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Dynamic period detection and maintenance optimization for the Wiener degradation dependence process systems

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Abstract: The implementation of timely monitoring and preventive maintenance plays a fundamental role to ensure the reliable operation of complex systems. Condition-based maintenance strategy offers an effective means to leverage system remaining life information, enabling the application of targeted measures to reduce maintenance costs and elevate overall operational efficiency. This study delves into a performance degradation system affected by external random shocks, utilizing the Wiener process model to characterize the continuous degradation process. Within this framework, two distinct condition-based monitoring schemes are proposed: one is the real-time condition monitoring and the other is the dynamic periodic monitoring. Through the optimization of maintenance strategies for each scheme based on the long-term average cost, the study aims to optimize the preventive maintenance threshold for system failure. The Monte Carlo simulation algorithm is adopted to solve the optimization problem. Finally, a comprehensive numerical example is provided to validate the efficiency of both the models and the proposed maintenance strategies.

Keywords: Wiener process, shock effect, real-time condition monitoring, dynamic periodic monitoring, optimization strategy.

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1. Introduction

The lives of complex systems or key equipment, such as aviation, aerospace, ships and other fields are getting longer and longer, and the failure data of these systems is difficult to obtain. The traditional reliability analysis methods based on statistical analysis of failure data are encountering many problems. The reliability model based on performance degradation can better obtain the information of system degradation process and then predict the remaining life of the system [1]. As we know, Ye et al. [2] did a good review on stochastic modelling which comprehensively summarized the degradation models. Performance degradation failure means that the key indicators characterizing the system performance will gradually decline as the component operating time increases. When the degradation continues to decline and exceeds the specified threshold, it is considered that the component loses function and fails, such as fatigue cracks and mechanical wear [3]. The performance state of the system deteriorates with the use time, and the probability of failure increases. Reasonable preventive maintenance can effectively improve the availability of the equipment. However, the existing preventive maintenance is often executed by experience, so it is necessary to study the preventive maintenance strategy of the random degradation system.

For the failure process of complex equipment or components, a variety of performance degradation models have been established at home and abroad, such as general degradation trajectory model [4], degradation quantity distribution model, Markov process model and stochastic process model, etc. Among them, stochastic process model is an important method to describe system degradation. For example, Wiener process, Gamma process, inverse Gaussian process and other models are widely used in different scenarios. Wiener process has been applied frequently in research which can well describe non-monotonic incremental degradation process [1,3]. In addition, due to the complexity of the system operating environment, the system is vulnerable to shocks. Existing studies generally assume that the shocks will cause a sudden increase in the amount of system degradation, and even lead to instantaneous collapse or failure of the system [5]. The preventive maintenance model has been applied in many fields [6]. In [7], A model of multi-criteria decision-making to prioritize

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medical devices according to their criticality is presented. Papadopoulos et al. [8] proposed the stochastic holistic opportunistic scheduler, which is a maintenance scheduling approach tailored to address the unique challenges and uncertainties in offshore wind farms.

Detecting the operating state of complex system and carrying out maintenance activities is an important means to improve the system reliability and ensure the safe of the system. Compared with passive maintenance or replacement after failure, preventive maintenance in advance has become a common maintenance strategy. In the existing literature, there are two kinds of preventive maintenance methods that have been studied. One is condition-based maintenance (CBM) [9−11]. The other one is called the time-based maintenance (TBM). In the TBM strategy, the periodic maintenance strategy is widely used in current engineering practice. This strategy can facilitate the arrangement of maintenance plans, convenient to predict the maintenance period and cost in advance, but the decision of specific maintenance activities is dominated by experience, and it is difficult to flexibly adjust the maintenance cycle according to the system status. A too long maintenance cycle will lead to "insufficient maintenance", so that the equipment operation failure rate is high, resulting in serious losses. A too short maintenance cycle will lead to "maintenance excess", increase maintenance costs, and cause unnecessary waste. Wu et al. [12] considered an imperfect degradation-based model to optimize the interval of condition monitoring for a single component system. They considered the total cost as the objective. Cui et al. [13] considered the minimal repair and perfect repair policy for a monotonic failure rate repairable system, in which an optimal allocation problem is studied under some resource constraints. Huynh et al. [14] investigated age-based maintenance policies with minimal repairs, comparing time-based and condition-based approaches for repair decisions to minimize maintenance costs. They gave an effective approach for making a minimal repair decision can lead to minimum maintenance cost. CBM is the focus of research in the field of maintenance, and it is an effective method to solve the problem of component maintenance, which can make up for the deficiency of regular maintenance strategy to a certain extent.

The maintenance model of a performance degradation system based on stochastic process has been widely used. In addition, significant and meaningful research has been done on reliability and maintenance policies for systems subject to multiple degradation processes [15]. Lucia et al. [16] analyzed a system subject to different deterioration processes in which a maintenance strategy is implemented in this system and the state of the system is checked in inspection times. Researchers developed maintenance policies from different point of view. Trying to reduce the cost of the action of maintenance is a most common way for traditional research. In addition to cost, we can use other factors as the objective of the model. Liao et al. [17] considered a CBM model for continuously degrading systems under continuous monitoring. They try to get the maximal availability through optimizing the preventive threshold. Choosing an appropriate model for a given failures, dataset is an important practical issue. El et al. [18] proposed a general goodness-of-fit test for imperfect repair models based on sequential (or on-line) assessment of times-to-failures forecasts.

In addition to the continuous performance degradation process, the system was often affected by shocks. The failure caused by the degradation process such as wear, fatigue and corrosion are called the degradation failure process, while the sudden functional damage caused by the impact load is a sudden failure process. The impact effect is manifested as the sudden increase in the amount of system degradation, which will lead to the instantaneous failure of the system in serious cases. Furthermore, competing failure models are widely used to describe the multiple failure processes in which the hard failure is caused by the shocks and the soft failure is caused by the degradation. There exist many works under the assumption that the multiple failure processes are independent. For example, Klutke et al. [19] computed the availability of an independent competing failure system and proposed effective inspection strategies. Cao et al. proposed a vine-Copula-based reliability evaluation method to estimate the reliability of system components with multiple dependent performance characteristics [20]. Huang et al. [21] analyzed the system reliability of the electronic devices involving an independent degradation failure and a catastrophic failure. The dependency among the multiple failure processes presents a challenging issue in reliability modeling. Even so, there has been considerable amount of literature published in the literature. To be specific, Peng et al. [22] proposed the reliability models and preventive maintenance policies for systems which are subject to the multiple dependent competing failure processes. They built the dependence relation by considering the effects to both the soft and hard failure processes caused by the same external shocks. Many works are done to extend the dependent competing failure degradation models and optimization policies [23,24]. Yang et al. [25] constructed a novel weather-centered framework for wind turbines by considering both the positive and negative impacts (maintenance delays) of wind conditions.

In the selection of maintenance scheme, different optimization objectives will affect the selection of maintenance strategy, and engineers will formulate the optimal maintenance strategy from different objectives. Reducing the cost of maintenance behavior is the most common optimization goal in existing studies. Besides cost, other parameters can also be used as the model goal, such as maintenance rate, reliability, revenue, etc. Marais et al. [26] proposed a maintainability framework to capture and quantify the value of maintenance activities and compare it with existing cost-centric maintainability models. In literature, two classification criteria have been used for imperfect maintenance models. The first one is based on the failure intensity. This model class contains the governed models by the initial failure intensity and the not governed ones. The second criterion considers the effect of the maintenance on the system. Yeh et al. [27] studied a preventive maintenance model, which assumed that since the preventive maintenance actions are prepared in advance, their effect can be controlled. Laggoune et al. [28] proposed a model that assumes that the maintenance efficiency is linearly proportional to the cost of the maintenance action.

To sum up, the existing studies are generally based on the single failure process such as progressive degradation failure or shock failure, and the maintenance modeling of the degradation system with impact effect is insufficient. There are few studies on the integrated modeling of the degradation system with impact consideration in the literature. In maintenance modeling, a constant cycle inspection is often used to carry out maintenance, which lacks the use of degradation process information. For example, based on continuous periodic monitoring, Liu et al. [29] studied the CBM strategy under the condition that multiple sudden failures may occur in the degradation process of the system. Tang et al. [30] used periodic detection to obtain the degradation information of the system and estimate the remaining life, so as to formulate the best preventive maintenance strategy. Later, there already have some scholars carried out research on non-periodic detection methods. For example, Lam et al. [31] proposed a CBM strategy under aperiodic observation on the basis of periodic detection. The strategy is to determine the time for the next detection at the same time of each detection, with the goal of minimizing the total maintenance cost. In practical engineering applications, the key equipment can be monitored in real time to reduce the huge loss caused by system failure. In CBM, it is necessary to dynamically adjust the condition detection cycle of the system according to the degradation state of the system.

Therefore, in order to avoid the deficiency of periodic detection, a degraded system affected by external random shocks is considered in this paper, and a reliability model of system degradation is established. The contribution of the paper is as follows: firstly, aiming at the system, this paper proposes two kinds of maintenance decision schemes for state detection, one is state real-time monitoring scheme and the other is dynamic periodic detection strategy. Secondly, for the latter, a dynamic cycle detection strategy based on average failure time is proposed, and a situational maintenance strategy is proposed for the two schemes respectively, and the preventive maintenance threshold of system failure is optimized. In addition, in this paper, a Monte Carlo simulation method is used to simulate the system degradation and monitoring maintenance process, and the algorithm of parameter optimization is given.

The main contributions of the paper could be summarized as follows: Firstly, this paper proposed a Wiener process degradation model which addresses the effect of external random shocks on system degradation, filling a gap in existing literature that often focuses on single failure processes. Secondly, two distinct maintenance decision schemes for state detection: a real-time monitoring scheme and a dynamic periodic detection strategy has been constructed. This dual approach allows for more flexible and responsive maintenance practices. Especially for the dynamic periodic detection strategy, the paper introduces a method that adjusts the detection cycle based on the average failure time, enhancing the adaptability of maintenance schedules to the actual condition of the system. Thirdly, from the prospective of economic efficiency, the study optimizes the preventive maintenance threshold to minimize the average cost rate of the system, ensuring that maintenance actions are both timely and cost-effective.

The remainder of this paper is organized as follows. The system assumptions, shock process, and system reliability are described in Section 2. The maintenance policies and the two maintenance strategy frameworks are formulated in Section 3. The simulation procedures of the two proposed maintenance models are presented in Section 4. The illustrative example is used to demonstrate the proposed policy in Section 5. Finally, the conclusion is discussed in Section 6.

2. Reliability modeling of performance degradation system with shocks

2.1 System overall assumptions

In order to exhibit the system and the maintenance model clearly, we give the overall assumptions in advance as follows:

(i) The degradation process of the system is governed by a linear Wiener process with external shocks.

(ii) The arrival of the shock process follows the homogeneous Poisson process.

(iii) Each external shock will cause a certain degree of damage to the system, and the shocks will increase the degradation of the system.

(iv) The total degradation of the system with shocks consists of two parts, one is the damage caused by continuous degradation and the other is the sudden damage caused by shocks.

(v) The system will fail when the total degradation value of the system reaches the given failure threshold for the first time.

2.2 Wiener degradation process modeling

In this paper, the definition of Wiener process and shock process is given first, and then the corresponding system performance detection approach and maintenance strategy are proposed according to the system failure process. The expression for a linear Wiener process is

$$X(t) = x_0 + \mu t + \sigma B(t),\ \ t \geqslant 0 \tag{1}$$

where $X(t)$ is the degradation value of the system at time point t. μ is the drift coefficient used to describe the degradation rate. σ is the diffusion coefficient used to describe the degradation volatility. $B(t)$ is a standard Brownian motion. x_0 is the initial degradation value of the system.

One possible operation process of the system has been exhibited in Fig. 1, where D_f is the failure threshold and D_p is the preventive maintenance threshold of the system. The operation of the system can be divided into three stages, called the safe operation stage, potential risk stage and the failure stage. In specific, when the degradation of the system is less than the preventive maintenance threshold, the system is in the safe operation stage. When the degradation is greater than the preventive maintenance threshold and smaller than the failure threshold, the system is in the potential risk stage. When the system degradation exceeds the failure threshold, the system fails, then it goes into the failure state. The purpose of this study is to minimize the probability of potential risk or even failure of the system and ensure the safe operation of the system.

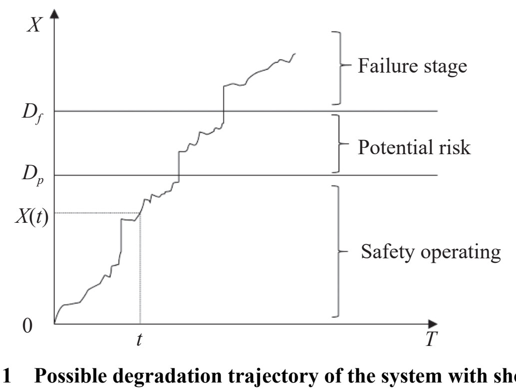

Fig. 1 Possible degradation trajectory of the system with shocks

Combined with the definition of the Wiener process, the mean and variance of degradation of system performance at any time t are shown as follows:

$$\begin{cases} \mathrm{E}(X(t)) = \mu t \\ \mathrm{Var}(X(t)) = \sigma t^2 \end{cases}.$$

For the performance degradation process, the lifetime of the system is a random variable, denoted as T. T can be expressed by the first passage time (FPT), that is, the time when the failure threshold is reached for the first time. Then the lifetime of the system can be expressed as $T = \inf\{t\colon X(t) \geqslant D_f\}$, and the reliability of the system at any time point t can be given by

$$R(t) = P\{T > t\}.$$

Then, according to [32], we know that the FPT of a linear Wiener process follows the inverse Gaussian distribution, which is recorded as

$$F_T = 1 - \Phi\left(\frac{D_f - x_0 - \mu t}{\sigma\sqrt{t}}\right) + \exp\left\{\frac{2\mu(D_f - x_0)}{\sigma^2}\right\}\Phi\left(\frac{-(D_f - x_0) - \mu t}{\sigma\sqrt{t}}\right) \tag{2}$$

where $\Phi(x) = \dfrac{1}{\sqrt{2\pi}}\displaystyle\int_{-\infty}^{x} \mathrm{e}^{-t^2/2}\mathrm{d}t$ is the distribution function of a standard normal variable. In this case, the expectation of the FPT of the system without shocks can be expressed as follows:

$$\mathrm{E}[T] = \frac{D_f - x_0}{\mu}. \tag{3}$$

2.3 Calculation of the FPT of the shock process

Let $\{H_j\}$ represents the time interval between any two successive shocks for $j = 1, 2, \cdots$. It is assumed that $H_1, H_2, \cdots$ are non-negative, independent, and random variables. They are subject to the same distribution $\{F(t), t \geqslant 0\}$. Each external shock will cause a certain degree of damage, which will increase the degradation of the system. The increment of the degradation caused by the $j$thshock is Y_j for $j = 1, 2, \cdots$, where $Y_1, Y_2, \cdots$ are non-negative, independent, and random variables. Denote $Y(t)$ as the overall degradation increment caused by the shocks at any time point t. $Y(t)$ can be expressed as follows:

$$Y(t) = \begin{cases} \displaystyle\sum_{j=1}^{N(t)} Y_j, & N(t) \geqslant 1 \\ 0, & N(t) = 0 \end{cases} \tag{4}$$

where $N(t)$ is the number of external shocks until time point t.

Define that the $k-$convolution of the distribution function $F(t)$ of the shock time interval is $F^{(k)}(t)$, where

$k=1,2,\cdots$. Since $H_1,H_2,\cdots$ are independent and have the same distribution, $F^{(k)}(t)$ corresponds to the distribution of the arrival time of the kth shock. The probability of the exactly occurring of k shocks within $[0,t]$ is

$$P(N(t)=k)=P\{T_k\leqslant t,T_{k+1}>t\}= \\ F^{(k)}(t)-F^{(k+1)}(t),\ \ k=0,1,2,\cdots. \tag{5}$$

It is assumed that the shock process follows the homogeneous Poisson process with an arrival rate λ. Then the probability $P(N(t)=k)=\dfrac{\exp(-\lambda t)(\lambda t)^k}{k!}$, where $k=1,2,\cdots$ represents the number of the shocks.

Theorem 1 Assume that the damage caused by each shock has a limit mean μ_s and a variance σ_s^2, then the mean value of the remaining lifetime for the compound Poisson process of the shocks until reaching threshold l is $\mathrm{E}[T_l]=\dfrac{1}{\lambda}\left[\dfrac{l}{\mu_s}+\dfrac{\sigma_s^2/\mu_s{}^2+1}{2}\right]$, where λ is the arrival rate of the shocks.

Proof Denote $\{G(t),t\geqslant 0\}$ as the distribution function of the amount of damage caused by shock Y_j. Define the $k-$convolution of the increased value of the degradation quantity Y_j, caused by shock as $G^{(k)}$. Then, within $[0,t]$, the probability that there occurs exactly k times of shocks and the cumulative damage caused will not exceed l is

$$P(Y(t)\leqslant l,N(t)=k)=P\{Y(t)\leqslant l|N(t)=k\}P(N(t)=k)= \\ G^{(k)}(l)[F^{(k)}(t)-F^{(k+1)}(t)],\ \ k=0,1,2,\cdots \tag{6}$$

Using the total probability formula, the probability that the cumulative damage within $[0,t]$ does not exceed l is

$$P(Y(t)\leqslant l)=\sum_{k=0}^{\infty}P\{Y(t)\leqslant l|N(t)=k\}P(N(t)=k)= \\ \sum_{k=0}^{\infty}G^{(k)}(l)[F^{(k)}(t)-F^{(k+1)}(t)]. \tag{7}$$

Since the degradation process of the system caused by shock is strictly regular, the event of $Y(t)\leqslant l$ is equivalent to $T_l\geqslant t$. Thus, the distribution of system failure time can be expressed as

$$F_T(t)=P\{T_l\leqslant t\}=P\{Y(t)\geqslant l\}= \\ \sum_{k=0}^{\infty}\left[G^{(k)}(l)-G^{(k+1)}(l)\right]F^{(k+1)}(t). \tag{8}$$

The Laplace-Stitcher's transformation of (8) is

$$\varPhi^*(s)=\int_0^{\infty}\mathrm{e}^{-st}\mathrm{d}F_T(t)=\sum_{k=0}^{\infty}[G^{(k)}(l)-G^{(k+1)}(l)][F^*(s)]^{(k+1)}. \tag{9}$$

The mean failure time is as follows:

$$\mathrm{E}(T_l)=\int_0^{\infty}t\mathrm{d}P(T_l\leqslant t)=-\frac{\mathrm{d}\varPhi^*(s)}{\mathrm{d}s}|s=0= \\ \frac{1}{\lambda}\sum_{k=0}^{\infty}G^{(k)}(l)=\frac{1}{\lambda}[1+M_G(l)] \tag{10}$$

where $M_G(l)=\sum_{k=1}^{\infty}G^{(k)}(l)$, representing the mean number of shocks before the total damage exceeds the failure threshold.

Due to that the damage $Y_j(j=1,2,\cdots)$ caused by the jth shock has a limit mean μ_s and a variance σ_s^2, (10) can be expressed as

$$\mathrm{E}[T_l]=\frac{1}{\lambda}\left[\frac{l}{\mu_s}+\frac{\sigma_s^2/\mu_s{}^2+1}{2}\right]. \tag{11}$$

After obtaining the remaining lifetime for the compound Poisson process of the shocks, then in next section, we could get the mean lifetime of the whole degradation process with shocks, which is the core foundation to construct the maintenance strategy.

2.4 Lifetime calculation of the degradation process with shocks

The degradations of a Wiener process considering shocks consist of two parts, one is the damage caused by continuous degradation and the other is the sudden damage caused by shocks. Assuming that the total degradation of the system is $Z(t)$. Then, we have

$$Z(t)=X(t)+Y(t)$$

where $X(t)$ is given in (1) and $Y(t)$ is given in (4). Since the degradation process of the system is a Wiener process, and Wiener is a non-monotonic continuous degradation process. Thus, it is difficult to obtain an analytical expression for the distribution of the first arrival time of the Wiener process considering shock effects.

In engineering applications, it is often assumed that the degradation increment follows a normal distribution, denoted as $Y_j\sim N(\mu_s,\sigma_s^2)$ for $j=1,2,\cdots$. This paper presents a method for calculating the mean time to failare (MTTF). The mean life of system failure due to continuous degradation and shocks are $\mathrm{E}[T_l^1]$ and $\mathrm{E}[T_l^2]$, respectively. According to (3) and (11), the average lifetime of the system reaching the failure threshold l can be obtained as follows:

$$1/\mathrm{E}[T_l]=1/\mathrm{E}[T_l^1]+1/\mathrm{E}[T_l^2]= \\ \left[\frac{\mu}{l}+\frac{\lambda}{l/\mu_s+\sigma_s^2/2\mu_s{}^2+1/2}\right]. \tag{12}$$

Further, the mean value of the FPT can be expressed as

$$\mathrm{E}[T_l]=\frac{1}{\left[\dfrac{\mu}{l}+\dfrac{\lambda}{l/\mu_s+\sigma_s^2/2\mu_s{}^2+1/2}\right]}. \tag{13}$$

According to the proposed average system lifetime of Wiener process with shock effects, a dynamic periodic detection maintainability scheme based on the average FPT is proposed in the next section. This scheme can make better use of the degradation process information of the system, save maintenance costs to the greatest extent, and prevent sudden system failures.

3. Optimization of maintenance strategies

For the important key equipment, once the system fails, there will be significant losses. Thus, it is necessary to strengthen the monitoring of system performance status. Two maintenance strategies are proposed in this paper, one is the real-time monitoring strategy and the other is the dynamic periodic detection maintenance strategy.

3.1 Real-time monitoring scheme

The real-time condition-based monitoring scheme is usually aimed at the key safety components of large complex systems, and the failure of the components may lead to huge losses to the system operation. Therefore, an advanced sensing technology is needed to monitor the operating status of the system in real time. Due to the shock effect, the degradation of the system will increase abruptly. Therefore, preventive measures should be set to prevent the sudden failures. The operation process of the system under this scheme can be illustrated in Fig. 2.

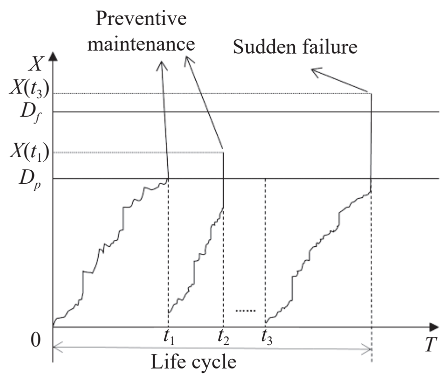

Fig. 2 Possible degradation path of the real-time monitoring

Based on the above considerations, for the sake of simplicity, the real-time monitoring model and maintenance strategy could be given as follows:

(i) The failure threshold of the system is a fixed value D_f, which is determined by the structural characteristics of the system and the external environment. Consider that the preventive maintenance threshold is D_p and $D_p < D_f$. In this case, it is necessary to select a reasonable preventive maintenance threshold. If the threshold is too large, the risk of system failure will increase; if it is too small, the detection frequency will increase and the detection cost will increase.

(ii) At any time t, there are four ways to maintain the system, including no maintenance, preventive maintenance, preventive replacement and replacement after failure. The principle of maintenance operation is if $Z(t) < D_p$, no maintenance operation; if $D_p \leqslant Z(t) < D_f$, preventive maintenance is carried out, and the cost of preventive maintenance is C_{PM}; if the system fails instantaneously due to shocks, that is, $Z(t) \geqslant D_f$. Then, replace the system after the failure, and the replacement cost is C_{FR}. There is a maximum number of maintenances on the part because the maintenance is imperfect. Suppose that the maximum number of preventive maintenances is $N_{\max}(N_{\max} \geqslant 1)$, when the number of maintenances exceeds $N_{\max}$, preventive replacement is carried out, and the cost of preventive replacement is C_{PR}. Obviously, there is $C_{\mathrm{PR}} < C_{\mathrm{FR}}$.

(iii) It is assumed that preventive maintenance is not perfect, and the state of the system after maintenance is certain random. After a preventive maintenance, it is assumed that the degradation amount of the system is random. This paper assumes that after the ith$(i=1,2,\cdots)$ maintenance, the degradation amount of the system follows a uniform distribution from 0 to D_p. That is, $Z(T_i^+) \sim \mathrm{U}(0, D_p)$, where T_i^+ is the right-hand limit after the ith maintenance.

(iv) The cost of the real-time monitoring per unit time is C_{moni}.

The purpose of this model is to find a suitable preventive maintenance threshold D_p, which can minimize the average cost rate of the system in a renewal cycle.

3.2 Dynamic periodic detection scheme

Compared with the real-time monitoring scheme, the periodic detection scheme is usually aimed at the case that the real-time monitoring cost is high, the running status of the component is concerned, and the loss degree is acceptable after the component failure. In the existing research, the cycle detection is usually used. Although the detection cycle can be optimized to a certain extent, it is often unable to detect the near failure state of the component efficiently.

The key innovation point of the dynamic cycle detection scheme proposed in this paper lies in the calculation of the average FPT reaching the preventive maintenance threshold according to the degradation of the system. Then, adjusts the time of the next detection cycle by the FPT to find the best maintenance opportunity for the system and reduce the waste of detection costs to the greatest extent.

A possible operation of the system under this scheme can be illustrated in Fig. 3.

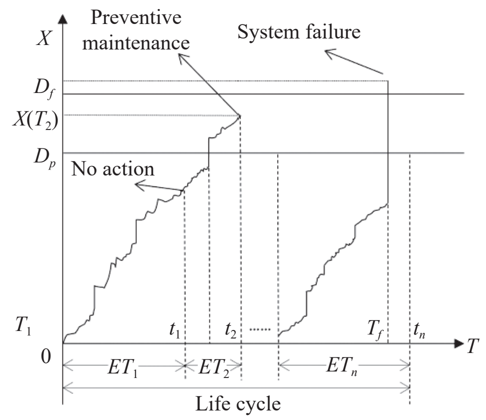

Fig. 3 Possible degradation path of dynamic periodic detection

The assumptions of the dynamic periodic inspection maintenance scheme could be summarized as follows:

(i) The failure threshold of the system is a fixed value D_f, which is determined by the structural characteristics of the system and the external environment. Consider the preventive maintenance threshold as D_p, and $D_p < D_f$. Similar to the real-time monitoring scheme, it is necessary to select a reasonable preventive maintenance threshold. If the threshold is too large, the risk of system failure will increase during detection; if it is too small, the detection frequency will increase and the detection cost will increase.

(ii) $T_1, T_2, \cdots$ are defined as the periodic state detection time points of the system. At the initial time, the system is in a perfect state and the initial degradation is 0. After the detection is completed at a certain state detection time $T_i (i = 1, 2, \cdots)$, the remaining time when the system reaches the preventive maintenance threshold is defined as L_i, and $E(L_i)$ is the average FPT. Consider $E(L_i)$ as the time interval between the ith and the $(i+1)$thdetections, and the value of $E(L_i)$ could be calculated by (13).

(iii) After carrying out any state detection, the detected degradation performance of the system is $Z(T_i)$. There are four actions to maintain the system: no maintenance, preventive maintenance, preventive replacement and after-failure replacement. The principles of maintenance operation are as follows: If $Z(T_i) < D_p$, no maintenance operation is performed; If $D_p \leqslant Z(T_i) < D_f$, preventive maintenance is carried out, and the cost of preventive maintenance is C_{PM}; If the system fails instantaneously due to shocks, that is, $Z(T_i) \geqslant D_f$. Then, replace the system after the failure. The replacement cost is C_{FR}.

(iv) Since the preventive maintenance is imperfect, the degradation performance after maintenance treatment is a random value. Like the real-time monitoring strategy, this paper assumes that it follows a uniform distribution from 0 to D_p, that is, $Z(T_i^+) \sim \mathrm{U}(0, D_p)$, where T_i^+ is the right-hand limit after the ith maintenance.

(v) The maximum number of preventive maintenances is $N_{\max}$. When the number of maintenances exceeds $N_{\max}$, preventive replacement is carried out, and the cost of preventive replacement is C_{PR}. Obviously, the cost of preventive replacement is lower than the cost of replacement after failure, that is, $C_{\mathrm{PR}} < C_{\mathrm{FR}}$.

Similarly, the purpose of this strategy is to find a suitable maintainability threshold D_p to minimize the average cost rate of the system in an update cycle.

4. Simulation of maintenance strategy

Now the simulation algorithm procedures of the above two maintenance strategies are proposed, respectively.

4.1 Real-time monitoring of maintenance strategy

For the real-time monitoring scheme, the key of simulation is to simulate the effect of the shock process. Because of the real-time monitoring, the burst failure of the system can only be brought out by the shocks.

As we can see, there are many parameters in the research which should be set by designer in advance. Certainly, it is crucial both in literature and in practice for the parameter setting issue. For the model parameters, they can be mainly divided into two kinds, one is the model parameters related to the system failure mechanism, such as $\mu, \sigma, x_0, D_f, \lambda, \mu_s, \sigma_s^2$. These parameters are determined based the internal degradation mechanism. In literature, there could be estimated by using the parameter estimation method after collecting the real degradation data. Similar parameter setting has been used in [1–3, 23] and [24], etc.

The other is the economic parameters, such as C_{PM}, C_{PR}, C_{FR}, C_{moni}, C_{Test}, $N_{\max}$. The parameter values could be determined by the engineering practice and the real cost based on the products. It should be noted that the economic parameters are not absolute. They will change due to different monitoring methods, equipment types, operating environments, etc. Similar parameter setting has been used in [17,26] and [29], etc.

In addition to the above two kinds of model parameters, there exist some additional parameters in the optimization procedure of the algorithm, such as maximum iteration number M, iteration time interval Δt. Generally, they are determined based on the balance of the efficiency and cost of the algorithm without absolute values. By the sensitivity analysis, better parameter values could be given when the results are convergent. Specifically in this research, based on the algorithm scale, we use $M = 10\,000$ and $\Delta t = 0.05$ which could get a better output both on the efficiency and time cost.

For clarity, the real-time monitoring maintenance scheme in Algprithm 1 has also been described by the simulation process, as shown in Fig. 4.

Algorithm 1 Simulation procedure of real-time monitoring maintenance scheme

Input Set parameters $x_0, \mu, \sigma, \lambda, \mu_s, \sigma_s, D_f, D_p, C_{\mathrm{PM}}, C_{\mathrm{FR}}, C_{\mathrm{PR}}, C_{\mathrm{moni}}, N_{\max}, M, \Delta t$

Output Compare the cost rates and get the optimal preventive maintenance threshold D_p

Initialization set the preventive maintenance D_p, iterations number $i = 1$;

While $i \leqslant M$,

Let $t = 0$;

Calculate the degradation performance $Z(t) = Z(t-\Delta t) + X(\Delta t) + Y(\Delta t)$;

Let $t \leftarrow t + \Delta t$;

if $Z(t) \geqslant D_f$, then execute replacement after failure, and break;

if $D_p \leqslant Z(t) < D_f$, then go to the next step:

if $N \leqslant N_{\max}$, execute the non-perfect maintenance, let $N \leftarrow N + 1$;

if $N > N_{\max}$, execute the preventive replacement, let $N \leftarrow N + 1$ and break;

if $Z(t) < D_p$, then execute no-action, and let $t \leftarrow t + \Delta t$;

End

Calculate the average cost rate of the system;

Iteration change the value of D_p and recalculate the average cost rate.

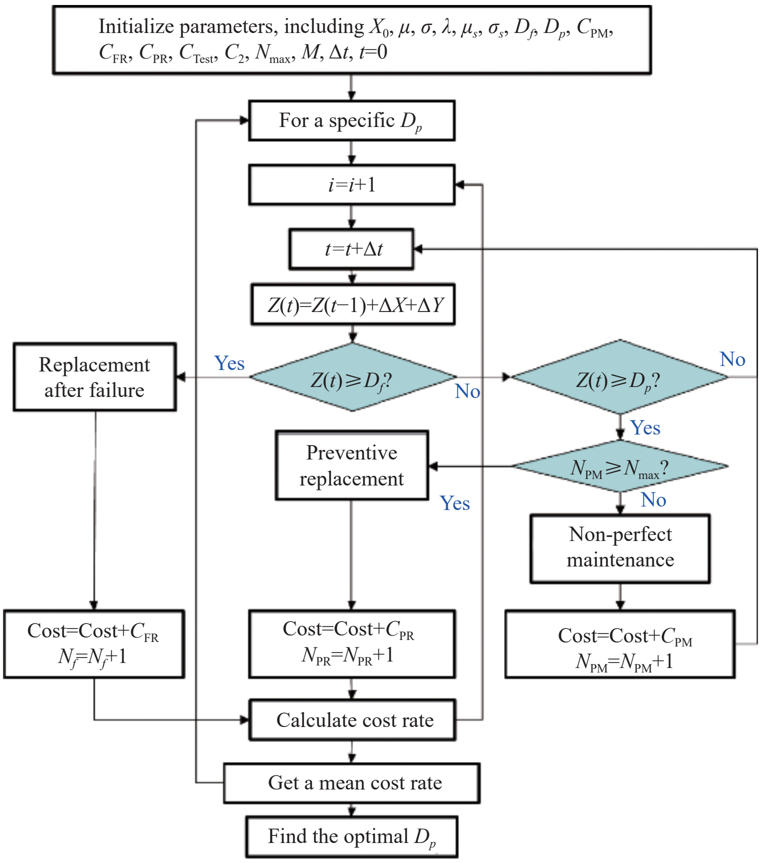

Fig. 4 Simulation flow of real-time monitoring maintenance scheme

4.2 Dynamic periodic detection maintenance strategy

For the dynamic periodic detection maintenance, the algorithm procedure and the flow chart are shown as Algorithm 2.

For clarity, the maintainability model algorithm in Algorithm 2 is expressed as the simulation process, as shown in Fig. 5.

Algorithm 2 Algorithm of dynamic periodic maintenance scheme

Input Set parameters $x_0, \mu, \sigma, \lambda, \mu_s, \sigma_s, D_f, D_p, C_{\mathrm{PM}}, C_{\mathrm{FR}}, C_{\mathrm{PR}}, C_{\mathrm{Test}}, N_{\max}, M, \Delta t$

Output Compare the cost rates and get the optimal preventive maintenance threshold D_p

Initialization Set the preventive maintenance value D_p, iterations number $i = 1$;

While $i \leqslant M$

Let $t = 0$; Calculate the next system state monitoring interval;

At the state monitoring time point, calculate the degradation performance $Z(t) = Z(t-\Delta t) + X(\Delta t) + Y(\Delta t)$, and let $t \leftarrow t + \Delta t$;

Judge:

if $Z(t) \geqslant D_f$, then execute replacement after failure and break;

if $D_p \leqslant Z(t) < D_f$, then go to the next step:

if $N \leqslant N_{\max}$, execute the non-perfect maintenance, let $N \leftarrow N+1$;

if $N > N_{\max}$, execute the preventive replacement, let $N \leftarrow N+1$ and break;

if $Z(t) < D_p$, then execute no-action, and let $t \leftarrow t + \Delta t$;

End

Calculate the average cost rate of the system;

Iteration change the value of D_p and recalculate the average cost rate.

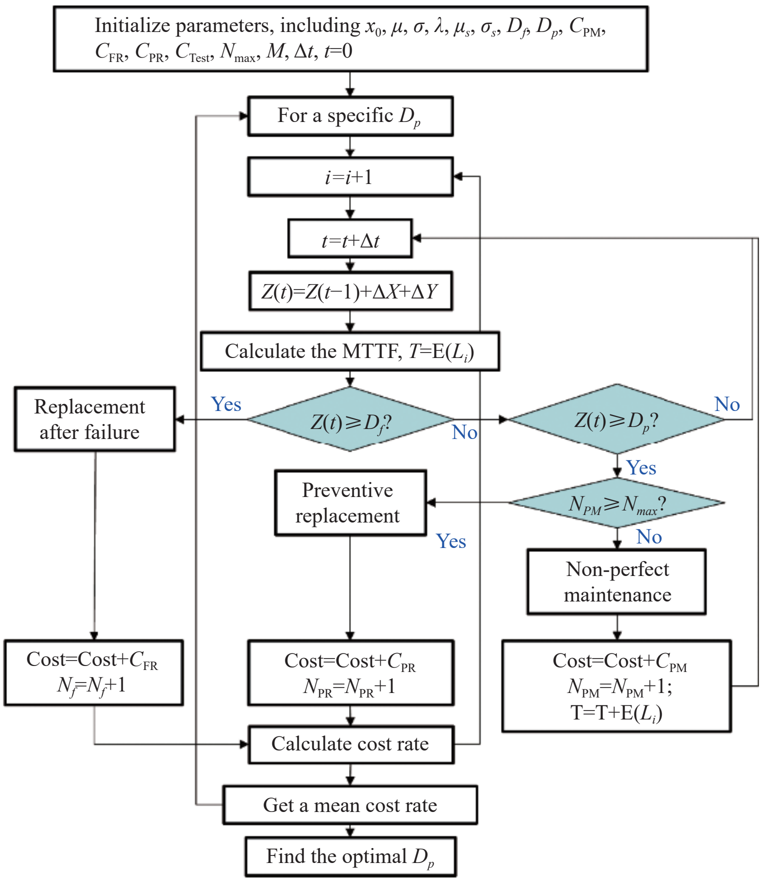

Fig. 5 Simulation process of the dynamic periodic detection maintenance scheme

5. Numerical example

A warship is a complex system composed of a variety of sub-platforms. Different units play different roles in the complex system. And the maintenance of a warship is an important factor to ensure the navigation safety, especially in a long-distance navigation. For the hull, circuit and most mechanical equipment of a warship, it is necessary to check the operating states of the equipment regularly or irregularly, which can correspond to the two degradation detection schemes proposed in this paper. The maintenance strategy of the important equipment in a ship is considered. In order to verify the proposed condition real-time monitoring scheme and dynamic period monitoring scheme, and to improve the ship maintenance and economy, appropriate parameters are selected for analysis.

For the parameters, they can be mainly divided into two kinds, one is the model parameters related to the system failure mechanism. In engineering, these parameters are determined based the internal degradation mechanism. The other is the economic parameters. The parameter values could be determined by the engineering practice and the real cost based on the products.

In this example, first, we assume the common parameters as follows. For the Wiener process, the degradation rate $\mu = 0.3$, the coefficient of fluctuation $\sigma = 0.1$, and the initial degradation value is $x_0 = 0$. The arrival rate of the shock process is $\lambda = 3$. The jth shock magnitude $Y_j \sim \mathrm{N}(\mu_s, \sigma_s^2)$, $\mu_s = 3$, $\sigma_s = 0.5$, and $D_f = 10$. For the maintenance parameters, the preventive maintenance cost $C_{\mathrm{PM}} = 20$, the preventive replacement cost $C_{\mathrm{PR}} = 100$, and the cost of emergency replacement after component failure is $C_{\mathrm{FR}} = 4\,000$.

For the real-time monitoring strategy, it is necessary to set up enough sensing equipment to monitor the system state, and different monitoring cost parameters have a great effect on the results.

It is assumed that the monitoring cost per unit time is $C_{\mathrm{Moni}} = 20$, and the cost for a single periodic inspection scheme is $C_{\mathrm{Test}} = 10$. The maximum number of preventive maintenances $N = 5$. The optimization schemes are shown in Fig. 6 and Fig. 7.

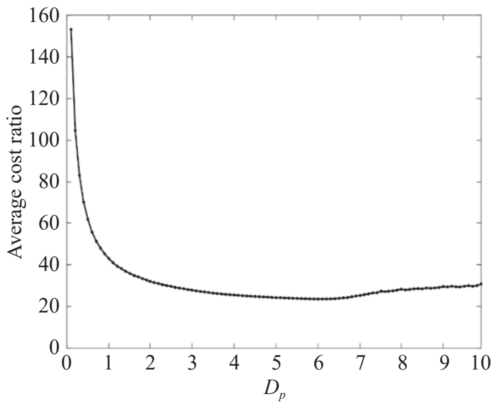

Fig. 6 Optimization scheme of real-time status monitoring strategy

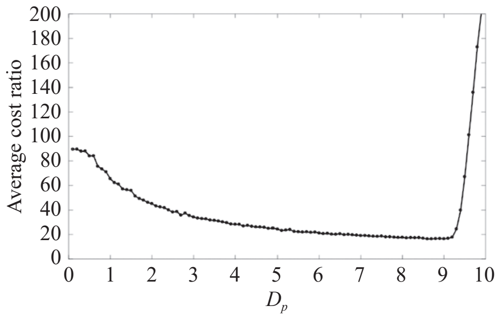

Fig. 7 Optimization scheme of state dynamic periodic detection strategy

For the real-time monitoring scheme, the optimal preventive maintenance threshold can be obtained from the simulation results $D_p = 6.2$, and the minimum cost rate is 21.52.

For the dynamic cycle detection scheme, the optimal preventive maintenance threshold $D_p = 8.6$, and the minimum maintenance cost rate in the long cycle is 16.34.

As can be seen from the figures, with the increase of the preventive maintenance threshold, the average cost rate of the two schemes showed a trend of first decreasing and then increasing. This is because when the preventive threshold is small, the system will incur a higher cost rate due to more frequent maintenance. When the preventive threshold is large or even close to the failure threshold, the cost of system failure will increase due to the increased probability of sudden system failure. Therefore, the trend of first decreasing and then increasing appears in the figure.

As shown in Fig. 6, when the preventive maintenance threshold is large, the cost rate of the system fluctuates less. This is because for the real-time condition-based monitoring, the performance state information of the system is clear, and the probability of sudden failure of the system is small. Therefore, the cost rate rises slowly.

In order to clearly compare the results of the optimization strategy and illustrate the results, the sensitivity analysis of the failure threshold for the dynamic periodic detection strategy has been given in Fig.8. By the sensitivity analysis, with the increase of the failure threshold D_f, the optimal preventive threshold D_p changes. This is consistent with reality. The larger the failure threshold, the larger the survival probability of the system, the larger the optional preventive maintenance threshold space, and the smaller the average cost rate. It is worth noting that parameters such as the failure threshold are related to the intrinsic mechanism of system degradation and can be estimated by parameter estimation through historical data.

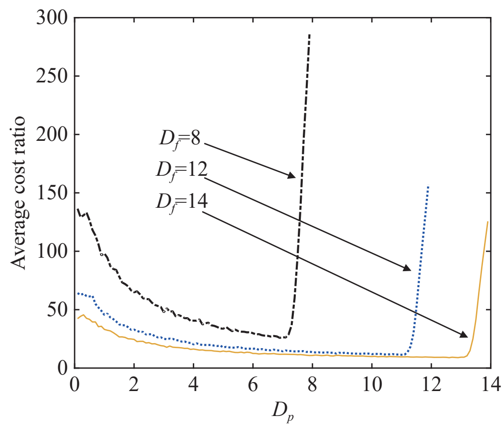

Fig. 8 Sensitivity analysis for the dynamic periodic detection strategy

For the comparison of the two schemes, it can be seen that the dynamic periodic monitoring is better than the real-time monitoring scheme under the above parameter Settings. Of course, if the cost of condition monitoring per unit time and the cost of a single monitoring change, it may have a relatively large impact on the optimization result of the system.

6. Conclusion

In this paper, a Wiener degradation process model with shock effect is considered for system preventive maintenance. Two maintenance strategies, including the real-time monitoring maintenance and the dynamic periodic monitoring maintenance, are constructed respectively, to minimize the long-term operation and maintenance costs. In the cycle detection scheme, a dynamic cycle detection strategy based on average remaining life is proposed. Monte Carlo simulation is used to solve the model, and the optimization problems of preventive maintenance under different strategies are deeply discussed. In conclusion, the optimization results provide significant reference value for reliability engineers in parameter optimization and maintenance decision-making. By implementing the proposed dynamic detection strategies and real-time monitoring schemes, engineers can develop more effective maintenance plans that adapt to actual operating conditions and external shocks. The optimized maintenance thresholds offer clear decision-making guidelines, enabling engineers to balance maintenance costs and system reliability, ultimately enhancing overall efficiency and effectiveness in complex system maintenance management.

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