功能梯度材料变转速旋转悬臂板的非线性动力学研究

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摘要 以航空发动机压气机叶片为实际工程背景,将叶片简化为功能梯度材料的旋转悬臂板模型。基于 Reddy 的高阶剪切变形理论和 von Karman 的大变形理论,考虑了变转速和离心力的作用,由一阶活塞理论得到气动力的表达式,利用 Hamilton 原理建立了系统的非线性动力学方程。应用 Galerkin 离散法进行二阶离散得到系统的常微分控制方程。考虑系统 1:1 内共振和主参数共振的情况,利用渐进摄动法得到了旋转悬臂板系统四维直角坐标形式的平均方程。通过数值仿真研究了变转速旋转悬臂板结构的复杂非线性振动响应。结果表明,叶片转速的变化对系统动力学特性有着重要的影响,在不同的转速下,系统存在着周期运动、多倍周期运动和混沌运动等多种复杂非线性动力学行为。

关键词 旋转悬臂板;变转速;功能梯度材料;混沌

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Nonlinear Dynamics of Rotating Cantilever FGM Rectangular Plate with Varying Rotating Speed

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Abstract In the study of the aero-engine compressor blades, the blade is simplified as a rotating cantilever FGM plate with varying rotating speed. Based on the Reddy´s high-order shear deformation theory and the von Karman type equations for the geometric nonlinearity, the nonlinear governing partial differential equations of motion are derived by using the Hamilton´s principle. The aerodynamic load is determined by using the first-order piston theory. The Galerkin approach is used to transform the nonlinear partial differential governing equations of motion into a two-degree-of-freedom nonlinear system. The principal parametric resonance and the 1:1 internal resonance are considered. The asymptotic perturbation method is used to obtain a four-dimensional nonlinear averaged equation. The numerical method is used to find the nonlinear dynamic responses of the rotating cantilever FGM plate. It is found the rotating speed has an important influence on its nonlinear dynamic behavior. It is shown that, at different rotating speeds, there exist the chaotic, periodic and quasi-periodic motions for the rotating cantilever FGM plate.

Keywords rotating cantilever plate; varying rotating speed; functionally graded materials; chaos

0 引言

航空发动机压气机第一级叶片是高速旋转机构,受到横向剪切、气动力等因素的影响,会发生大幅非线性振动,使叶片的可靠性下降,直接影响发动机和飞机的安全性。研究此类板结构的非线性动力学特性,对工程设计有重要的理论和应用价值。2003 年,盛国刚[1]利用 Hamilton 原理建立了支承运动情况下变转速旋转梁的振动方程,运用梁端部弹性振动的相轨迹分析了该时变系统的稳定性,给出了变转速情况下旋

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转梁的稳定性分析算例。Oh 和 Librescu[2]建立了功能梯度材料旋转薄壁梁的热弹性模型,考虑了材料的陶瓷金属含量在厚度方向的变化。邓峰岩等[3]针对旋转 Timoshenko 梁结构,在梁的纵向、横向变形中均考虑横向弯曲以及轴向伸缩的耦合作用,同时在横向弯曲中考虑剪切变形的影响,利用 Hamilton 原理建立非线性变形模式下的一种新的 Timoshenko 梁动力学方程。王旭东和汤文成[4]建立了轴流风机叶轮结构的有限元模型,计算得到了叶轮的固有频率和振型。Hashemi 等[5]将类似叶片的旋转机构简化为矩形板,利用 Mindlin 板理论和凯恩法得到了旋转厚板的非线性运动方程。Piovan 和 Sampaio[6]研究了旋转梁的动力学特性, 梁的材质是功能梯度材料;文章研究了剪切变形和非线性应变之间的关系,其中忽略了热效应,着重于阻尼效应和几何刚度,用有限元法离散模型得到运动方程的数值近似结果,但没有考虑气动载荷。冯景等[7]考虑水平轴风力发电机的叶片在旋转过程中所受的气动力、弹性力和惯性力的综合作用,建立了脉动风速下叶片的旋转悬臂梁力学模型。刘雄等[8]把叶片简化为悬臂梁,利用二结点梁单元进行离散化建模,获得了水平轴风力机叶片在时变载荷作用下的动力响应。Farhadi[9]研究了旋转矩形板在超音速流下的动力学行为,利用一阶活塞理论得到气动载荷,继而利用 Mindlin 板理论和凯恩法得到了旋转板动力学方程。Yao M.H.等[10]将高速旋转叶片简化成固定在刚性轮毂上旋转的薄壁梁模型,并考虑叶片的预扭转角和预安装角、大变形的几何非线性、离心力和气动载荷的影响,用数值方法分析了高速旋转叶片系统的非线性动力学行为。

功能梯度材料作为一种新兴材料, 相对于传统材料,有许多无法比拟的优越性,因此越来越广泛地应用于航空航天结构的工程实际中,与其相关的研究和应用也得到了迅速发展。Reddy[11]研究了功能梯度材料板的振动特性,基于高阶剪切理论和 von Karman 大变形理论,得到了在厚度方向变化的功能梯度材料板的有限元模型。李永[12]考虑材料非均质性,采用分层剪切变形理论对功能梯度材料层梁进行分析,建立了功能梯度材料悬臂梁模型,得出了梁结构的变形与应力之间的关系。2003 年,Yang 等[13]研究了具有预应力的功能梯度材料层合板的大幅振动, 基于 Reddy 的高阶剪切变形板理论,得到了功能梯度材料层合板的运动控制方程,同时考虑了横向剪应变和转动惯量。Kitipornchai 等[14]研究了具有缺陷的剪切变形层合矩形板(包括一个均匀中间层和两个功能梯度材料表层)的非线性振动。此研究基于 Reddy 高阶剪切变形板理论,包含了中面转动、挠度以及应力函数;用半解析法与 Galerkin 法将偏微分方程化为常微分方程, 并求得各种边界条件下的解。Chen[15]给出了具有非均匀初始应力的功能梯度材料矩形板非线性振动的非线性偏微分方程。功能梯度材料矩形板的材料特性沿厚度方向呈梯度变化,体分比沿梯度方向变化为幂律分布。方程中包含了剪切变形和转动惯量的影响。Woo[16]研究了功能梯度材料板的非线性自由振动,利用 von Karman 大变形理论得到了功能梯度材料矩形薄板的动力学方程,研究表明非线性耦合效应在功能梯度板的振动中有重要影响。郭翔鹰等[17]研究了四边简支纤维增强复合材料层合薄板在受到 x 轴方向面内激励和横向激励联合作用下的非线性动力学问题。Hao Y.X.等[18]研究了功能梯度材料悬臂板在横向外激励作用下的非线性特性。结果表明,随着外激励的增加,板出现混沌、周期、概周期等不同的运动形式。

以上研究中,将叶片处理为考虑气动力、离心力和变转速的功能梯度材料旋转悬臂板模型的文献还不多见。本文首先利用 Hamilton 原理, 基于高阶剪切理论、von Karman 大变形理论和一阶活塞理论建立了旋转悬臂板系统的非线性动力学方程; 然后利用 Galerkin 方法和渐进摄动法得到了 1:1 内共振情况下的平均方程;最后通过数值模拟分析旋转叶片的混沌和周期运动。对于旋转机械,转速在系统的振动特性中起着重要作用,因此选取转速为控制参量。结果表明,在不同的转速下,旋转叶片存在周期和混沌等不同的运动形式。

1 功能梯度材料变转速旋转悬臂板的动力学模型

本文以航空涡轮发动机压气机叶片为工程背景,将叶片简化为图 1 所示的旋转悬臂板结构。功能梯度板位于带基座的轮毂上,基座平面距轮毂中心距离为 R_0,轮毂旋转速度为 Ω_r。转速受到周期的微扰动,因此,转速可以表示为 $\Omega_r=\Omega_0+f\cos(\Omega t)$,其中 f 和 Ω 分别为速度扰动的幅值和频率。功能梯度板的长、宽、高分别为 a、b、h。直角坐标系 oxy 位于功能梯度板的中性面内,z 轴向上,e_1,e_2 和 e_3 分别为 x,y 和 z 轴的方向向量。气动力方向如图 2 所示,相对于叶片旋转运动方向。

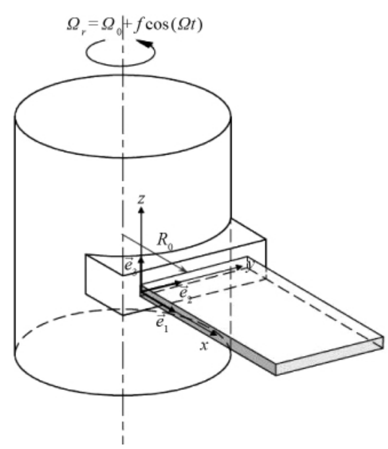


图 1 模型示意图

Fig. 1 The profile of the model

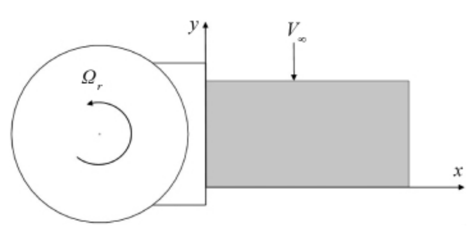


图 2 气流相对于叶片的运动方向

Fig. 2 Direction of air flow

根据 Reddy 高阶剪切变形板理论，功能梯度材料矩形板的位移场可以表示为

$$u(x,y,z,t)=u_0(x,y,t)+z\phi_x(x,y,t)-c_1z^3\left(\phi_x+\frac{\partial w_0}{\partial x}\right) \quad (1)$$

$$v(x,y,z,t)=v_0(x,y,t)+z\phi_y(x,y,t)-c_1z^3\left(\phi_y+\frac{\partial w_0}{\partial y}\right) \quad (2)$$

$$w(x,y,z,t)=w_0(x,y,t) \quad (3)$$

其中设 u,v 和 w 分别为板内任意一点沿 x,y 和 z 方向的位移；u_0,v_0,w_0 分别为板中面内任意一点在 3 个坐标方向的位移；ϕ_x,ϕ_y 表示板中面法线相对 y,x 轴的转角；$c_1=\frac{4}{3h^2}$。

由于结构产生几何大变形，所以应变与位移的关系取为 von Karman 型非线性几何关系

$$\varepsilon_{xx}=\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2,\quad \varepsilon_{yy}=\frac{\partial v}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2,$$

$$\gamma_{xy}=\frac{1}{2}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right),$$

$$\gamma_{yz}=\frac{1}{2}\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right),\gamma_{zx}=\frac{1}{2}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right) \quad (4)$$

功能梯度材料板的本构关系为

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{yz}\\ \sigma_{zx}\\ \sigma_{xy}\end{Bmatrix}=\begin{bmatrix}Q_{11}&Q_{12}&0&0&0\\ Q_{21}&Q_{22}&0&0&0\\ 0&0&Q_{44}&0&0\\ 0&0&0&Q_{55}&0\\ 0&0&0&0&Q_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{yz}\\ \gamma_{zx}\\ \gamma_{xy}\end{Bmatrix} \quad (5)$$

功能梯度材料板的薄膜刚度 A_{ij}、耦合刚度 B_{ij}、弯曲刚度 D_{ij} 及高阶刚度 E_{ij}、F_{ij}、H_{ij} 由下式计算

$$(A_{ij},B_{ij},D_{ij},E_{ij},F_{ij},H_{ij})=\int_{-h/2}^{h/2}Q_{ij}(1,z,z^2,z^3,z^4,z^6)\mathrm{d}z,\quad i,j=1,2,6 \quad (6)$$

$$(A_{ij},D_{ij},F_{ij})=\int_{-h/2}^{h/2}Q_{ij}(1,z^2,z^4)\mathrm{d}z,\ i,j=4,5 \quad (7)$$

由 Hamilton 原理，得到由广义位移 u_0,v_0,w_0,ϕ_x,ϕ_y 表示的功能梯度材料板的非线性动力学方程为

$$\begin{aligned}&A_{11}\frac{\partial^2u_0}{\partial x^2}+A_{66}\frac{\partial^2u_0}{\partial y^2}+(A_{12}+A_{66})\frac{\partial^2v_0}{\partial x\partial y}+A_{11}\frac{\partial w_0}{\partial x}\frac{\partial^2w_0}{\partial x^2}\\&+A_{66}\frac{\partial w_0}{\partial x}\frac{\partial^2w_0}{\partial y^2}+(A_{12}+A_{66})\frac{\partial w_0}{\partial y}\frac{\partial^2w_0}{\partial x\partial y}-c_1E_{11}\frac{\partial^3w_0}{\partial x^3}\\&-c_1(E_{12}+2E_{66})\frac{\partial^3w_0}{\partial x\partial y^2}+(B_{11}-c_1E_{11})\frac{\partial^2\phi_x}{\partial x^2}\\&+(B_{66}-c_1E_{66})\frac{\partial^2\phi_x}{\partial y^2}+(B_{12}-c_1E_{12}+B_{66}-c_1E_{66})\frac{\partial^2\phi_y}{\partial x\partial y}\\&=-2\dot{\Omega}_rI_0\dot{v}_0-2\Omega_r(I_1-c_1I_3)\dot{\phi}_y+\dot{\Omega}_rc_1I_3\frac{\partial w_0}{\partial y}+2\Omega_rc_1I_3\frac{\partial\dot{w}_0}{\partial y}\\&-\dot{\Omega}_r[I_0(y-\frac{b}{2})+I_0v_0+(I_1-c_1I_3)\phi_y]\end{aligned} \quad (8)$$

$$\begin{aligned}&A_{22}\frac{\partial^2v_0}{\partial y^2}+A_{66}\frac{\partial^2v_0}{\partial x^2}+(A_{66}+A_{21})\frac{\partial^2u_0}{\partial x\partial y}+A_{22}\frac{\partial w_0}{\partial y}\frac{\partial^2w_0}{\partial y^2}\\&+A_{66}\frac{\partial w_0}{\partial y}\frac{\partial^2w_0}{\partial x^2}+(A_{21}+A_{66})\frac{\partial w_0}{\partial x}\frac{\partial^2w_0}{\partial x\partial y}-c_1E_{22}\frac{\partial^3w_0}{\partial y^3}\\&-c_1(E_{21}+2E_{66})\frac{\partial^3w_0}{\partial x^2\partial y}+(B_{22}-c_1E_{22})\frac{\partial^2\phi_y}{\partial y^2}\\&+(B_{66}-c_1E_{66})\frac{\partial^2\phi_y}{\partial x^2}+(B_{21}-c_1E_{21}+B_{66}-c_1E_{66})\frac{\partial^2\phi_x}{\partial x\partial y}\\&=-c_1I_3\frac{\partial\ddot{w}_0}{\partial y}+I_0\ddot{v}+(I_1-c_1I_3)\ddot{\phi}_y+\dot{\Omega}_r(R_0+x)\end{aligned} \quad (9)$$

$$\begin{aligned}&(A_{55}-2c_2D_{55}+c_2^2F_{55})\frac{\partial^2w_0}{\partial x^2}+(A_{44}-2c_2D_{44}+c_2^2F_{44})\frac{\partial^2w_0}{\partial y^2}\\&+c_1(2E_{66}-E_{12}-E_{21})\frac{\partial^2w_0}{\partial x^2}\frac{\partial^2w_0}{\partial y^2}\\&+c_1(E_{12}+E_{21}-2E_{66})\left(\frac{\partial^2w_0}{\partial x\partial y}\right)^2\\&+(4A_{66}+A_{21}+A_{12})\frac{\partial^2w_0}{\partial x\partial y}\frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}\\&+\left(\frac{1}{2}A_{12}+A_{66}\right)\frac{\partial^2w_0}{\partial x^2}\left(\frac{\partial w_0}{\partial y}\right)^2\\&+\left(\frac{1}{2}A_{21}+A_{66}\right)\left(\frac{\partial w_0}{\partial x}\right)^2\frac{\partial^2w_0}{\partial y^2}+\frac{3}{2}A_{22}\left(\frac{\partial w_0}{\partial y}\right)^2\frac{\partial^2w_0}{\partial x^2}\\&+\frac{3}{2}A_{11}\left(\frac{\partial w_0}{\partial x}\right)^2\frac{\partial^2w_0}{\partial x^2}-c_1^2H_{11}\frac{\partial^4w_0}{\partial x^4}-c_1^2H_{22}\frac{\partial^4w_0}{\partial y^4}\\&-c_1^2(H_{12}+H_{21}+4H_{66})\frac{\partial^4w_0}{\partial x^2\partial y^2}+A_{11}\frac{\partial w_0}{\partial x}\frac{\partial^2u_0}{\partial x^2}\\&+A_{66}\frac{\partial w_0}{\partial x}\frac{\partial^2u_0}{\partial y^2}+(A_{12}+A_{66})\frac{\partial^2v_0}{\partial x\partial y}\frac{\partial w_0}{\partial x}\\&+(B_{11}-c_1E_{11})\frac{\partial w_0}{\partial x}\frac{\partial^2\phi_x}{\partial x^2}+(B_{66}-c_1E_{66})\frac{\partial w_0}{\partial x}\frac{\partial^2\phi_x}{\partial y^2}\\&+(B_{12}-c_1E_{12}+B_{66}-c_1E_{66})\frac{\partial w_0}{\partial x}\frac{\partial^2\phi_y}{\partial x\partial y}+A_{22}\frac{\partial w_0}{\partial y}\frac{\partial^2v_0}{\partial y^2}\\&+A_{66}\frac{\partial w_0}{\partial y}\frac{\partial^2v_0}{\partial x^2}+(A_{22}+A_{66})\frac{\partial w_0}{\partial y}\frac{\partial^2u_0}{\partial x\partial y}\\&+(B_{22}-c_1E_{22})\frac{\partial w_0}{\partial y}\frac{\partial^2\phi_y}{\partial y^2}+(B_{66}-c_1E_{66})\frac{\partial w_0}{\partial y}\frac{\partial^2\phi_y}{\partial x^2}\\&+(B_{21}-c_1E_{21}+B_{66}-c_1E_{66})\frac{\partial w_0}{\partial y}\frac{\partial^2\phi_x}{\partial x\partial y}+2A_{66}\frac{\partial v_0}{\partial x}\frac{\partial^2w_0}{\partial x\partial y}\\&+2A_{66}\frac{\partial u_0}{\partial y}\frac{\partial^2w_0}{\partial x\partial y}+2(B_{66}-c_1E_{66})\frac{\partial\phi_x}{\partial y}\frac{\partial^2w_0}{\partial x\partial y}\\&+2(B_{66}-c_1E_{66})\frac{\partial\phi_y}{\partial x}\frac{\partial^2w_0}{\partial x\partial y}+c_1E_{11}\frac{\partial^3u_0}{\partial x^3}+c_1E_{22}\frac{\partial^3v_0}{\partial y^3}\\&+(c_1F_{11}-c_1^2H_{11})\frac{\partial^3\phi_x}{\partial x^3}+(c_1F_{22}-c_1^2H_{22})\frac{\partial^3\phi_y}{\partial y^3}\\&+A_{11}\frac{\partial u_0}{\partial x}\frac{\partial^2w_0}{\partial x^2}+A_{21}\frac{\partial u_0}{\partial x}\frac{\partial^2w_0}{\partial y^2}+A_{12}\frac{\partial v_0}{\partial y}\frac{\partial^2w_0}{\partial x^2}\\&+A_{22}\frac{\partial v_0}{\partial y}\frac{\partial^2w_0}{\partial y^2}+(B_{11}-c_1E_{11})\frac{\partial\phi_x}{\partial x}\frac{\partial^2w_0}{\partial x^2}\\&+(B_{21}-c_1E_{21})\frac{\partial\phi_x}{\partial x}\frac{\partial^2w_0}{\partial y^2}+(A_{55}-2c_2D_{55}+c_2^2F_{55})\frac{\partial\phi_x}{\partial x}\\&+(B_{12}-c_1E_{12})\frac{\partial\phi_y}{\partial y}\frac{\partial^2w_0}{\partial x^2}+(B_{22}-c_1E_{22})\frac{\partial\phi_y}{\partial y}\frac{\partial^2w_0}{\partial y^2}\\&+(A_{44}-2c_2D_{44}+c_2^2F_{44})\frac{\partial\phi_y}{\partial y}+c_1(E_{21}+2E_{66})\frac{\partial^3u_0}{\partial x\partial y^2}\\&+c_1(E_{12}+2E_{66})\frac{\partial^3v_0}{\partial x^2\partial y}+c_1(F_{21}-c_1H_{21}+2F_{66}-2c_1E_{66})\frac{\partial^3\phi_x}{\partial x\partial y^2}\end{aligned}$$

$$+c_1(F_{12}-c_1H_{12}+2F_{66}-2c_1H_{66})\frac{\partial^3\phi_y}{\partial x^2\partial y}+\Delta P$$

$$=-2\Omega_r c_1 I_3\frac{\partial\dot{v}_0}{\partial x}+c_1I_3\frac{\partial\ddot{v}_0}{\partial y}-2\Omega_r c_1(I_4-c_1I_4)\frac{\partial\dot{\phi}_y}{\partial x}$$

$$+c_1(I_4-c_1I_6)\frac{\partial\ddot{\phi}_y}{\partial y}+2c_1I_6\Omega_r\frac{\partial\dot{w}_0}{\partial x\partial y}-c_1^2I_6\frac{\partial\ddot{w}_0}{\partial y}+I_0\ddot{w}_0$$

$$-\dot{\Omega}_r c_1I_3\frac{\partial v_0}{\partial x}-\dot{\Omega}_r c_1(I_4-c_1I_6)\frac{\partial\phi_y}{\partial x}+c_1^2I_6\dot{\Omega}_r\frac{\partial_2 w_0}{\partial x\partial y}$$

$$-c_1^2I_6\frac{\partial^2\ddot{w}_0}{\partial y^2} \qquad (10)$$

$$(B_{11}-c_1E_{11})\frac{\partial^2u_0}{\partial x^2}+(B_{66}-c_1E_{66})\frac{\partial^2u_0}{\partial y_2}$$

$$+(B_{12}-c_1E_{12}+B_{66}-c_1E_{66})\frac{\partial^2v_0}{\partial x\partial y}+(B_{11}-c_1E_{11})\frac{\partial w_0}{\partial x}\frac{\partial^2w_0}{\partial x^2}$$

$$+(B_{66}-c_1E_{66})\frac{\partial w_0}{\partial x}\frac{\partial^2w_0}{\partial y^2}$$

$$+(B_{12}-c_1E_{12}+B_{66}-c_1E_{66})\frac{\partial w_0}{\partial y}\frac{\partial^2w_0}{\partial x\partial y}-(c_1^2H_{11}-c_1F_{11})\frac{\partial^3w_0}{\partial x^3}$$

$$+c_1(c_1H_{12}+2c_1F_{66}-F_{12}-2F_{66})\frac{\partial^3w_0}{\partial x\partial y^2}$$

$$-(A_{55}-2c_2D_{55}+c_2^2F_{55})\frac{\partial w_0}{\partial x}+(D_{11}-2c_1F_{11}+c_1^2H_{11})\frac{\partial^2\phi_x}{\partial x^2}$$

$$+(D_{66}-2c_1F_{66}+c_1^2H_{66})\frac{\partial^2\phi_x}{\partial y^2}$$

$$+(D_{12}-2c_1F_{12}+c_1^2H_{12}+D_{66}-2c_1F_{66}+c_1^2H_{66})\frac{\partial^2\phi_y}{\partial x\partial y}$$

$$-(A_{55}-2c_2D_{55}+c_2^2F_{55})\phi_x$$

$$=-2c_1\Omega_r(I_4-c_1I_6)\frac{\partial\dot{w}_0}{\partial y}-2\Omega_r(I_1-c_1I_3)\dot{v}_0$$

$$-2\Omega_r(I_2-2c_1I_4+c_1^2I_6)\dot{\phi}_y$$

$$+c_1\dot{\Omega}_r(I_4-c_1I_6)\frac{\partial w_0}{\partial y}-\dot{\Omega}_r(I_1-c_1I_3)(y-\frac{b}{2}+v_0) \qquad (11)$$

$$(B_{22}-c_1E_{22})\frac{\partial^2v_0}{\partial y^2}+(B_{66}-c_1E_{66})\frac{\partial^2v_0}{\partial x^2}$$

$$+(B_{21}-c_1E_{21}+B_{66}-c_1E_{66})\frac{\partial^2u_0}{\partial x\partial y}+(B_{22}-c_1E_{22})\frac{\partial w_0}{\partial y}\frac{\partial^2w_0}{\partial y^2}$$

$$+(B_{66}-c_1E_{66})\frac{\partial w_0}{\partial y}\frac{\partial^2w_0}{\partial x^2}$$

$$+(B_{21}-c_1E_{66}+B_{66}-c_1E_{21})\frac{\partial w_0}{\partial x}\frac{\partial^2w_0}{\partial x\partial y}+(c_1^2H_{22}-c_1F_{22})\frac{\partial^3w_0}{\partial y^3}$$

$$+c_1(c_1H_{21}+2c_1H_{66}-F_{21}-2F_{66})\frac{\partial^3w_0}{\partial x^2\partial y}$$

$$-(A_{44}-2c_2D_{44}+c_2^2F_{44})\frac{\partial w_0}{\partial y}+(D_{22}-2c_1F_{11}+c_1^2H_{22})\frac{\partial^2\phi_y}{\partial y^2}$$

$$+(D_{66}-2c_1F_{66}+c_1^2H_{66})\frac{\partial^2\phi_y}{\partial x^2}+(D_{21}-2c_1F_{21}+c_1^2H_{21}$$

$$+D_{66}-2c_1F_{66}+c_1^2H_{66})\frac{\partial^2\phi_y}{\partial x\partial y}-(A_{44}-2c_2D_{44}+c_2^2F_{44})\phi_y$$

$$=-c_1(I_4-c_1I_6)\frac{\partial\ddot{w}_0}{\partial y}+(I_1-c_1I_3)\ddot{v}_0+(I_2-2c_1I_4+c_1^2I_6)\ddot{\phi}_y$$

$$+(I_1-c_1I_3)(R_0+x)\dot{\Omega}_r \qquad (12)$$

其中功能梯度材料板的各阶广义惯量的表达式为

$$I_i=\int_{-h/2}^{h/2}z^i\rho(z)\mathrm{d}z,\quad i=0,1,2,3,4,6 \qquad (13)$$

2 Galerkin 离散

本文主要研究功能梯度材料变转速旋转悬臂板的横向非线性振动响应,取二阶模态进行 Galerkin 截断。

设横向位移 w 的模态函数为

$$w_0=w_1(t)X_1(x)Y_1(y)+w_2(t)X_2(x)Y_2(y) \qquad (14)$$

其中

$$X_i(x)=\sin\lambda_i x-\sinh\lambda_i x+\alpha_i(\cosh\lambda_i x-\cos\lambda_i x),\quad i,j=1,2 \qquad (15)$$

$$Y_i(y)=\sin\mu_j x+\sinh\mu_j x-\beta_i(\cosh\mu_j x+\cos\mu_j x),\quad i,j=1,2 \qquad (16)$$

$$\cos\lambda_i a\cosh\lambda_i a+1=0,\ \cos\mu_j b\cosh\mu_j b-1=0,\quad i,j=1,2 \qquad (17)$$

$$\alpha_i=\frac{\sinh\lambda_i a+\sin\lambda_i a}{\cosh\lambda_i a+\cos\lambda_i a},\ \mu_j=\frac{\sinh\mu_j b-\sin\mu_j b}{\cosh\mu_j b-\cos\mu_j b},\quad i,j=1,2 \qquad (18)$$

在下面的研究中,采用文献[19]、[20]中的方法,忽略方程(8),(9),(11),(12)中的惯性项,将式(14)—式(18)代入方程(8),(9),(11),(12) 中, 可以将 u,v,ϕ_x,ϕ_y 用变量 w 表示出来,最后将所有量代入式(10)中,运用 Galerkin 法得到了系统无量纲形式的非线性运动控制方程

$$\ddot{w}_1+(g_1+f_{11}\sin(\Omega t))w_1+(g_2\Omega_0+f_{12}\cos(\Omega t))\dot{w}_1+g_3w_1^2+g_4w_1w_2$$
$$+g_5w_1w_2^2+g_6w_1^2w_2+g_7w_1^3+g_8w_2^2+g_9w_2^3+(g_{10}+f_{13}\sin(\Omega t))w_2$$
$$+(g_{11}+f_{14}\cos(\Omega t))\dot{w}_2+g_{12}\ddot{w}_2=0 \qquad (19)$$

$$\ddot{w}_2+(h_1+f_{21}\sin(\Omega t))w_2+(h_2\Omega_0+f_{22}\cos(\Omega t))\dot{w}_2+h_3w_2^2+h_4w_1w_2$$
$$+h_5w_1w_2^2+h_6w_1^2w_2+h_7w_2^3+h_8w_1^2+h_9w_1^3$$
$$+(h_{10}+f_{23}\sin(\Omega t))w_1+(h_{11}+f_{24}\cos(\Omega t))\dot{w}_1$$
$$+h_{12}\ddot{w}_1=0 \qquad (20)$$

3 摄动分析

由于系统非线性运动控制方程中既有平方非线性项也有立方非线性项, 所以本文应用渐进摄动法进行摄动分析。引入小参数 ε, 考虑功能梯度材料变转速旋转悬臂板的共振关系为 1:1 内共振-主参数共振-1/2 亚谐共振情况。

$$\omega_1\approx\omega_2,\omega_1=\frac{\Omega_1}{2}+\varepsilon^2\sigma_1,\omega^2=\frac{\Omega_2}{2}+\varepsilon^2\sigma_2,\Omega_2=\Omega_1=\Omega \qquad (21)$$

式中 ω_1 和 ω_2 是两个不同的线性频率,σ_1 和 σ_2 是两个不同的调谐参数。

运用渐进摄动法得到系统直角坐标下的平均方程为

$$\dot{x}_1=(-\mu_1\Omega_0+\alpha_1)x_1+\sigma_1x_2+(-\mu_2\Omega_0+\alpha_3)x_3+\alpha_2x_4+\alpha_4x_2(x_3^2-x_4^2)$$
$$+\alpha_5x_4(x_1^2-x_2^2)-\alpha_6x_2(x_1^2+x_2^2)-\alpha_7x_4(x_3^2+x_4^2)-\alpha_8x_4(x_1^2+x_2^2)$$
$$-\alpha_9x_2(x_3^2+x_4^2)-2\alpha_4x_1x_3x_4-2\alpha_5x_1x_2x_3 \qquad (22)$$

$$\dot{x}_2=-\sigma_1x_1-(\mu_1\Omega_0+\alpha_1)x_2+\alpha_2x_3-(\mu_2\Omega_0+\alpha_3)x_4+\alpha_4x_1(x_3^2-x_4^2)$$
$$+\alpha_5x_3(x_1^2-x_2^2)+\alpha_6x_1(x_1^2+x_2^2)+\alpha_7x_3(x_3^2+x_4^2)+\alpha_8x_3(x_1^2+x_2^2)$$
$$+\alpha_9x_1(x_3^2+x_4^2)+2\alpha_4x_2x_3x_4+2\alpha_5x_1x_2x_4 \qquad (23)$$

$$\dot{x}_3=(-\mu_4\Omega_0+\beta_3)x_1-\beta_2x_2+(-\mu_3\Omega_0+\beta_1)x_3+\sigma_2x_4+\beta_4x_4(x_1^2-x_2^2)$$
$$+\beta_5x_2(x_3^2-x_4^2)-\beta_6x_4(x_3^2+x_4^2)-\beta_7x_2(x_1^2+x_2^2)-\beta_8x_2(x_3^2+x_4^2)$$

$$-\beta_9x_4(x_1^2+x_2^2)-2\beta_4x_1x_2x_3-2\beta_5x_1x_3x_4 \tag{24}$$

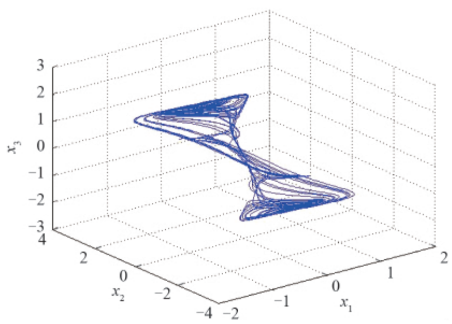
$$\begin{aligned}\dot{x}_4=&\beta_2x_1-(\mu_4\Omega_0+\beta_3)x_2-\sigma_2x_3-(\mu_3\Omega_0+\beta_1)x_4+\beta_4x_3(x_1^2-x_2^2)\\&+\beta_5x_1(x_3^2-x_4^2)+\beta_6x_3(x_3^2+x_4^2)+\beta_7x_1(x_1^2+x_2^2)+\beta_8x_1(x_3^2+x_4^2)\\&+\beta_9x_3(x_1^2+x_2^2)+2\beta_4x_1x_2x_4+2\beta_5x_2x_3x_4\end{aligned} \tag{25}$$

4 数值模拟

本节利用 Runge-Kutta 法对功能梯度材料旋转悬臂板系统在主参数共振、1:1 内共振情况下的平均方程(22)—(25)进行数值计算。为了研究转速在旋转悬臂板系统振动中所起的作用,选定一组初始参数,以转速 Ω_0 作为控制参数,研究其对旋转悬臂板非线性振动响应的影响。

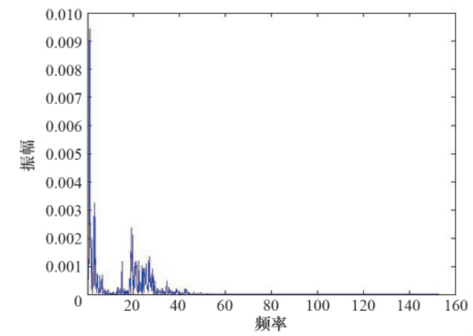
图 3 所示为变转速旋转悬臂板在参数为 $\mu_1=4.5$,$\mu_2=2.49$,$\mu_3=-0.85$,$\mu_4=-4.56$,$\alpha_1=-7.72$,$\alpha_2=4.46$,$\alpha_3=3.78$,$\alpha_4=2$,$\alpha_5=-0.4$,$\alpha_6=-1.41$,$\alpha_7=-3.25$,$\alpha_8=3.41$,$\alpha_9=-5.13$,$\beta_1=6.22$,$\beta_2=0.6$,$\beta_3=-0.59$,$\beta_4=0.75$,$\beta_5=-1.64$,$\beta_6=-0.9$,$\beta_7=4.63$,$\beta_8=-1.37$,$\beta_9=-5.24$,$\sigma_1=1.75$,$\sigma_2=1.84$,初始条件为 $x_1=0.2403$,$x_2=-0.2118$,$x_3=-0.1176$,$x_4=0.0828$,转速为 $\Omega_0=4$ 时的混沌运动。图 3(a)表示系统(22)—(25)的三维相图,图 3(b)为频谱图,图 3(c)为平面(x_1,x_2)上的相图,图 3(d)表示(t,x_1)上的波形图。根据混动运动描述方法,从相图、波形图都可以判断变转速旋转悬臂板在这组参数作用下产生混沌运动。

继续增大转速 Ω_0,当 $\Omega_0=5$ 时,系统的三维相图、功率谱、平面相图和波形图如图 4 所示,系统仍处于混沌运动。



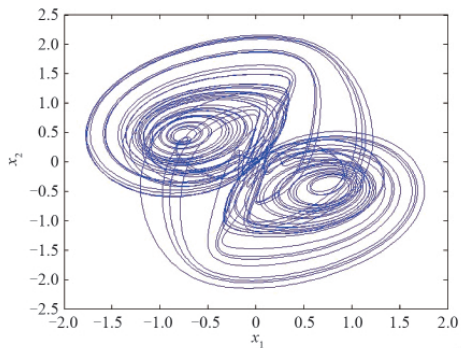
(a) 三维相图

(a) Three-dimensional phase portrait



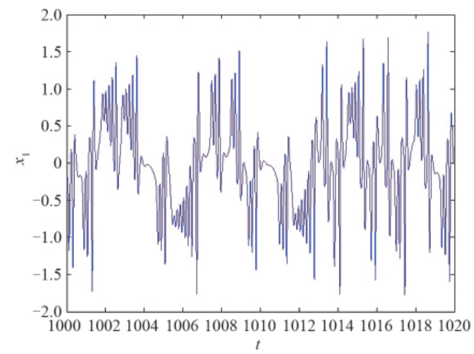
(b) 频谱图

(b) Frequency spectrum



(c) (x_1,x_2)平面二维相图

(c) Two-dimensional phase portrait in plane (x_1,x_2)

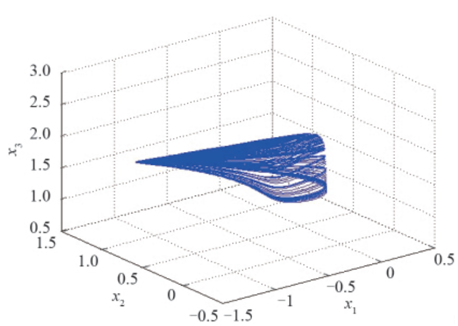


(d) (t,x_1)平面波形图

(d) Wave form portrait in plane (t,x_1)

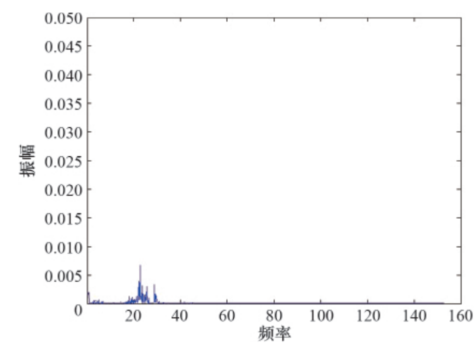
图 3 混沌运动

Fig. 3 Chaotic motion



(a) 三维相图

(a) Three-dimensional phase portrait

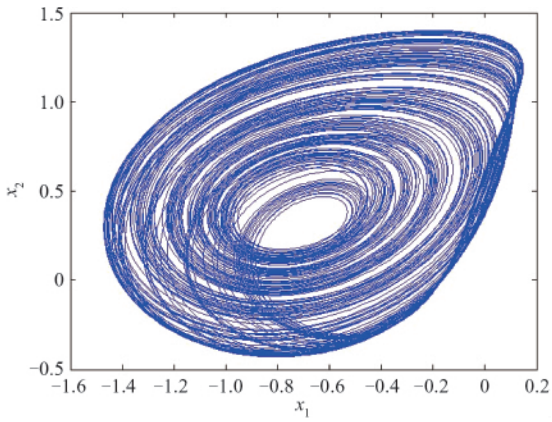


(b) 频谱图

(b) Frequency spectrum

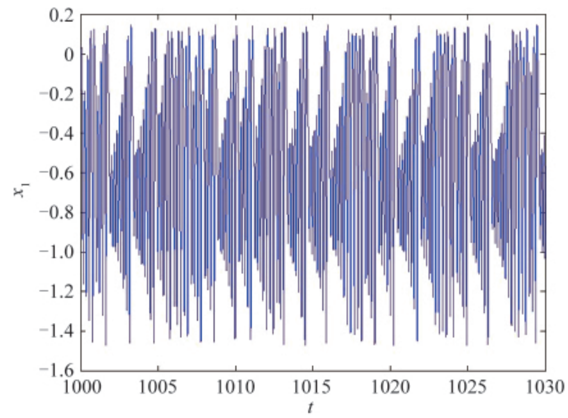
图 4 混沌运动

Fig. 4 Chaotic motion



(c) (x_1,x_2)平面二维相图

(c) Two-dimensional phase portrait in plane (x_1,x_2)



(d) (t,x_1)平面波形图

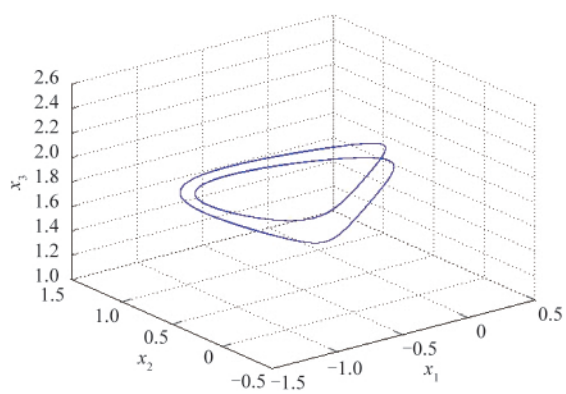
(d) Wave form portrait in plane (t,x_1)

图4　混沌运动(续)

Fig. 4　Chaotic motion (continued)

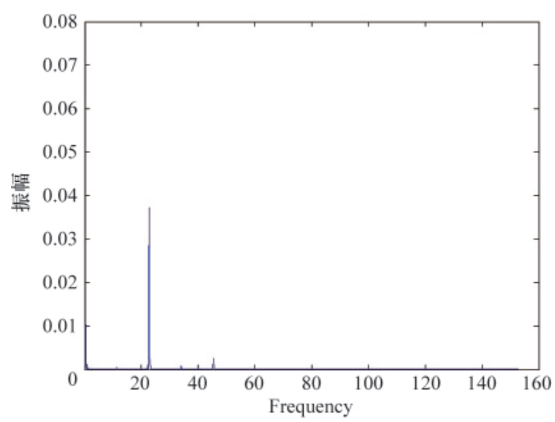
改变转速 Ω_0,当 Ω_0=5.2 时,系统的三维相图、功率谱、平面相图和波形图如图 5 所示,系统产生二倍周期运动。

改变转速 Ω_0,当 Ω_0=5.5 时,系统的三维相图、功率谱、平面相图和波形图如图 6 所示,系统产生一倍周期运动。



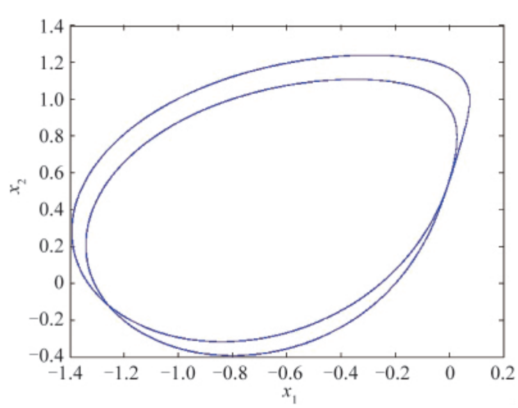
(a) 三维相图

(a) Three-dimensional phase portrait



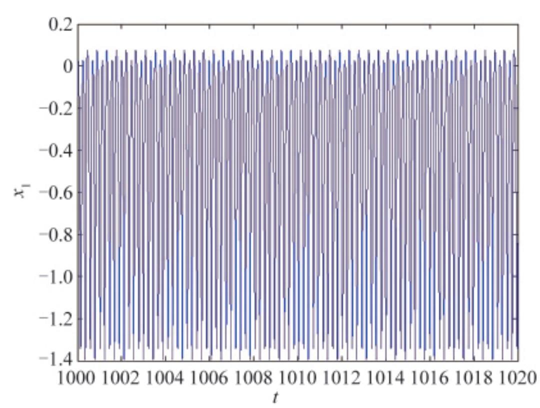
(b) 频谱图

(b) Frequency spectrum



(c) (x_1,x_2)平面二维相图

(c) Two-dimensional phase portrait in plane (x_1,x_2)

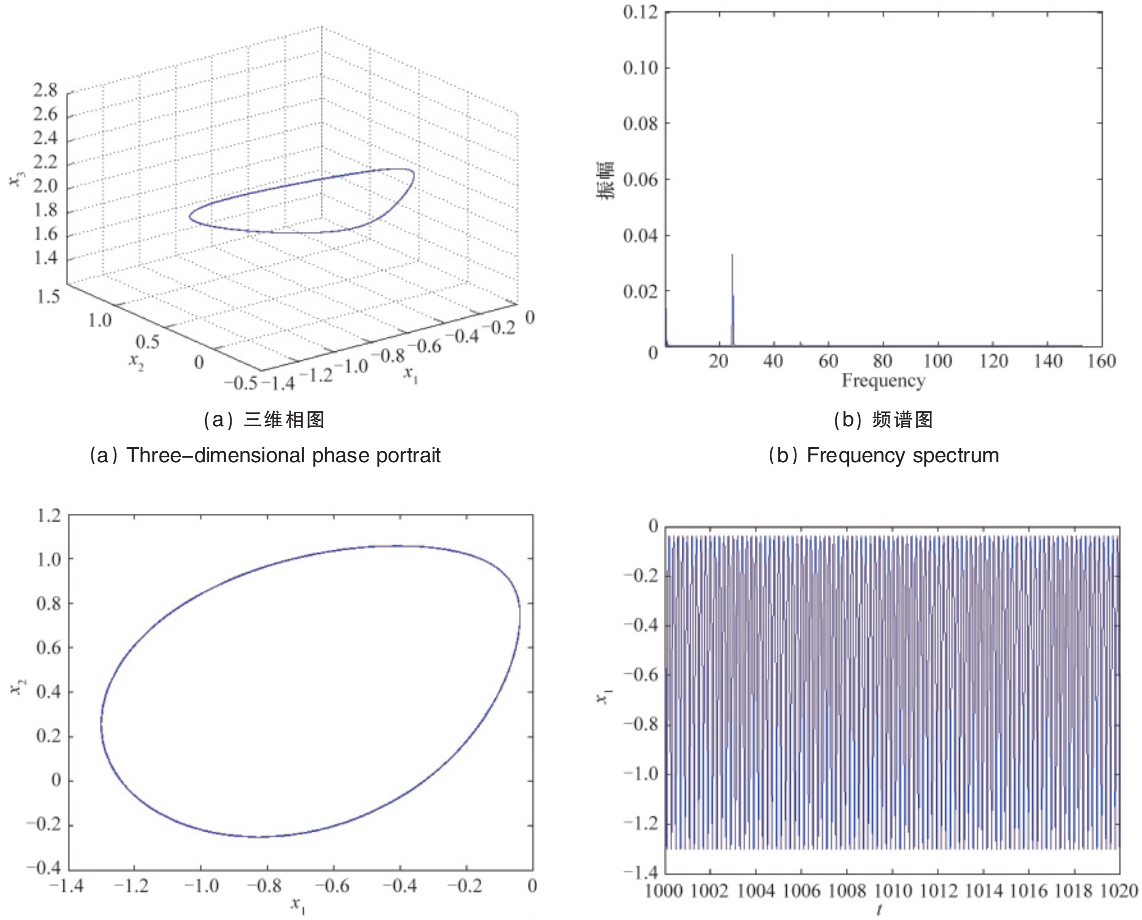


(d) (t,x_1)平面波形图

(d) Wave form portrait in plane (t,x_1)

图5　二倍周期运动

Fig. 5　Period-two motion



(a) 三维相图

(a) Three-dimensional phase portrait

(b) 频谱图

(b) Frequency spectrum

(c) (x_1,x_2)平面二维相图

(c) Two-dimensional phase portrait in plane (x_1,x_2)

(d) (t,x_1)平面波形图

(d) Wave form portrait in plane (t,x_1)

图 6 一倍周期运动

Fig. 6 Period motion

5 结论

本文利用 Reddy 的高阶剪切理论、一阶活塞理论和 Hamilton 原理建立了功能梯度材料变转速旋转悬臂板的非线性偏微分运动控制方程，利用 Galerkin 法离散后得到常微分控制方程，之后用渐进摄动法进行摄动分析，得到了系统的四维平均方程，然后进行数值模拟，研究其在不同转速下的非线性动力学行为。

通过数值分析方法得到了不同转速下系统的三维相图、二维相图、波形图和频谱图。研究表明，转速的变化对系统的动力学行为有着很大的影响。随着转速的变化，系统出现了周期和混沌等不同的运动形式。由此可见，转速是影响系统非线性动力学行为的主要因素之一，通过改变转速，可以对变转速旋转悬臂板的非线性动力学行为产生显著影响。

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