、加工成型方便等特点,已逐步取代木材及金属合金的材料板,广泛应用于航空航天、导弹、交通、船舶以及建筑等领域。但是随着复合材料层合板结构不断向大型化和柔性化方向发展,板的振动问题显得日益突出,尤其是非线性动力学响应对板的控制起着极为重要的作用。因此,研究复合材料层合板的非线性动力学特性对板的设计和控制具有非常重要的工程意义。国内外很多学者对复合材料层合板的非线性动力学特性进行了大量研究,但对于复合材料层合悬臂板的研究相对很少。

本文利用高阶剪切变形理论对复合材料层合悬臂板模型的非线性动力学行为进行研究。经典层合板壳理论是基于Kirchhoff-Love假设的经典理论及一阶剪切变形理论发展而来。但经典理论忽略了横向剪切变形和转动惯量,在处理相对较厚的板以及较高的弹性模量与剪切模量比值的情况时并不适用。为了考虑横向剪应力的影响,国外学者于20世纪70年代提出了高阶剪切变形理论。

1980年,Levinson[1]最早用高阶剪切变形理论研究了弹性板的静力学和动力学问题。1984年,Reddy[2-4]提出三阶剪切变形板理论,可以很好地描述沿厚度方向剪切变形的变化,而后又将此理论推广到包含几何非线性的复合材料层合板中。1989年,Noor和Burton[5]讨论了不同的层合板建模方法。1994年,Reddy和Rpbbin[6]综述了等效单层和Layer-wise层合板理论并且提出了他们的有限元模型。同年,Liew等[7-8]总结了正交层合板的高阶理论,并通过所研究板的不同形状对以往研究加以分类。1996年,Chandiramani等[9]用Galerkin法和连续弧长方法研究了层合板的颤振和几何不完全剪切变形。2000年,White等[10]研究了各种设计参数对复合材料夹层板自由振动的影响。2002年,Wang等[11]用FSDT方法对复合材料层合板的弯曲变形、自由振动和弹性屈服进行了分析。2005年,Ye等[12]利用全局摄动法研究了复合材料层合板的非线性动力学行为。2007年,Yao等[13]分析了在参数激励和横向激励共同作用下薄板的全局动力学。2008年,Zhang等[14]研究了1:3内共振情况下复合材料压电层合板的动力学响应。同年,Singh[15]用高阶剪切变性理论研究了不同弹性支撑条件下的几何非线性层合板。

在层合板的研究中,大多数使用了简支等边界条件较简单的板模型进行研究。虽然对边简支板的横向振动可以求得精确解,但是对于其他支撑情况,仍很难求得满意的结果,只能用各种近似解来求解。其中较典型的是一边固定、三边自由的悬臂板问题。随着悬臂板结构在工程实际中的应用越来越广泛,学者们提出了很多方法,其中得到广泛应用的有Leissa等[16]提出的基于悬臂梁形函数的Rayleigh-Ritz方法,Bhat[17]提出的带有逐阶正交特性的多项式法,以及Rossi等[18]使用的有限元方法(FEM)。

本文以面内激励和横向外激励联合作用下的复合材料层合悬臂板为研究对象,考虑高阶横向剪切变形和横向阻尼对系统的影响,综合运用Galerkin方法、多尺度方法以及数值仿真方法研究复合材料层合悬臂板的非线性动力学行为。

1 复合材料悬臂板的动力学模型

本文以飞机飞行中机翼颤振为工程背景,将机翼简化为如图1所示的复合材料悬臂板结构。模型同时受到y方向的面内载荷与横向面外载荷联合作用,振动过程中考虑横向阻尼的影响。夹层板的长、宽、高分别为a、b、h,直角坐标Oxy位于层合板的中性面内,z轴向下,设板内任一点沿x、y和z方向的位移分别为u、v和w,沿着x方向作用的面内载荷为$F=F_0+F_1\cos\Omega_1 t$,横向载荷为$P=P_0(x,y)\cos\Omega_2 t$,其中,F_0为面内载荷的平均值,F_1为面内载荷的周期扰动载荷的幅值,Ω_1为扰动载荷的频率;P_0为横向载荷的幅值,Ω_2为横向载荷的频率,t为时间变量。本文考虑一块复合材料层合薄板,由多层单层板粘合在一起,假设其层与层之间粘结良好,等厚且层间变形连续,总厚度符合薄板假设。

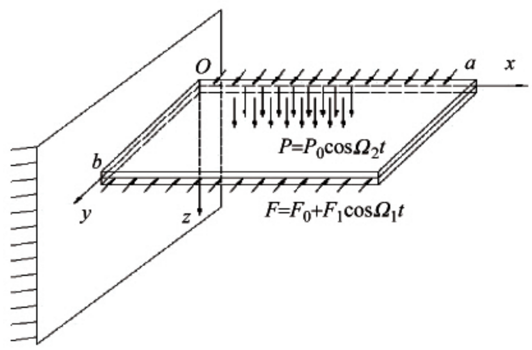


图1 受横向与y方向的面内载荷联合作用下的层合悬臂板模型

Fig. 1 Model of the plate subjected to in-plane and transverse excitations

采用Reddy的三阶剪切变形理论建立板的非线性动力学方程。由Reddy的三阶剪切变形板理论可知,板的位移场为

$$u(x,y,z,t)=u_0(x,y,t)+z\phi_x(x,y,t)-z^3\frac{4}{3h^2}\left(\phi_x+\frac{\partial w_0}{\partial x}\right) \quad (1a)$$

$$v(x,y,z,t)=v_0(x,y,t)+z\phi_y(x,y,t)-z^3\frac{4}{3h^2}\left(\phi_y+\frac{\partial w_0}{\partial y}\right) \quad (1b)$$

$$w(x,y,z,t)=w_0(x,y,t) \quad (1c)$$

其中u_0、v_0、w_0为中性面上的位移,ϕ_x、ϕ_y为中性面法线相对于x、y轴的转角,h为板的高度。

应变与位移的关系即几何关系为非线性关系

$$\varepsilon_{xx}=\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2,\ \varepsilon_{yy}=\frac{\partial v}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2,\ \varepsilon_{zz}=\frac{\partial w}{\partial z},$$

$$\gamma_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y},\ \gamma_{yz}=\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y},\ \gamma_{xz}=\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x} \quad (2)$$

根据Hamilton原理,得到系统广义位移形式的偏微分动

力学方程为

$$A_{11}\frac{\partial^2 u_0}{\partial x^2}+A_{66}\frac{\partial^2 u_0}{\partial y^2}+(A_{12}+A_{66})\frac{\partial^2 v_0}{\partial x\partial y}+A_{11}\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x^2}+$$
$$A_{66}\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial y^2}+(A_{12}+A_{66})\frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial x\partial y}=$$
$$I_0\ddot{u}_0+J_1\ddot{\phi}_x-c_1I_3\frac{\partial\ddot{w}_0}{\partial x} \tag{3a}$$

$$A_{66}\frac{\partial^2 v_0}{\partial x^2}+A_{22}\frac{\partial^2 v_0}{\partial y^2}+(A_{21}+A_{66})\frac{\partial^2 u_0}{\partial x\partial y}+A_{66}\frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial x^2}+$$
$$A_{22}\frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial y^2}+(A_{21}+A_{66})\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x\partial y}=$$
$$I_0\ddot{v}_0+J_1\ddot{\phi}_y-c_1I_3\frac{\partial\ddot{w}_0}{\partial y} \tag{3b}$$

$$A_{11}\frac{\partial u_0}{\partial x}\frac{\partial^2 w_0}{\partial x^2}+A_{21}\frac{\partial u_0}{\partial x}\frac{\partial^2 w_0}{\partial y^2}+2A_{66}\frac{\partial u_0}{\partial y}\frac{\partial^2 w_0}{\partial x\partial y}+A_{11}\frac{\partial^2 u_0}{\partial x^2}\frac{\partial w_0}{\partial x}+$$
$$(A_{21}+A_{66})\frac{\partial^2 u_0}{\partial x\partial y}\frac{\partial w_0}{\partial y}+A_{66}\frac{\partial^2 v_0}{\partial x^2}\frac{\partial w_0}{\partial y}+(A_{12}+A_{66})\frac{\partial^2 v_0}{\partial x\partial y}\cdot$$
$$\frac{\partial w_0}{\partial x}+A_{22}\frac{\partial^2 v_0}{\partial y^2}\frac{\partial w_0}{\partial y}+\frac{3}{2}A_{11}\left(\frac{\partial w_0}{\partial x}\right)^2\frac{\partial^2 w_0}{\partial x^2}+$$
$$(\frac{1}{2}A_{21}+A_{66})\left(\frac{\partial w_0}{\partial x}\right)^2\frac{\partial^2 w_0}{\partial y^2}+(A_{12}+A_{21}+4A_{66})\frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}\cdot$$
$$\frac{\partial^2 w_0}{\partial x\partial y}+\frac{3}{2}A_{22}\left(\frac{\partial w_0}{\partial y}\right)^2\frac{\partial^2 w_0}{\partial y^2}+(\frac{1}{2}A_{12}+A_{66})\left(\frac{\partial w_0}{\partial y}\right)^2\cdot$$
$$\frac{\partial^2 w_0}{\partial x^2}+(A_{55}-2c_2D_{55}+c_2^2F_{55})\frac{\partial^2 w_0}{\partial x^2}-(F_0+F_1\cos\Omega_1 t)\frac{\partial^2 w_0}{\partial y^2}+$$
$$(A_{44}-2c_2D_{44}+c_2^2F_{44})\frac{\partial^2 w_0}{\partial y^2}-c_1^2H_{11}\frac{\partial^4 w_0}{\partial x^4}-c_1^2(H_{21}+4H_{66}+$$
$$H_{12})\frac{\partial^4 w_0}{\partial x^2\partial y^2}+(A_{55}-2c_2D_{55}+c_2^2F_{55})\frac{\partial\phi_x}{\partial x}-c_1^2H_{22}\frac{\partial^4 w_0}{\partial y^4}+$$
$$c_1(F_{11}-c_1H_{11})\frac{\partial^3\phi_x}{\partial x^3}+c_1(F_{21}+2F_{66}-c_1H_{21}-2c_1H_{66})\frac{\partial^3\phi_x}{\partial x\partial y^2}+$$
$$(A_{44}-2c_2D_{44}+c_2^2F_{44})\frac{\partial\phi_y}{\partial y}+c_1(F_{12}+2F_{66}-c_1H_{12}-2c_1H_{66})\cdot$$
$$\frac{\partial^3\phi_y}{\partial x^2\partial y}+c_1(F_{22}-c_1H_{22})\frac{\partial^3\phi_y}{\partial y^3}+P\cos\Omega_2 t-\gamma\dot{w}_0=I_0\ddot{w}_0-c_1^2I_6\cdot$$
$$\left(\frac{\partial^2\ddot{w}_0}{\partial x^2}+\frac{\partial^2\ddot{w}_0}{\partial y^2}\right)+c_1I_3\left(\frac{\partial\ddot{u}_0}{\partial x}+\frac{\partial\ddot{v}_0}{\partial y}\right)+c_1J_4\left(\frac{\partial\ddot{\phi}_x}{\partial x}+\frac{\partial\ddot{\phi}_y}{\partial y}\right) \tag{3c}$$

$$(D_{11}-2F_{11}c_1+H_{11}c_1^2)\frac{\partial^2\phi_x}{\partial x^2}+(D_{66}-2F_{66}c_1+H_{66}c_1^2)\frac{\partial^2\phi_x}{\partial y^2}-$$
$$c_1(F_{11}-H_{11}c_1)\frac{\partial^3 w_0}{\partial x^3}-(F_{55}c_2^2-2D_{55}c_2+A_{55})\frac{\partial w_0}{\partial x}+(D_{12}+$$
$$D_{66}+H_{66}c_1^2-2F_{66}c_1+H_{12}c_1^2-2F_{12}c_1)\frac{\partial^2\phi_y}{\partial y\partial x}-$$
$$c_1(2F_{66}+F_{12}-2H_{66}c_1-H_{12}c_1)\frac{\partial^3 w_0}{\partial y^2\partial x}+(2D_{55}c_2-A_{55}-F_{55}c_2^2)\cdot$$
$$\phi_x=J_1\ddot{u}_0+K_2\ddot{\phi}_x-c_1J_4\frac{\partial\ddot{w}_0}{\partial x} \tag{3d}$$

$$(D_{66}-2F_{66}c_1+H_{66}c_1^2)\frac{\partial^2\phi_y}{\partial x^2}-c_1(F_{21}+2F_{66}-H_{21}c_1-2H_{66}c_1)\frac{\partial^3 w_0}{\partial y\partial x^2}-$$
$$c_1(F_{22}-H_{22}c_1)\frac{\partial^3 w_0}{\partial y^3}+(H_{21}c_1^2+D_{66}+D_{21}-2F_{21}c_1+H_{66}c_1^2-$$
$$2F_{66}c_1)\frac{\partial^2\phi_x}{\partial y\partial x}+(H_{22}c_1^2+D_{22}-2F_{22}c_1)\frac{\partial^2\phi_y}{\partial y^2}-$$
$$(F_{44}c_2^2-2D_{44}c_2+A_{44})\frac{\partial w_0}{\partial y}+(2D_{44}c_2-F_{44}c_2^2-A_{44})\phi_y=$$
$$J_1\ddot{v}_0+K_2\ddot{\phi}_y-c_1J_4\frac{\partial\ddot{w}_0}{\partial y} \tag{3e}$$

其中,$c_1=\dfrac{4}{3h^2}$,$c_2=3c_1$,γ 为横向振动阻尼系数,

$$(A_{ij},B_{ij},D_{ij},E_{ij},F_{ij},H_{ij})=\sum_{k=1}^{3}\int_{z_k}^{z_{k+1}}Q_{ij}^k(1,z,z^2,z^3,z^4,z^6)\mathrm{d}z,$$
$$(i,j=1,2,6)$$
$$(A_{ij},D_{ij},F_{ij})=\sum_{k=1}^{3}\int_{z_k}^{z_{k+1}}Q_{ij}^k(1,z^2,z^4)\mathrm{d}z,\ (i,j=4,5)$$
$$I_i=\sum_{k=1}^{3}\int_{z_k}^{z_{k+1}}\rho_k(z)^i\mathrm{d}z,\ (i=0,1,2,\cdots,6)$$
$$J_i=I_i-c_1I_{i+2},\ K_2=I_2-2c_1I_4+c_1^2I_6$$

2 Galerkin 离散

通过 Hamilton 原理所得到的广义位移运动控制方程组(3)为偏微分形式。为了方便后续的分析和研究,需要使用Galerkin 方法将偏微分方程离散为常微分方程组。

对于面内载荷与横向面外载荷联合作用下一边固支、三边自由的复合材料层合板,横向非线性振动是其主要振动形式,且远大于其面内振动,因此很多学者对悬臂板进行研究时只考虑其横向振动位移,以方便进一步分析[19-20]。本文也将采取这种处理方法,并对横向非线性振动的位移 w 进行二阶离散。

设横向位移 w 模态的函数为

$$w_0=w_1(t)X_1(x)Y_1(y)+w_2(t)X_3(x)Y_1(y) \tag{4}$$

把式(4)带入方程(3c),并在方程两边分别乘以相对应的模态函数后,在整个板内积分,对方程进行无量纲化,利用三角函数的正交性,得到两自由度非线性动力学方程为

$$\ddot{w}_1+\gamma_1\dot{w}_1+\omega_1^2w_1+a_1f_1\cos(\Omega_1 t)w_1+b_1w_2-$$
$$t_2w_1^3-t_4w_2w_1^2-t_3w_2^2w_1-t_1w_2^3=r_1p_1\cos\Omega_2 t \tag{5a}$$
$$\ddot{w}_2+\gamma_2\dot{w}_2+\omega_2^2w_2+a_2f_1\cos(\Omega_1 t)w_2+b_2w_{12}-$$
$$t_5w_2^3-t_7w_1w_2^2-t_6w_1^2w_2-t_4w_1^3=r_2p_2\cos\Omega_2 t \tag{5b}$$

3 摄动分析

采用多尺度法对偏微分方程进行摄动分析,引入小参数 ε,并设方程的解为

$$w_1(t,\varepsilon)=w_{10}(T_0,T_1)+\varepsilon w_{11}(T_0,T_1)w_2(t,\varepsilon)=$$
$$w_{20}(T_0,T_1)+\varepsilon w_{21}(T_0,T_1) \tag{6}$$

其中 $T_0=t$,$T_1=\varepsilon t$。

引入微分算子

$$\frac{\mathrm{d}}{\mathrm{d}t}=\frac{\partial}{\partial T_0}\frac{\partial T_0}{\partial t}+\frac{\partial}{\partial T_1}\frac{\partial T_1}{\partial t}=D_0+\varepsilon D_1 \tag{7a}$$

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}=\frac{\partial^2}{\partial T_0^2}\frac{\partial T_0}{\partial t}+2\varepsilon\frac{\partial^2}{\partial T_0\partial T_1}+\varepsilon^2\frac{\partial^2}{\partial T_1^2}=(D_0+\varepsilon D_1)^2 \tag{7b}$$

考虑主参数共振、1/2亚谐共振和1:2内共振，即

$$\Omega_1=\Omega_2,\omega_1\approx\frac{1}{2}\omega_2\approx\frac{1}{2}\Omega,\omega_1^2=\frac{1}{4}\Omega^2+\varepsilon\sigma_1,\omega_2^2=\Omega^2+\varepsilon\sigma_2 \quad (8)$$

其中 σ_1 和 σ_2 为调谐参数。

得到1:2内共振情况下直角坐标形式的平均方程为

$$\dot{x}_1=-\frac{1}{2}c_1x_1-\left(\sigma_1-\frac{1}{2}a_1f_1\right)x_2-2t_3x_2(x_3^2+x_4^2)+3t_2x_2(x_1^2+x_2^2) \quad (9a)$$

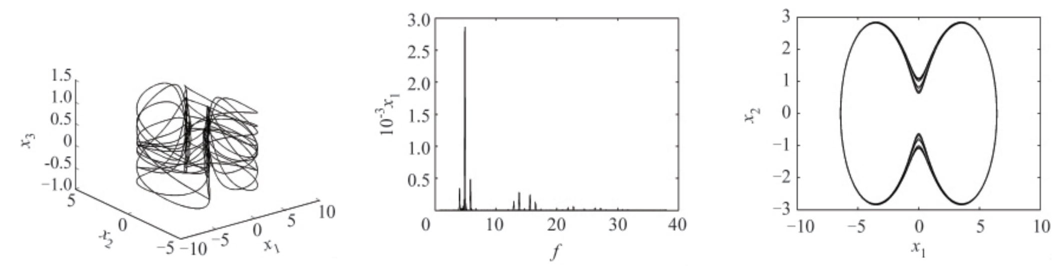
$$\dot{x}_2=-\frac{1}{2}c_1x_2+\left(\sigma_1+\frac{1}{2}a_1f_1\right)x_1-2t_3x_1(x_3^2+x_4^2)-3t_2x_1(x_1^2+x_2^2) \quad (9b)$$

$$\dot{x}_3=-\frac{1}{2}c_2x_3-\frac{1}{2}\sigma_2x_4+\frac{3}{2}t_5x_3(x_3^2-x_4^2)-t_8x_4(x_1^2+x_2^2) \quad (9c)$$

$$\dot{x}_4=-\frac{1}{2}c_2x_4+\frac{1}{2}\sigma_2x_3-\frac{3}{2}t_5x_4(x_4^2-x_3^2)-t_8x_3(x_1^2+x_2^2)-\frac{1}{4}r_2p_2 \quad (9d)$$

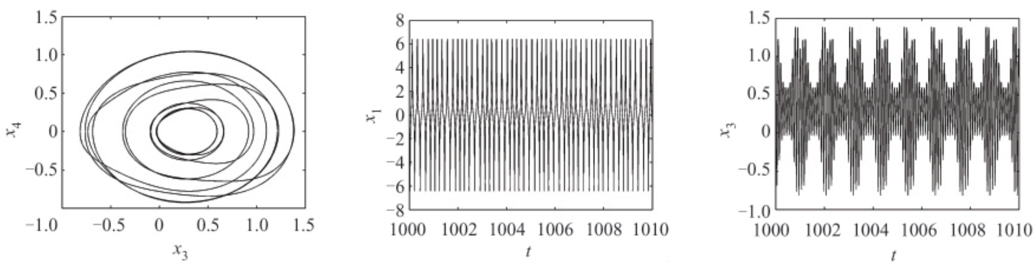
4 数值模拟

数值模拟是研究系统非线性动力学特性的有效方法，本文利用相图、波形图和频谱图反映系统的非线性动力学行为，随参数变化的相图和波形图反映了参数对系统非线性动力学行为的影响。利用Runge-Kutta法对复合材料层合板系统在1/2亚谐共振、1:2内共振情况下的平均方程进行数值仿真，主要研究外激励对复合材料层合板运动的影响。为了研究外激励在复合材料层合板结构的混沌运动中所起的作用，选定一组初始参数，改变外激励 p_2 的大小，研究激励幅值对复合材料层合板运动的影响。所选取的参数和初值分别为 $\mu_1=0.0107$，$\mu_2=0.0529$，$\sigma_1=30$，$\sigma_2=120.098$，$f_1=18.5936$，$a_1=6.2$，$t_2=1.42$，$t_3=0.19$，$t_5=0.25$，$t_8=0.8$，$r_2=1.5$，$x_{10}=0.44$，$x_{20}=0.798$，$x_{30}=0.35$，$x_{40}=0.5$，然后改变外激励 p_2，当激励 $p_2=45.5$ 时系统发生多倍周期运动，如图2所示。持续增大激励幅值到 $p_2=280.5$ 时，系统发生混沌现象，如图3所示。从以上结果能明显地看出，改变横向外激励 p 的幅值可以显著改变系统的振动状态。



(a) 三维相图　(b) 频谱图　(c) 平面(x_1, x_2)二维相图

(a) Three-dimensional phase portrait　(b) Frequency spectrum　(c) Two-dimensional phase portrait in the plane(x_1, x_2)

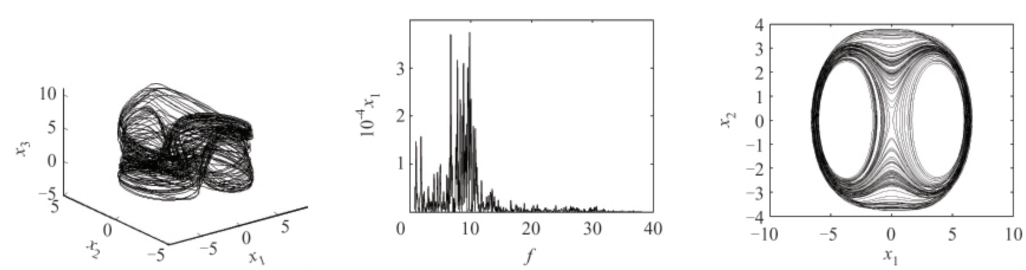


(d) 平面(x_3, x_4)二维相图　(e) 波形图　(f) 波形图

(d) Two-dimensional phase portrait in the plane (x_3, x_4)　(e) Wave form in the plane(t, x_1)　(f) Wave form in the plane(t, x_3)

图2　多倍周期运动

Fig. 2　Period-n solution of the composite laminated cantilever plate

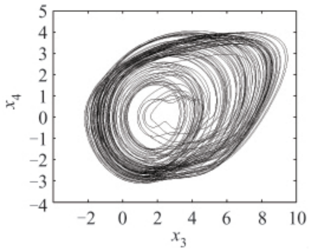


(a) 三维相图　(b) 频谱图　(c) 面(x_1, x_2)二维相图

(a) Three-dimensional phase portrait　(b) Spectrum　(c) Two-dimensional phase portrait in the plane(x_1, x_2)

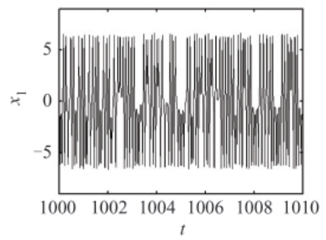
图3　混沌运动

Fig. 3　Chaotic motion of the composite laminat cantilever plate



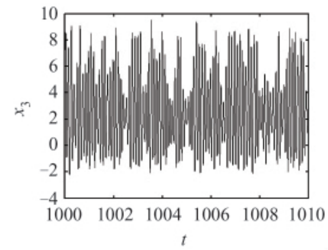
(d) 面(x_3, x_4)二维相图

(d) Two-dimensional phase portrait in the plane (x_3, x_4)



(e) 波形图

(e) Wave form in the plane(t, x_1)



(f) 波形图

(f) Wave form in the plane(t, x_3)

图 3 混沌运动(续)

Fig. 3 Chaotic motion of the composite laminated cantilever plate (continuted)

5 结论

本文利用 Reddy 的高阶剪切理论和 Hamilton 原理建立了面内激励和横向外激励联合作用下的复合材料悬臂板的非线性偏微分运动控制方程，综合运用 Galerkin 方法和多尺度法，得到了系统的平均方程，并用数值方法研究了其非线性动力学行为。

通过分析数值方法获得的三维相图、二维相图、波形图和频谱图，在确定几何和材料参数的情况下，外激励的变化对系统的动力学行为有非常大的影响。在不同取值范围内，可以出现不同的非线性动力学现象。随着激励幅值的变化，系统的动力学响应会呈现多倍周期运动、周期运动、概周期运动，以及混沌运动。由此可见，横向外激励是影响系统非线性动力学行为的主要因素之一，通过改变横向外激励可以对复合材料悬臂板的非线性动力学行为产生显著影响。

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