非线性变速轴向运动黏弹性梁稳态响应

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摘要 研究了非线性变速轴向运动梁稳态幅频响应。变速轴向运动梁的控制微分方程被建立,黏弹性本构关系引入了物质时间导数,考虑了由均匀轴向运动梁变形的影响而导致梁轴向伸长而引起的附加力,并以轴向张力平均值代替梁上各点的精确值,建立了积分-偏微分非线性轴向运动梁的控制方程。轴向运动梁两端的边界为带有扭转弹簧的套筒铰支的混杂边界条件,同时认为轴向运动速度在平均速度附近做微小简谐脉动。应用渐进摄动法直接求解非线性变速轴向运动梁的控制方程并导出了当扰动速度的频率接近未扰系统任意两个固有频率之和时所发生的组合参数共振的稳态幅频响应方程和振幅方程。数值结果给出了轴向运动梁的黏弹性、扰动振幅、非线性对稳态幅频响应的影响。结果显示,轴向运动梁的材料的黏弹性增大时,零平衡位置的失稳区域会减小;当梁的轴向扰动速度幅值增大时,零平衡位置的失稳区域随之增大;稳定及非稳定的两条非零解曲线的振幅都会因为非线性系数的增大而减小。零解失稳范围则不受非线性项的影响。

关键词 轴向变速运动梁;黏弹性;渐进法;参数共振;稳态幅频响应

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Analysis of the Steady-State Response of Nonlinear Axially Moving Viscoelastic Beams with Time-Dependent Velocity

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Abstract Axially accelerating beams can be used to simulate many engineering devices. As a parametric vibration is excited by the variation of the beam tension or the beam axial speed, a large transverse motion of axially moving beams may occur under certain conditions. In this paper, the steady-state response of nonlinear axially moving beams with time-dependent velocity is analyzed. The material time derivative is used in the viscoelastic constitutive relation. Asymptotic analysis is proposed to investigate the governing equation of an axially accelerating viscoelastic beam. Beams are constrained by elastic joints at both ends in all time. The supporting conditions may be formulated as simple supports with torsion springs. The axial speed is characterized as a simple harmonic variation about a constant mean speed. If the axial speed variation frequency approaches the sum of any two natural frequencies, the summation parametric resonance may occur. The stability-state equation and amplitude equation are obtained for the combined parametric resonance via the asymptotic analysis. Numerical examples show the effects of the viscoelasticity, the amplitude of fluctuation and the nonlinearity of the beam on stability-state response.

Keywords axially accelerating beam; asymptotic analysis; parametric resonance; viscoelasticity; stability-state response

0 引言

轴向运动梁可作为多种工程装置的力学模型[1-2]。由于轴

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向速度变化的影响，梁可能出现大幅度的振动，因此，轴向变速运动梁的稳定性问题受到众多学者的关注。非线性变速轴向运动梁稳态响应已有大量的研究。Chen 和 Yang 对积分型轴向运动梁稳态响应和非线性振动做了一系列研究[3-4]。Chen 和 Ding 对于积分型轴向运动梁的稳态响应做了数值验证[5]。本文利用渐进法分析了非线性变速轴向运动梁稳态响应。

1 控制方程

考虑一个均匀黏弹性梁，密度为 ρ、横截面积为 A，惯性矩为 I、初始拉力为 P，在支承两端间长度为 l 上以轴向传输速度 $\gamma(t)$运动，认为梁在平面内只有横向位移 $v(x,\ t)$的弯曲振动，这里 t 为时间，x 为轴向坐标。由于梁变形影响导致梁轴向伸长而引起附加力 $A\sigma$，其中 σ 为由于运动产生的扰动应力，这里以轴向张力平均值代替梁上各点的精确值。黏弹性本构关系选为式(1a)，由牛顿第二定律得到控制方程(1b)

$$\sigma(x,\ z,\ t)=E\varepsilon(x,\ z,\ t)+\eta[\varepsilon(x,\ z,\ t)_{,t}+\gamma\varepsilon(x,\ z,\ t)_{,x}] \tag{1a}$$

$$\rho A(v_{,tt}+\dot{\gamma}v_{,x}+2\gamma v_{,xt}+\gamma^2 v_{,xx})+M_{,xx}=\left[\left(P+\frac{1}{l}\int_0^l A\sigma\mathrm{d}x\right)v_{,x}\right]_{,x} \tag{1b}$$

其中，E 为梁的弹性模量，η 为黏弹性系数。梁横截面上的弯矩 $M(x,\ t)$和轴向应变 $\varepsilon(x,\ z,\ t)$分别定义为

$$M(x,\ t)=-\int_A z\sigma(x,\ z,\ t)\mathrm{d}A \quad \varepsilon(x,\ z,\ t)=-z\frac{\partial^2 v(x,\ t)}{\partial x^2} \quad \varepsilon_L=\frac{1}{2}v_{,x}^2 \tag{2}$$

其中，平面$(z,\ x)$为弯曲主平面，$\sigma(x,z,\ t)$为分布正应力。将式(2)代入式(1)，得到无量纲控制方程

$$v_{,tt}+\dot{c}v_{,x}+2cv_{,xt}+(c^2-1)v_{,xx}+k_f^2 v_{,xxxx}+\varepsilon\alpha v_{,xxxxt}+\varepsilon\alpha c v_{,xxxxx}=v_{,xx}\cdot$$
$$\int_0^l\left(\frac{1}{2}\varepsilon k_1^2 v_{,x}^2+\varepsilon^2 k_2 v_{,x}v_{,xt}+\varepsilon^2 k_2\gamma v_{,x}v_{,xx}\right)\mathrm{d}x \tag{3}$$

其中，

$$\begin{cases} v\leftrightarrow\dfrac{v}{\sqrt{\varepsilon l}} \quad x\leftrightarrow\dfrac{x}{l} \quad t\leftrightarrow t\sqrt{\dfrac{P}{\rho A l^2}} \quad c=\gamma\sqrt{\dfrac{\rho A}{P}} \\ \alpha=\dfrac{\eta I}{\varepsilon l^3\sqrt{P\rho A}} \quad v_f^2=\dfrac{EI}{Pl^2} \quad k_1=\sqrt{\dfrac{AE}{P}} \quad k_2=\dfrac{A\eta}{\varepsilon l\sqrt{\rho AP}} \end{cases} \tag{4}$$

式中，ε 为无量纲小参数。考虑梁的两端为带有扭转弹簧的套筒铰支的混杂无量纲边界条件

$$\begin{cases} v(0,\ t)=0 \quad v_{,xx}(0,\ t)-kv_{,x}(0,\ t)=0 \\ v(1,\ t)=0 \quad v_{,xx}(1,\ t)+kv_{,x}(1,\ t)=0 \end{cases} \tag{5}$$

假设轴向运动速度在平均速度附近做微小简谐脉动

$$c(t)=c_0+\varepsilon c_1\sin\omega t \tag{6}$$

其中，c_0 为平均轴向速度，c_1 和 ω 分别为轴向速度变化的振幅和频率。

将式(6)代入式(3)得

$$Mv_{,tt}+Gv_{,t}+Kv=\varepsilon(\omega c_1\cos\omega t v_{,x}+2c_1\sin\omega t v_{,xt}+$$
$$c_1\sin\omega t^2 v_{,xx}+2c_0c_1\sin\omega t v_{,xx}+\alpha v_{,xxxxt}+\alpha\gamma v_{,xxxxx})-$$
$$v_{,xx}\int_0^l\left[\frac{1}{2}\varepsilon k_1^2 v_{,x}^2+\varepsilon^2 k_2 v_{,x}v_{,xt}+\varepsilon^2 k_2(c_0+\varepsilon c_1\sin\omega t)v_{,x}v_{,xx}\right]\mathrm{d}x \tag{7}$$

其中，质量算子 M、陀螺算子 G 及刚度算子 K 定义为

$$M=I \quad G=2c_0\frac{\partial}{\partial x} \quad K=(c_0^2-1)\frac{\partial^2}{\partial x^2}+v_f^2\frac{\partial^4}{\partial x^4} \tag{8}$$

这里算子 M 和 K 为对称正定算子，陀螺算子 G 为斜对称算子。

2 渐进分析

在边界条件(5)下，关于方程(7)的线性系统

$$Mv_{,tt}+Gv_{,t}+Kv=0 \tag{9}$$

的固有频率 $\omega_k(k=m,\ n)$可以被确定。如果轴向速度振动频率 ω 接近系统任意两个固有频率之和，将会发生组合共振。为了定量描述 ω 和 $\omega_m+\omega_n$ 的接近程度，引进一个由

$$\omega=\omega_m+\omega_n+\varepsilon\sigma \tag{10}$$

定义的解谐参数 $\mu=O(1)$。假设式(7)解的形式为

$$v(x,\ t)=\psi_m(x,\ \tau;\ \varepsilon)\mathrm{e}^{\mathrm{i}\omega_m t}+\psi_n(x,\ \tau;\ \varepsilon)\mathrm{e}^{\mathrm{i}\omega_n t}+cc \tag{11}$$

其中，cc 表示前面所有项的共轭；引入慢变时间尺度 $\tau=\varepsilon t$；函数 $\psi_k(x,\ \tau;\ \varepsilon)$取决于参数 ε，假设 ψ_k 在 $\varepsilon\to 0$ 时的极限存在且有界，可将 $\psi_k(x,\tau,\varepsilon)$展开成 ε 的幂级数形式为

$$\psi_k=\psi_{0k}+\varepsilon\psi_{1k} \quad (k=m,\ n) \tag{12}$$

将式(11)代入式(5)，并令 $\mathrm{e}^{\mathrm{i}\omega_k t}$ 的系数分别在 ε^0 和 ε^1 相等，分别导出 ε^0 和 ε^1 的边界条件为

$$\begin{cases} \psi_{0k}(0,\ \tau)=0 \quad \psi''_{0k}(0,\ t)-k\psi'_{0k}(0,\ t)=0 \\ \psi_{0k}(1,\ \tau)=0 \quad \psi''_{0k}(1,\ t)+k\psi'_{0k}(1,\ t)=0 \end{cases} \quad (k=m,\ n) \tag{13}$$

将式(11)代入式(7)，并令 $\mathrm{e}^{\mathrm{i}\omega_k t}$ 的系数分别在 ε^0 阶和 ε^1 阶相等，得到

$$-\omega_k^2 M\psi_{0k}+\mathrm{i}\omega_k G\psi_{0k}+K\psi_{0k}=0 \quad (k=m,\ n) \tag{14}$$

$$-\omega_k^2 M\psi_{1k}+\mathrm{i}\omega_k G\psi_{1k}+K\psi_{1k}=-2\mathrm{i}\omega_k\dot{\psi}_{0k}-2c_0\dot{\psi}'_{0k}+\frac{1}{2}k_1^2\bar{\psi}''_{0k}\int_0^1(\psi'_{0k})^2\mathrm{d}x+$$
$$k_1^2\bar{\psi}''_{0j}\int_0^1\psi'_{0j}\psi'_{0k}\mathrm{d}x+k_1^2\psi''_{0j}\int_0^1\psi'_{0k}\bar{\psi}'_{0j}\mathrm{d}x+k_1^2\psi''_{0k}\int_0^1\psi'_{0j}\bar{\psi}'_{0j}\mathrm{d}x+$$
$$k_1^2\psi''_{0k}\int_0^1\psi'_{0k}\bar{\psi}'_{0k}\mathrm{d}x+\frac{1}{2}[(\omega_j-\omega_k)\bar{\psi}'_{0j}+2\mathrm{i}c_0\bar{\psi}''_{0j}]\gamma_1\mathrm{e}^{\mathrm{i}\sigma\tau}-$$
$$\alpha\left(\mathrm{i}\omega_k\psi_{0k}^{(4)}+c_0\psi_{0k}^{(5)}\right) \quad (k=m,\ n;\ j=n,\ m) \tag{15}$$

设方程(14)的解为

$$\psi_{0k}(x,\ \tau)=q_k(\tau)\phi_k(x) \quad (k=m,\ n) \tag{16}$$

将方程(16)分别代入方程(13)~(15)，得到

$$\begin{matrix} \phi_k(0)=0 \quad \phi''_k(0)-k\phi'_k(0)=0 \\ \phi_k(1)=0 \quad \phi''_k(1)-k\phi'_k(1)=0 \end{matrix} \quad (k=m,\ n) \tag{17}$$

$$-\omega_k^2 M\phi_k+\mathrm{i}\omega_k G\phi_k+K\phi_k=0 \quad (k=m,\ n) \tag{18}$$

$$-\omega_k^2 M\psi_{1k}+\mathrm{i}\omega_k G\psi_{1k}+K\psi_{1k}=-2(\mathrm{i}\omega_k\phi_k+\gamma_0\phi'_k)\dot{q}_k-\mathrm{i}\omega_k\eta\phi_k^{(4)}q_k-$$
$$\gamma_0\eta\phi_k^{(5)}q_k+\frac{1}{2}k_1^2 q_k^2\bar{q}_k\left(2\phi''_k\int_0^1\phi'_k\bar{\phi}'_k\mathrm{d}x+\bar{\phi}''_k\int_0^1\phi'^2_k\mathrm{d}x\right)+$$
$$\frac{1}{2}\gamma_1\mathrm{e}^{\mathrm{i}t\sigma}[(\omega_j-\omega_k)\bar{\phi}'_j+2\mathrm{i}\gamma_0\bar{\phi}''_j]\bar{q}_j+k_1^2 q_j\bar{q}_j q_k\left(\phi''_k\int_0^1\phi'_j\bar{\phi}'_j\mathrm{d}x+\right.$$
$$\left.\phi''_j\int_0^1\phi'_k\bar{\phi}'_j\mathrm{d}x+\bar{\phi}''_j\int_0^1\phi'_k\phi'_j\mathrm{d}x\right)$$
$$(k=m,\ n;\ j=n,\ m) \tag{19}$$

其中,在边界条件(17)下,方程(18)的解 $\phi_k(x)$在文献[6]已经给出;方程(19)必须满足如下可解性条件才能得到有界解[7]。

$$\langle -\omega_k^2 M\psi_{1k}+\mathrm{i}\omega_k G\psi_{1k}+K\psi_{1k},\ \phi_k\rangle=0 \qquad (k=m,\ n) \tag{20}$$

根据可解性条件(20),使用函数 $\phi_k(x)$对方程(19)两边进行内积,得到

$$\dot{q}_k+(\eta\mu_k+k_1^2F_{kj}q_j\bar{q}_j)q_k+k_1^2\kappa_k q_k^2\bar{q}_k+\mathrm{e}^{\mathrm{i}t\sigma}\gamma_1\chi_k\bar{q}_j=0 \quad (k=m,\ n;\ j=n,\ m) \tag{21}$$

其中

$$\begin{cases}\mu_k=\dfrac{\gamma_0\int_0^1\bar{\phi}_k\phi_k^{(5)}\mathrm{d}x+\mathrm{i}\omega_k\int_0^1\bar{\phi}_k\phi_k^{(4)}\mathrm{d}x}{2\left(\gamma_0\int_0^1\bar{\phi}_k\phi_k'\mathrm{d}x+\mathrm{i}\omega_k\int_0^1\bar{\phi}_k\phi_k\mathrm{d}x\right)} \qquad \kappa_k=\dfrac{\int_0^1\bar{\phi}_k'\phi_k'\mathrm{d}x\int_0^1\bar{\phi}_k\phi_k''\mathrm{d}x+\frac{1}{2}\int_0^1\phi_k'^2\mathrm{d}x\int_0^1\bar{\phi}_k\bar{\phi}_k''\mathrm{d}x}{2\left(\gamma_0\int_0^1\bar{\phi}_k\phi_k'\mathrm{d}x+\mathrm{i}\omega_k\int_0^1\bar{\phi}_k\phi_k\mathrm{d}x\right)}\\[2ex] F_{kj}=\dfrac{\int_0^1\bar{\phi}_j'\phi_j'\mathrm{d}x\int_0^1\bar{\phi}_k\phi_k''\mathrm{d}x+\int_0^1\bar{\phi}_j'\phi_k'\mathrm{d}x\int_0^1\bar{\phi}_k\phi_j''\mathrm{d}x+\int_0^1\phi_k'\phi_j'\mathrm{d}x\int_0^1\bar{\phi}_k\bar{\phi}_j''\mathrm{d}x}{2\left(\gamma_0\int_0^1\bar{\phi}_j\phi_j'\mathrm{d}x+\mathrm{i}\omega_j\int_0^1\bar{\phi}_j\phi_j\mathrm{d}x\right)} \qquad \chi_k=\dfrac{\mathrm{i}\gamma_0\int_0^1\bar{\phi}_k\phi_j''\mathrm{d}x+\frac{1}{2}(\omega_j-\omega_k)\int_0^1\bar{\phi}_k\bar{\phi}_j'\mathrm{d}x}{2\left(\gamma_0\int_0^1\bar{\phi}_j\phi_j'\mathrm{d}x+\mathrm{i}\omega_j\int_0^1\bar{\phi}_j\phi_j\mathrm{d}x\right)}\end{cases} \tag{22}$$

数值试验表明,各系数的实部和虚部满足

$$\begin{cases}\mathrm{Re}(\mu_n)>0 \quad \mathrm{Im}(\mu_n)=0\\ \mathrm{Re}(F_{jk})=0 \quad \mathrm{Im}(F_{jk})<0\\ \mathrm{Re}(\kappa_n)=0 \quad \mathrm{Im}(\kappa_n)<0\\ \dfrac{\mathrm{Im}(\chi_m)}{\mathrm{Im}(\chi_n)}=\dfrac{\mathrm{Re}(\chi_m)}{\mathrm{Re}(\chi_n)}=g\end{cases} \tag{23}$$

将判定方程(21)的解写成极坐标形式

$$q_m=\alpha_m\mathrm{e}^{\mathrm{i}\beta_m} \qquad q_n=\alpha_n\mathrm{e}^{\mathrm{i}\beta_n} \tag{24}$$

其中,α_k 和 β_k 分别表示第 k 阶亚谐波共振响应的幅值和相角,是有关 τ 的实数函数。将方程(24)代入方程(21)并分离实部和虚部得到

$$\dot{\alpha}_m=\gamma_1\alpha_n\mathrm{Im}(\chi_m)\sin\theta-\gamma_1\alpha_n\mathrm{Re}(\chi_m)\cos\theta-\eta\alpha_m\mathrm{Re}(\mu_m) \tag{25}$$

$$\dot{\alpha}_n=\gamma_1\alpha_m\mathrm{Im}(\chi_n)\sin\theta-\gamma_1\alpha_m\mathrm{Re}(\chi_n)\cos\theta-\eta\alpha_n\mathrm{Re}(\mu_n) \tag{26}$$

$$\alpha_n\dot{\beta}_m=-k_1^2\mathrm{Im}(\kappa_m)\alpha_m^3-k_1^2\mathrm{Im}(F_{mn})\alpha_m\alpha_n^2-\gamma_1\alpha_n\mathrm{Im}(\chi_m)\cos\theta-\gamma_1\alpha_n\mathrm{Re}(\chi_m)\sin\theta \tag{27}$$

$$\alpha_i\dot{\beta}_n=-k_1^2\mathrm{Im}(\kappa_n)\alpha_n^3-k_1^2\mathrm{Im}(F_{nm})\alpha_n\alpha_m^2-\gamma_1\alpha_m\mathrm{Im}(\chi_n)\cos\theta-\gamma_1\alpha_m\mathrm{Re}(\chi_n)\sin\theta \tag{28}$$

其中

$$\theta=\sigma t-\beta_m-\beta_n \tag{29}$$

3 组合共振稳态响应

对于稳态响应,振幅 $\alpha_k(k=m,\ n)$和新相角 θ 均为常数。令方程(25)和(26)中$\dot{\alpha}_k=0$,得到

$$\gamma_1\alpha_n\mathrm{Im}(\chi_m)\sin\theta-\gamma_1\alpha_n\mathrm{Re}(\chi_m)\cos\theta-\eta\alpha_m\mathrm{Re}(\mu_m)=0 \tag{30}$$

$$\gamma_1\alpha_m\mathrm{Im}(\chi_n)\sin\theta-\gamma_1\alpha_m\mathrm{Re}(\chi_n)\cos\theta-\eta\alpha_n\mathrm{Re}(\mu_n)=0 \tag{31}$$

及令$\dot{\theta}=0$,由方程(29)得到$\dot{\beta}_m+\dot{\beta}_n=\sigma$。令方程(27)两端乘 α_n 和方程(28)两端乘 α_m,之后并将两个导出的方程相加得到

$$\alpha_n\alpha_m\sigma=-k_1^2\mathrm{Im}(\kappa_m)\alpha_m^3\alpha_n-k_1^2\mathrm{Im}(F_{mn})\alpha_m\alpha_n^3-\gamma_1\alpha_n^2\mathrm{Im}(\chi_m)\cos\theta-\gamma_1\alpha_n^2\mathrm{Re}(\chi_m)\sin\theta-k_1^2\mathrm{Im}(\kappa_n)\alpha_n^3\alpha_m-k_1^2\mathrm{Im}(F_{nm})\alpha_n\alpha_m^3-\gamma_1\alpha_m^2\mathrm{Im}(\chi_n)\cos\theta-\gamma_1\alpha_m^2\mathrm{Re}(\chi_n)\sin\theta \tag{32}$$

将方程(23)代入(30)后,消去方程(30)和(31)中的 $\mathrm{Im}(\chi n)\sin\theta-\mathrm{Re}(\chi_n)\cos\theta$ 得到

$$\frac{\alpha_m^2}{\alpha_n^2}=g\frac{\mathrm{Re}(\mu_n)}{\mathrm{Re}(\mu_m)} \tag{33}$$

将方程(50)代入(49)后,消去方程(47)和(49)中的 θ,将方程(51)代入导出的方程,得到稳态响应的振幅方程:

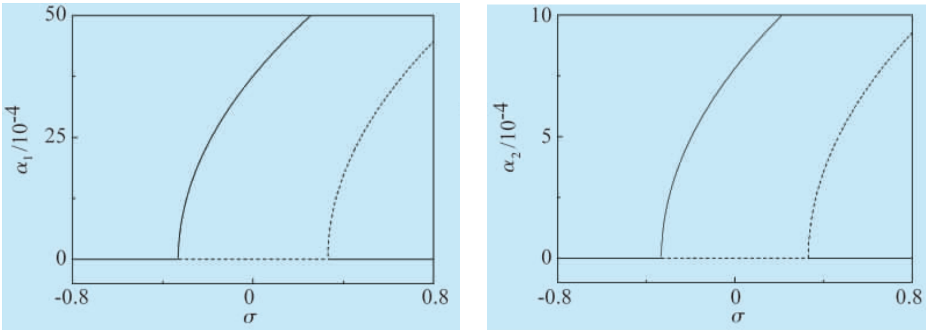
$$s_1a_n^{10}+s_2a_n^8+s_3a_n^6=0 \tag{34}$$

其中

$$\begin{cases}s_1=\dfrac{g^2k_1^4c_1^2}{(\mu_m^R)^4}(\mu_n^R)^2\left[(F_{mn}^I+\kappa_n^I)\mu_m^R+g(F_{nm}^I+\kappa_m^I)\mu_n^R\right]^2\\ s_2=\dfrac{2s_1\sigma\mu_m^R}{k_1^2\left[(F_{mn}^I+\kappa_n^I)\mu_m^R+g(F_{nm}^I+\kappa_m^I)\mu_n^R\right]}\\ s_3=\dfrac{g^2c_1^2\mu_n^R}{(\mu_m^R)^3}\left\{-gc_1^2(\mu_m^R+\mu_n^R)^2\left[(\chi_n^I)^2+(\chi_n^R)^2\right]+\right.\\ \qquad \left.\mu_m^R\mu_n^R\left[\sigma^2+\alpha^2(\mu_m^R+\mu_n^R)^2\right]\right\}\end{cases} \tag{35}$$

4 数值结果与结论

本文研究了非线性变速轴向运动梁稳态幅频响应。数值结果如图 1~图 4 所示。图中显示:① 梁材料的黏弹性增大时,零平衡位置的失稳区域会减小;② 当梁的轴向扰动速度幅值增大时,零平衡位置的失稳区域随之增大;③ 稳定和非稳定的两条非零解曲线的振幅都会因为非线性系数的增大而减小,零解失稳范围则不会受非线性项的影响。



(a) 第一阶

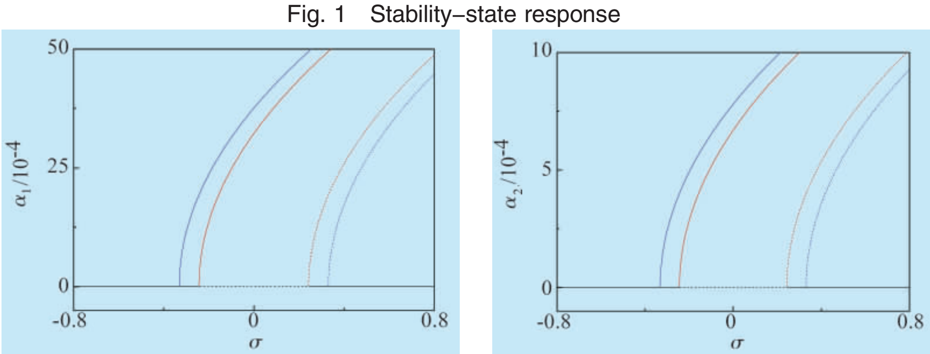
(a) The first step

(b) 第二阶

(b) The second step

图 1 稳态幅频响应

Fig. 1 Stability–state response



(a) 第一阶

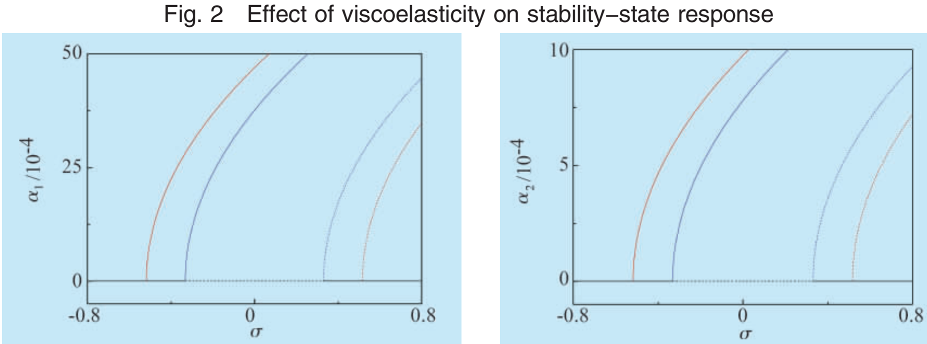
(a) The first step

(b) 第二阶

(b) The second step

图 2 稳态幅频响应黏弹性影响

Fig. 2 Effect of viscoelasticity on stability–state response



(a) 第一阶

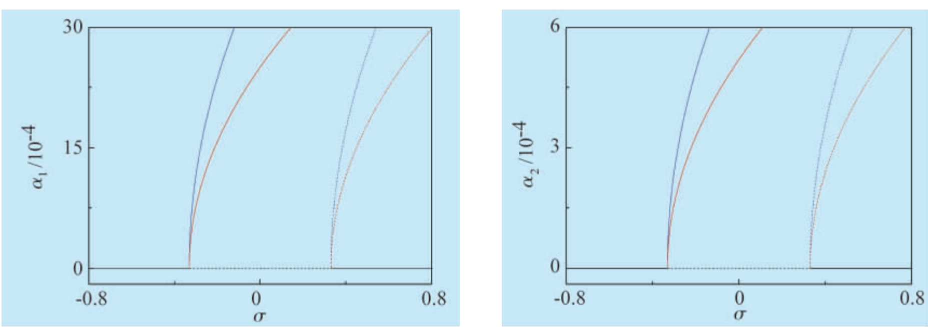
(a) The first step

(b) 第二阶

(b) The second step

图 3 稳态幅频响应扰动振幅影响

Fig. 3 Effect of the amplitude of fluctuation on stability–state response



(a) 第一阶

(a) The first step

(b) 第二阶

(b) The second step

图 4 稳态幅频响应非线性影响

Fig. 4 Effect of nonlinear term on stability–state response

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