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Combat task-oriented weapon portfolio selection method

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Abstract: Existing weapon portfolio selection methods do not sufficiently support specific combat tasks, with uncertainty in the decision information. Therefore, a combat task–oriented weapon portfolio selection method that adapts weapon capabilities to combat tasks is proposed. The approach is based on specific combat tasks and weapon background, using fuzzy interval values to describe indicators and the applicability of a weapon portfolio. In addition, an interval entropy weighting method is applied to obtain weight information of indicators. Meanwhile, we define the similarity measure of fuzzy interval values and use the interval fuzzy collaborative filtering algorithm to calculate the fitness of the residual weapons. Furthermore, the interval fuzzy set clustering algorithm clusters the tasks to inform the decision of weapon portfolio. Finally, we verify the method's feasibility and advancement by comparing actual combat tasks as examples with the traditional methods. The contributions of this paper include improvements to the accuracy and reliability of decision-making from the perspective of adapting weapon capability to combat tasks. At the same time, this paper accounts for the method's shortcomings by considering the hesitancy and ambiguity of the indicator data.

Keywords: weapon portfolio selection, interval-value hesitation fuzzy theory, collaborative filtering algorithm, entropy weighting method.

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1. Introduction

Weapon and equipment selection is a complex component of system engineering that determines the future development direction, scale structure, and capability level of various types of weapons and equipment [1]. It also relates to the success of national security and future military struggle, having significant military and research value [2]. Furthermore, the development trend of modern warfare is moving toward joint warfare and system confrontation [3,4]. On this basis, weapon and equipment development decisions are no longer limited to selecting a single high-precision piece of equipment from multiple alternatives but are more concerned with a cross-domain system portfolio assessment and decisions [5–7].

Portfolio selection theory [8] was proposed by Markowitz in 1952. However, in the military field, the first application of portfolio selection was suggested by Buede and Bresnick in 1992 to solve the investment decision problem of the US Navy's equipment program [9]. In recent years, some researchers have proposed novel ways to optimize the weapon portfolio decision method. For example, Liu et al. proposed a weapon portfolio decision method based on the contribution rate of the weapon system [10]. In addition, Xiang et al. proposed a weapon portfolio decision method based on an expert trust network, which improves the scientific nature of the decision method by analyzing expert social networks to assign weights to the decision results of experts [11].

These studies assessed capabilities and made choices from the perspective of the weapon portfolio. Combat tasks are the goals and responsibilities that armed forces need to achieve in combat, and their specific needs and autonomous requirements for equipment guide the selection of equipment combinations [12]. Thus, a traditional method can be considered to optimize the configurations of weapon portfolios and combat tasks.

For instance, collaborative filtering is a simple recommendation method that can achieve the optimal configurations of weapon portfolios and combat tasks. After a long development period, collaborative filtering methods have been used in many fields [13]. The developers of Tapestry [14], one of the earliest recommendation systems, proposed a recommendation algorithm for collaborative filtering. The basic assumption of collaborative filtering algorithms is that the target users prefer items that are liked by users similar to themselves [15]. Similarity computation between users or objects is critical in both user- and item-based approaches. Well-known similarity calculation algorithms include the Pearson product-

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moment correlation coefficient (PCC) [16] and vector space similarity (VSS) algorithms [17].

To address the problem of data sparsity in collaborative filtering methods, Wang et al. proposed a generative probabilistic framework that allows more available data in the user-item matrix for recommendations [18]. Furthermore, Xue et al. introduced a smoothing-based approach that addresses the problem of data sparsity by predicting all missing data in the user-item matrix [19].

Although existing methods can also optimize the configuration between some weapon portfolios and combat tasks, they have limitations. With increasingly complex and uncertain operational environments, weapon portfolio decisions are becoming more difficult [20]. Traditional decision-making methods seldom consider the hesitancy and fuzziness of indicator data. Therefore, standardizing the degree of hesitancy and fuzziness of performance parameters, dealing with uncertain indicators, and developing scientific calculation methods for indicator weights are the critical issues studied in this paper.

To facilitate the description and processing of imprecise and incomplete information, Zadeh established the theory of fuzzy sets and defined the corresponding operation rules [21], after which various extended forms of fuzzy sets were proposed, such as interval [22], intuitionistic [23], and L-fuzzy sets [24]. For example, Eulalia et al. proposed an intuitionistic fuzzy set between similarity measures [25]. Zhang et al. gave an axiomatic definition of the entropy of interval fuzzy sets based on distance measures [26]. Additionally, Xu et al. studied intuitionistic fuzzy multiple-indicator decision-making, cluster decision-making, and clustering [27–33]. Similarly, Mikhailov studied a fuzzy planning method that converts fuzzy planning problems into standard linear planning problems to solve [34]. Finally, Herrera-Viedma et al. studied a process to measure the degree of consistency of fuzzy preference relations based on error analysis [35].

This paper proposes a combat task-oriented weapon portfolio selection method, using fuzzy interval values to describe the suitability of the indicator and the weapon portfolio under a specific combat task. Firstly, we use the interval entropy weight method to obtain the weight of indicators. Secondly, we define the similarity measure of fuzzy interval values and apply the interval fuzzy collaborative filtering algorithm to calculate the residual suitability. Subsequently, the tasks are clustered by using the interval fuzzy set clustering algorithm, and the suitability is integrated on this basis by the internal-valued hesitant fuzzy weighted averaging (IVHFWA) operator. Finally, an illustrative example comparing the method with traditional methods demonstrates its feasibility and progressiveness.

2. Determination of indicator weights

In practical applications, multi-attribute decision-making often requires the consideration of multiple indicators, which often have different levels of importance [36]. Decision-makers (DMs) often hold different attitudes toward the same indicator and find agreement on subjectively determined indicator weights difficult. Additionally, considering the complexity of weapons and the cognitive uncertainty of DMs, the interval hesitation fuzzy theory is applied to the whole process to reflect the opinions of DMs accurately.

2.1 Interval-valued hesitant fuzzy theory

Definition 1 Let X be the reference set and $D[0,1]$ be a closed subset of $[0,1]$. The interval-valued hesitant fuzzy set (IVHFS) with respect to X [37] is

$$\tilde{A}=\left\{\left\langle x_i,\tilde{h}_{\tilde{A}}(x_i)\right\rangle | x_i\in X, i=1,2,\cdots,n\right\} \tag{1}$$

where $\tilde{h}_{\tilde{A}}(x_i)=\left\{\tilde{\gamma}|\tilde{\gamma}\in\tilde{h}_{\tilde{A}}(x_i)\right\}$ is the interval-valued hesitant fuzzy element (IVHFE), which denotes the set of possible intervals of the set X, of which the elements x belong to $\tilde{A}$. In addition, $\tilde{\gamma}=[\tilde{\gamma}^{\mathrm{L}},\tilde{\gamma}^{\mathrm{U}}]$ is a closed subset of $[0,1]$, and $\tilde{\gamma}^{\mathrm{L}}$ and $\tilde{\gamma}^{\mathrm{U}}$ represent the upper and lower bounds of the interval, respectively.

For example, if $X=\{x_1,x_2\}$ is a reference set, the two IVHFEs $\tilde{h}_{\tilde{A}}(x_1)=[0.1,0.3],[0.4,0.6]$ and $\tilde{h}_{\tilde{A}}(x_2)=[0.2,0.4],[0.5,0.7]$ are the elements of the IVHFS in $\tilde{A}$, respectively, denoted as

$$\tilde{A}=\{\langle x_1,\{[0.1,0.3],[0.4,0.6]\}\rangle,\\ \langle x_2,\{[0.2,0.4],[0.5,0.7]\}\rangle\}.$$

Definition 2 Assuming that $\tilde{a}=[\tilde{a}^{\mathrm{L}},\tilde{a}^{\mathrm{U}}]$ and $\tilde{b}=\left[\tilde{b}^{\mathrm{L}},\tilde{b}^{\mathrm{U}}\right]$ are two given interval values and $\lambda\geqslant 0$, the interval operation rule is

(i) $\tilde{a}=\tilde{b}\Leftrightarrow\tilde{a}^{\mathrm{L}}=\tilde{b}^{\mathrm{L}}$ and $\tilde{a}^{\mathrm{U}}=\tilde{b}^{\mathrm{U}}$;

(ii) $\tilde{a}+\tilde{b}=\left[\tilde{a}^{\mathrm{L}}+\tilde{b}^{\mathrm{L}},\tilde{a}^{\mathrm{U}}+\tilde{b}^{\mathrm{U}}\right]$;

(iii) $\lambda\tilde{a}=[\lambda\tilde{a}^{\mathrm{L}},\lambda\tilde{a}^{\mathrm{U}}]$.

Definition 3 Let $\tilde{a}=[\tilde{a}^{\mathrm{L}},\tilde{a}^{\mathrm{U}}]$ and $\tilde{b}=[\tilde{b}^{\mathrm{L}},\tilde{b}^{\mathrm{U}}]$ be two given interval values, $l_{\tilde{a}}=\tilde{a}^{\mathrm{U}}-\tilde{a}^{\mathrm{L}}$, $l_{\tilde{b}}=\tilde{b}^{\mathrm{U}}-\tilde{b}^{\mathrm{L}}$. Then, the possible degree of $\tilde{a}\geqslant\tilde{b}$ is

$$P\left(\tilde{a}\geqslant\tilde{b}\right)=\max\left(1-\max\left(\frac{\tilde{b}^{\mathrm{U}}-\tilde{a}^{\mathrm{L}}}{l_{\tilde{a}}+l_{\tilde{b}}},0\right),0\right). \tag{2}$$

Definition 4 Let $\tilde{h}(x)=\left\{\tilde{\gamma}(x)\middle|\tilde{\gamma}(x)\in\tilde{h}(x)\right\}$ be an IVHFE, then the score function $s\left(\tilde{h}(x)\right)$ of $\tilde{h}(x)$ is

$$s\left(\tilde{h}(x)\right)=\frac{\sum\limits_{\tilde{\gamma}\in\tilde{h}}\tilde{\gamma}}{l_{\tilde{h}}}=\left[\frac{\sum\limits_{\tilde{\gamma}\in\tilde{h}}\tilde{\gamma}^{\mathrm{L}}}{l_{\tilde{h}}},\frac{\sum\limits_{\tilde{\gamma}\in\tilde{h}}\tilde{\gamma}^{\mathrm{U}}}{l_{\tilde{h}}}\right] \tag{3}$$

where $l_{\tilde{h}}$ is the number of intervals in $\tilde{h}(x)$. The score $s\left(\tilde{h}(x)\right)$ of $\tilde{h}(x)$ is obviously also a closed subset of

$[0,1]$. Thus, the superiority of the IVHFEs $\tilde{h}(x_1)$ and $\tilde{h}(x_2)$ is defined [38] as folows:

(i) If $s\left(\tilde{h}(x_1)\right) > s\left(\tilde{h}(x_2)\right)$, then $\tilde{h}(x_1)$ is better than $\tilde{h}(x_2)$, noted as $\tilde{h}(x_1) \succ \tilde{h}(x_2)$.

(ii) If $s\left(\tilde{h}(x_1)\right) = s\left(\tilde{h}(x_2)\right)$, then no superiority relationship exists between $\tilde{h}(x_1)$ and $\tilde{h}(x_2)$, which is written as $\tilde{h}(x_1) \sim \tilde{h}(x_2)$.

To calculate the distance between IVHFEs, they first need to be normalized. For the case of different numbers of interval values in the IVHFEs, the one with less is complemented. The widely used complementary principles are pessimistic and optimistic principles [39]. This paper uses the interval fuzzy collaborative filtering algorithm to complement the missing suitability.

Definition 5 Suppose two intervals have equal lengths of l. The IVHFEs $\tilde{\alpha}$ and $\tilde{\beta}$ have been arranged from the smallest to the largest, $\tilde{\alpha}_{\sigma(i)}$ and $\tilde{\beta}_{\sigma(i)}$, with $i = 1,2,\cdots,l$ denoting the number of intervals with the ith smallest of $\tilde{\alpha}$ and $\tilde{\beta}$. The distance between $\tilde{\alpha}$ and $\tilde{\beta}$ [40] is defined as

$$d\left(\tilde{\alpha},\tilde{\beta}\right) = \sqrt{\frac{1}{l}\sum_{i=1}^{l}\left(\left|\tilde{\alpha}^{\mathrm{L}}_{\sigma(i)} - \tilde{\beta}^{\mathrm{L}}_{\sigma(i)}\right|^2 + \left|\tilde{\alpha}^{\mathrm{U}}_{\sigma(i)} - \tilde{\beta}^{\mathrm{U}}_{\sigma(i)}\right|^2\right)}. \tag{4}$$

Definition 6 Let $\tilde{h}_j\,(j = 1,2,\cdots,n)$ be an IVHFE, $\boldsymbol{\omega} = (\omega_1,\omega_2,\cdots,\omega_n)^{\mathrm{T}}$ be the weight coefficient of the IVHFE with $\omega_i \in [0,1]$, and $\sum_{i=1}^{n}\omega_i = 1$, then the IVHFWA operator [41] is

$$\mathrm{IVHFWA}\left(\tilde{h}_1,\tilde{h}_2,\cdots,\tilde{h}_n\right) = \left\{\left[1-\prod\nolimits_{j=1}^{n}\left(1-\tilde{\gamma}^{\mathrm{L}}_j\right)^{w_j}, 1-\prod\nolimits_{j=1}^{n}\left(1-\widetilde{\gamma^{\mathrm{U}}_j}\right)^{w_j}\right] \middle| \tilde{\gamma}_1 \in \tilde{h}_1, \tilde{\gamma}_2 \in \tilde{h}_2, \cdots, \tilde{\gamma}_n \in \tilde{h}_n\right\}. \tag{5}$$

2.2 Interval-valued entropy weighting method

Weapons portfolio selection and configuration programs involve multiple indicators, which need to be comprehensively evaluated and weighted to better meet the needs of combat tasks. The entropy weight method is a common approach to weight determination that can objectively calculate the weight value of each indicator. However, the problem of the uncertainty of indicator data may affect the application of the entropy weight method. Therefore, Zhu et al. mixed the interval fuzzy algorithm with the entropy weight method and proposed the interval entropy weight method [42]. The calculation process of each indicator weight w is as follows:

(i) Establish the decision matrix.

Assume that the set of indicators of weapon utility or combat task requirement utility is $Q = Q_1,Q_2,\cdots,Q_m$. The set $S = S_1,S_2,\cdots,S_n$ contains the results of n measurements. The ith measurement obtains the value of the jth indicator Q_j as the interval value $\bar{a}_{ij} = \left[a^{\mathrm{L}}_{ij}, a^{\mathrm{U}}_{ij}\right]$, which constitutes the following decision matrix:

$$\bar{\boldsymbol{A}} = \begin{bmatrix} \bar{a}_{11} & \bar{a}_{12} & \cdots & \bar{a}_{1n} \\ \bar{a}_{21} & \bar{a}_{22} & \cdots & \bar{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{a}_{m1} & \bar{a}_{m2} & \cdots & \bar{a}_{mn} \end{bmatrix}.$$

(ii) Standardize the data.

The decision matrix is normalized to

$$\bar{\boldsymbol{P}} = \begin{bmatrix} \bar{p}_{11} & \bar{p}_{12} & \cdots & \bar{p}_{1n} \\ \bar{p}_{21} & \bar{p}_{22} & \cdots & \bar{p}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{p}_{m1} & \bar{p}_{m2} & \cdots & \bar{p}_{mn} \end{bmatrix}.$$

The calculation formula is as follows. If indicator Q_j is a benefit-type indicator,

$$\bar{p}_{ij} = \frac{\bar{a}_{ij}}{\sum_{k=1}^{m}\bar{a}_{kj}}, \tag{6}$$

then

$$\begin{cases} p^{\mathrm{L}}_{ij} = \dfrac{a^{\mathrm{L}}_{ij}}{\sum_{k=1}^{m} a^{\mathrm{U}}_{kj}} \\ p^{\mathrm{U}}_{ij} = \dfrac{a^{\mathrm{U}}_{ij}}{\sum_{k=1}^{m} a^{\mathrm{L}}_{kj}} \end{cases}. \tag{7}$$

If indicator Q_j is a cost-based indicator,

$$\bar{p}_{ij} = \frac{\dfrac{1}{a_{ij}}}{\sum_{k=1}^{m}\dfrac{1}{a_{kj}}} \tag{8}$$

then

$$\begin{cases} p^{\mathrm{L}}_{ij} = \dfrac{\dfrac{1}{a^{\mathrm{U}}_{ij}}}{\sum_{k=1}^{m}\dfrac{1}{a^{\mathrm{L}}_{kj}}} \\ p^{\mathrm{U}}_{ij} = \dfrac{\dfrac{1}{a^{\mathrm{L}}_{ij}}}{\sum_{k=1}^{m}\dfrac{1}{a^{\mathrm{U}}_{kj}}} \end{cases}. \tag{9}$$

Therefore,

$$\sum_{k=1}^{m} p^{\mathrm{L}}_{ij} \leqslant 1, \sum_{k=1}^{m} p^{\mathrm{U}}_{ij} \geqslant 1,\ \ j = 1,2,\cdots,n. \tag{10}$$

(iii) Compute the indicator entropy.

The entropy of indicator Q_j is

$$H_j = -k\sum_{i=1}^{m} p_{ij}\ln p_{ij},\ \ j = 1,2,\cdots,n \tag{11}$$

where $k = (\ln m)^{-1}$, assuming that $p_{ij}\ln p_{ij} = 0$ when $p_{ij} = 0$.

To compute the entropy of the indicator $H_j = \left[H_j^{\mathrm{L}}, H_j^{\mathrm{U}}\right]$, two optimization models can be developed:

$$H_j^{\mathrm{L}} = \min\left\{-k\sum_{i=1}^{m} p_{ij}\ln p_{ij}\right\}$$
$$\text{s.t.}\begin{cases} p_{ij}^{\mathrm{L}} \leqslant p_{ij} \leqslant p_{ij}^{\mathrm{U}},\ \ i = 1,2,\cdots,m \\ \displaystyle\sum_{i=1}^{m} p_{ij} = 1 \end{cases} \tag{12}$$

and

$$H_j^{\mathrm{U}} = \max\left\{-k\sum_{i=1}^{m} p_{ij}\ln p_{ij}\right\}$$
$$\text{s.t.}\begin{cases} p_{ij}^{\mathrm{L}} \leqslant p_{ij} \leqslant p_{ij}^{\mathrm{U}},\ \ i = 1,2,\cdots,m \\ \displaystyle\sum_{i=1}^{m} p_{ij} = 1 \end{cases}. \tag{13}$$

(iv) Calculate the entropy weight.

After obtaining the interval entropy value of the indicator $\bar{H}_j = \left[H_j^{\mathrm{L}}, H_j^{\mathrm{U}}\right]$ $(j = 1,2,\cdots,n)$, the interval entropy weight $\bar{\omega}_j = \left[\omega_j^{\mathrm{L}}, \omega_j^{\mathrm{U}}\right]$ of the indicator Q_j can be calculated by

$$\bar{\omega}_j = \frac{1-\bar{H}_j}{n-\displaystyle\sum_{j=1}^{n}\bar{H}_j},\ \ j = 1,2,\cdots,n, \tag{14}$$

which can be written as

$$\begin{cases} \omega_j^{\mathrm{L}} = \dfrac{1-H_j^{\mathrm{U}}}{n-\displaystyle\sum_{j=1}^{n}H_j^{\mathrm{L}}} \\ \omega_j^{\mathrm{U}} = \dfrac{1-H_j^{\mathrm{L}}}{n-\displaystyle\sum_{j=1}^{n}H_j^{\mathrm{U}}} \end{cases},\ \ j = 1,2,\cdots,n. \tag{15}$$

(v) Normalize the entropy.

The obtained entropy weights are normalized by the following formula:

$$\bar{\omega}_j' = \frac{\bar{\omega}_j}{\omega_j^{+}},\ \ j = 1,2,\cdots,n \tag{16}$$

where $\bar{\omega}_j = \left[\omega_j^{\mathrm{L}}, \omega_j^{\mathrm{U}}\right]$ and $\omega_j^{+} = \max\{\omega_j^{\mathrm{U}} | j = 1,2,\cdots,n\}$, and can be written as

$$\begin{cases} \omega_j^{\mathrm{L}\prime} = \dfrac{\omega_j^{\mathrm{L}}}{\omega_j^{+}} \\ \omega_j^{\mathrm{U}\prime} = \dfrac{\omega_j^{\mathrm{U}}}{\omega_j^{+}} \end{cases},\ \ j = 1,2,\cdots,n. \tag{17}$$

(vi) Obtain the assessment conclusion.

3. Calculation of portfolio suitability

The suitability of the weapon portfolio and combat tasks is an essential reference basis for achieving the optimal configuration of weapon and combat tasks. However, in practice, because of the high complexity of weapons and combat tasks, many values are often missing in the suitability matrix of historical tasks and weapons. Therefore, filling in the missing suitability based on the original suitability data is the key to achieving the weapon portfolio selection.

Collaborative filtering is a widely used method in re commender systems that predicts the missing preference information by analyzing similarities. In this paper, since the original suitability is expressed by the interval value, the interval collaborative filtering algorithm is proposed to calculate the missing suitability.

3.1 Similarity of portfolios and tasks

Assume that the set of m historical tasks is $T = t_1, t_2, \cdots, t_m$ and the set of n weapon portfolios is $E = e_1, e_2, \cdots, e_n$. The suitability of portfolio e_j to historical task t_i is the interval value $\bar{r}_{ij} = \left[r_{ij}^{\mathrm{L}}, r_{ij}^{\mathrm{U}}\right]$, which forms the following interval value suitability matrix:

$$\bar{\boldsymbol{R}} = \begin{bmatrix} \bar{r}_{11} & \bar{r}_{12} & \cdots & \bar{r}_{1n} \\ \bar{r}_{21} & \bar{r}_{22} & \cdots & \bar{r}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{r}_{m1} & \bar{r}_{m2} & \cdots & \bar{r}_{mn} \end{bmatrix}.$$

The PCC is used in many collaborative filtering recommender systems to measure similarity because it is easy to implement and highly accurate compared to other methods. Similarity is calculated based on the common suitability between two combat tasks a and u as

$$\mathrm{Sim}(a,u) = \frac{\displaystyle\sum_{i\in I(a)\cap I(u)} (r_{ai}-\bar{r}_a)\cdot(r_{ui}-\bar{r}_u)}{\sqrt{\displaystyle\sum_{i\in I(a)\cap I(u)} (r_{ai}-\bar{r}_a)^2}\cdot\sqrt{\displaystyle\sum_{i\in I(a)\cap I(u)} (r_{ui}-\bar{r}_u)^2}} \tag{18}$$

where $\mathrm{Sim}(a,u)$ denotes the similarity between combat tasks a and u, portfolio i belongs to the set of portfolios with common suitability between combat tasks a and u, r_{ai} denotes the suitability of portfolio i to combat task a, and $\bar{r}_a$ denotes the average suitability of the combat task a. Obviously, $\mathrm{Sim}(a,u) \in [0,1]$, and a larger $\mathrm{Sim}(a,u)$ means that the combat task a is more similar to u.

Calculating the similarity between portfolios in the portfolio-based collaborative filtering algorithm is similar to the above method. The specific formula is

$$\mathrm{Sim}(i,j)=\frac{\sum\limits_{u\in U(i)\cap U(j)}(r_{ui}-\bar{r}_i)\cdot(r_{uj}-\bar{r}_j)}{\sqrt{\sum\limits_{u\in U(i)\cap U(j)}(r_{ui}-\bar{r}_i)^2}\cdot\sqrt{\sum\limits_{u\in U(i)\cap U(j)}(r_{uj}-\bar{r}_j)^2}} \tag{19}$$

where $\mathrm{Sim}(i,j)$ denotes the similarity between portfolio i and portfolio j. Combat task u belongs to the set of combat tasks with common suitability between portfolio i and j; r_{ui} is the suitability of portfolio i to combat task u, and $\bar{r}_i$ is the average suitability of portfolio i. Obviously, $\mathrm{Sim}(i,j)\in[0,1]$, where a larger value of $\mathrm{Sim}(i,j)$ means that portfolio i is more similar to portfolio j.

The collaborative filtering algorithm based on the PCC can achieve better results compared to other algorithms because it considers the uniqueness factor of combat task suitability. However, the PCC may overestimate the similarity of combat tasks with fewer suitability records. Thus, Mclaughlin et al. proposed the method of adding a significance weight factor to reduce the weight of similarity with fewer suitability records [43]. The method uses the following improved formula:

$$\mathrm{Sim}'(a,u)=\frac{\mathrm{Min}(|I(a)\cap I(u)|,\gamma)}{\gamma}\cdot\mathrm{Sim}(a,u). \tag{20}$$

Similarly, the improved similarity between portfolios is calculated as

$$\mathrm{Sim}'(i,j)=\frac{\mathrm{Min}(|U(i)\cap U(j)|,\delta)}{\delta}\cdot\mathrm{Sim}(i,j) \tag{21}$$

where both γ and δ are significance adjustment parameters that can be appropriately scaled according to the sparsity of the original suitability matrix.

3.2 Suitability estimation

3.2.1 Similar nearest neighbor selection

Similar nearest neighbor selection is a key step in predicting the missing suitability. If the selected nearest neighbors have low similarity to the target combat task, the estimated value of the missing suitability is inaccurate, eventually affecting the prediction results. To overcome the shortcomings of the conventional Top-N nearest neighbor selection algorithm, a threshold value η is introduced. If the similarity is greater than η, it is selected as a similar nearest neighbor.

For each missing suitability r_{ui}, the set $S(u)$ consisting of similar nearest neighbors of the combat task u is defined as

$$S(u)=\left\{u_a\,\middle|\,\mathrm{Sim}'(u_a,u)>\eta,u_a\neq u\right\} \tag{22}$$

where $\mathrm{Sim}'(u_a,u)$ is obtained from (20) and η is the portfolio similarity threshold. Similarly, for each missing suitability r_{ui}, the set $S(i)$ consisting of similar nearest neighbors of portfolio i is defined as

$$S(i)=\left\{i_k\,\middle|\,\mathrm{Sim}'(i_k,i)>\theta,i_k\neq i\right\} \tag{23}$$

where $\mathrm{Sim}'(i_k,i)$ is obtained from (21) and θ is the portfolio similarity threshold. Clearly, the selection of thresholds η and θ is a critical factor that influences the effectiveness of the portfolio decision.

In practical applications, we can make appropriate adjustments according to the sparsity of the original fitness matrix and the prediction results. Specifically, when the original fitness matrix is dense, the similarity threshold should be increased to avoid too many similar nearest neighbors; when the original fitness matrix is sparse, the similarity threshold should be reduced to prevent an insufficient number of similar nearest neighbors. In addition, we can find the optimal similarity threshold value by repeated experiments and combining it with the actual application requirements.

3.2.2 Missing suitability estimates

Using only the combat mission- or the scenario-based method to estimate the missing suitability may ignore precious information that can make the estimation more accurate. Hence, the two methods are systematically combined in this paper.

First, for the missing suitability r_{ui}, the similar nearest neighbor sets $S(u)$ and $S(i)$ of combat task u and portfolio i are selected according to (22) and (23), respectively. The missing suitability is estimated as follows.

If $S(u)=\varnothing\wedge S(i)=\varnothing$, there are no similar nearest neighbors for both combat task u and portfolio i; thus, it is filled with $[0,0]$.

If $S(u)\neq\varnothing\wedge S(i)\neq\varnothing$, this means that similar nearest neighbors exist for both combat task u and portfolio i. The estimated value $P(r_{ui})$ of the missing suitability r_{ui} is then calculated as

$$P(r_{ui})=\lambda\left(\bar{u}+\frac{\sum\limits_{u_a\in S(u)}\mathrm{Sim}'(u_a,u)\cdot(r_{u_a i}-\bar{u}_a)}{\sum\limits_{u_a\in S(u)}\mathrm{Sim}'(u_a,u)}\right)+(1-\lambda)\left(\bar{i}+\frac{\sum\limits_{i_k\in S(i)}\mathrm{Sim}'(i_k,i)\cdot\left(r_{ui_k}-\bar{i}_k\right)}{\sum\limits_{i_k\in S(i)}\mathrm{Sim}'(i_k,i)}\right) \tag{24}$$

where λ is a parameter in the range $[0,1]$ that adjusts the percentage of both combat task- and portfolio-based methods.

If $S(u)=\varnothing\wedge S(i)\neq\varnothing$, there are no similar nearest neighbor exists for combat task u while similar nearest neighbor exists for portfolio i. Then, the estimated value

$P(r_{ui})$ of the missing suitability r_{ui} is calculated as

$$P(r_{ui}) = \bar{u} + \frac{\sum\limits_{u_a \in S(u)} \mathrm{Sim}'(u_a, u) \cdot (r_{u_a i} - \bar{u}_a)}{\sum\limits_{u_a \in S(u)} \mathrm{Sim}'(u_a, u)}. \tag{25}$$

If $S(u) \neq \varnothing \wedge S(i) = \varnothing$, there are similar nearest neighbors exist for combat task u while similar nearest neighbors do not exist for portfolio i, then the estimated value $P(r_{ui})$ of the missing suitability r_{ui} is calculated as

$$P(r_{ui}) = \bar{i} + \frac{\sum\limits_{i_k \in S(i)} \mathrm{Sim}'(i_k, i) \cdot \left(r_{ui_k} - \bar{i}_k\right)}{\sum\limits_{i_k \in S(i)} \mathrm{Sim}'(i_k, i)}. \tag{26}$$

4. Weapon portfolio selection

The prerequisite for making a weapon portfolio selection for a new task is to derive estimates of the suitability of each portfolio for the new task and rank the portfolios. Therefore, calculating the estimates of suitability is critical to the correctness of the selection results.

4.1 Interval-valued fuzzy set clustering algorithm

The principle of the interval-valued fuzzy set clustering algorithm can be summarized as a representation of samples and clusters, similarity measure method, and clustering strategy.

4.1.1 Representation of samples and clusters

Each sample is represented as a vector in the feature space, and each cluster is represented by its center of mass, i.e., the mean value of each sample in the cluster. The expression for the cluster center is according to

$$\mu\left(\tilde{h}_1, \tilde{h}_2, \cdots, \tilde{h}_n\right) = \left\{ \left[\frac{1}{n} \sum_{i=1}^{n} \tilde{h}_{ij}^{\mathrm{L}}, \frac{1}{n} \sum_{i=1}^{n} \tilde{h}_{ij}^{\mathrm{U}} \right] \Big/ j = 1, 2, \cdots, m \right\} \tag{27}$$

where the interval value $\tilde{h}_{ij} = \left[\tilde{h}_{ij}^{\mathrm{L}}, \tilde{h}_{ij}^{\mathrm{U}}\right]$ denotes the value of the indicator Q_j of the task t_i in the cluster and $\tilde{h}_i$ represents the tasks in the cluster.

4.1.2 Similarity measure method

The similarity measures between samples, between clusters, and between samples and clusters are expressed by using the weighted Euclidean distance of interval values. The whole cluster is replaced by the cluster center coordinates, of which the expression is

$$d\left(\tilde{\alpha}, \tilde{\beta}\right) = \sqrt{\sum_{i=1}^{m} \omega_i \left(\left| \tilde{\alpha}_i^{\mathrm{L}} - \tilde{\beta}_i^{\mathrm{L}} \right|^2 + \left| \tilde{\alpha}_i^{\mathrm{U}} - \tilde{\beta}_i^{\mathrm{U}} \right|^2 \right)} \tag{28}$$

where $\tilde{\alpha}$ and $\tilde{\beta}$ are two samples or clusters and ω_i is the indicator's weight. Similarity is geometrically equivalent to the distance between two vectors in the feature space, and a larger distance indicates a lower similarity and vice versa.

4.1.3 Clustering strategy

In the initial state of clustering, each sample is treated as a cluster individually, and at this time, the sample coordinates are at the center of mass of each cluster. Then, the similarity between two clusters is calculated by using the above similarity calculation (28), and the distance matrix is obtained.

In the distance matrix, the smallest similarity is found, and the two corresponding clusters are merged into a new cluster, while the cluster center is updated per (27).

The similarity between the new cluster and other clusters is calculated with (28), and the distance matrix is updated.

The above steps of merging the new clusters and updating the distance matrix are repeated until the clustering termination condition is reached. In this paper, the clustering termination condition is when the number of samples in the cluster with the highest similarity to the new task reaches or exceeds the threshold ϑ.

For ease of presentation, the above clustering process is shown in Fig. 1.

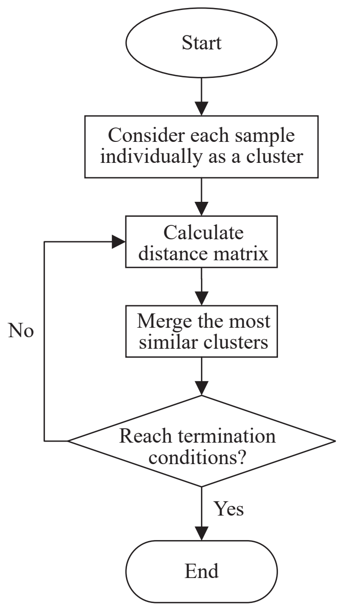

Fig. 1 Flow chart of interval fuzzy clustering algorithm

4.2 Portfolio ranking

After clustering, the cluster with the highest similarity to the new task can be used as a reference for the suitability estimation of each weapon portfolio for the new task. The suitability estimation is obtained by calculating the information integration operator [44] with

$$\hat{r}_{\text{new},j}=\sum_{t_i\in C(t_{\text{new}})}\left(\frac{d(t_i,t_{\text{new}})}{\sum\limits_{k\in C(t_{\text{new}})} d(t_k,t_{\text{new}})}\bar{r}_{ij}\right)$$

where $\hat{r}_{\text{new},j}$ denotes the estimated value of the new task t_{new} to the portfolio e_j and $C(t_{\text{new}})$ is the set consisting of the combat tasks in the cluster most similar to the new task.

Finally, the portfolio is ranked by calculating the possibility degree between the hesitant fuzzy sets using (2), and then the optimal weapon portfolio is derived. The specific process of the proposed selection method in this paper is shown in Fig. 2.

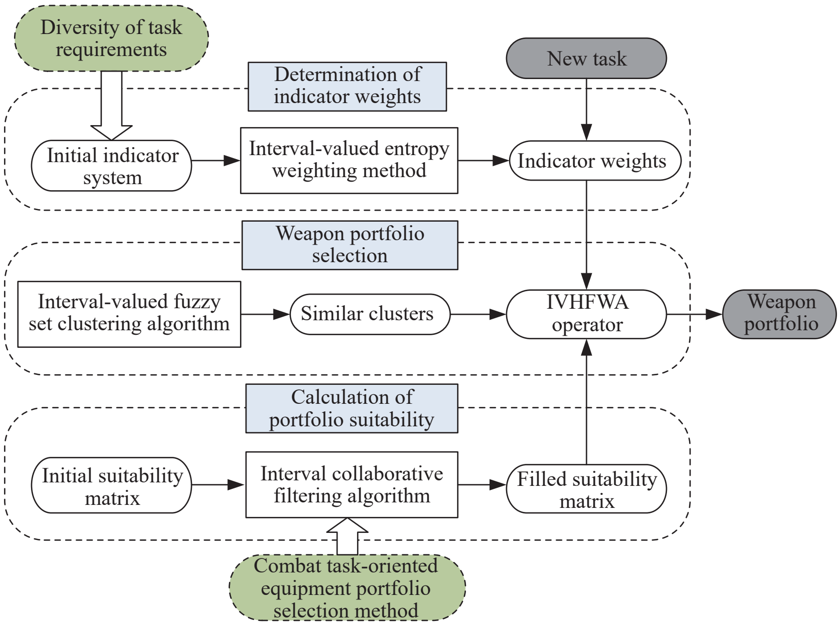

Fig. 2 Selection method process of combat task-oriented weapon portfolio

5. Illustrative example

In 2008, the US Army updated the Joint Capability Areas (JCAs) to summarize the capabilities of military weapons systematically. Twenty-two areas are considered, and each area is subdivided into 240 sub-capabilities. The proposal of JCAs can effectively eliminate problems caused by the inconsistent comprehension of capabilities in multiple sectors. To facilitate the research and analysis of the example, five of the more representative joint sub-capabilities are selected as the indicators $Q=\{q_1,q_2,q_3,q_4,q_5\}$ to be evaluated for the alternative weapon portfolio: joint ground capability q_1, joint air capability q_2, joint maritime capability q_3, specific deterrence capability q_4, and joint mobility capability q_5. Utility of weapon portfolios and combat task requirements are reflected through these five joint sub-capabilities to verify the practicality and feasibility of the method proposed in this paper. A new combat task is proposed, followed by expert analysis. The indicator t_{new} for this task is

$$t_{\text{new}}=[[0.27,0.52],[0.51,0.75],[0.17,0.38],[0.38,0.57],[0.5,0.73]].$$

Based on historical records, ten historical combat tasks $T=\{t_1,t_2,\cdots,t_{10}\}$ are evaluated, and their corresponding requirement utility is $C^A=\{c_1^A,c_2^A,\cdots,c_{10}^A\}$. In addition, in terms of weapon portfolios, ten portfolios $E=\{e_1,e_2,\cdots,e_{10}\}$ are available, and the utility corresponding to the portfolios is $C^B=\{c_1^B,c_2^B,\cdots,c_{10}^B\}$. Because of the uncertainty of combat and the ambiguity of human perception, the utility of the combat task requirements and weapon portfolios and the suitability of the portfolios to the combat tasks cannot be measured with numerical precision; the suitability information is lacking. Therefore, both the weapon portfolio and the combat task are evaluated under multiple indicators to obtain interval information, where the utility of the portfolio, the utility of the combat task requirements, and the suitability matrix of the portfolio to the combat task are shown in Tables 1–3.

Table 1 Efficiency of weapon portfolio

q	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}
q_1	[0.16,0.29]	[0.46,0.94]	[0.55,0.59]	[0.46,0.57]	[0.50,0.38]	[0.27,0.70]	[0.31,0.50]	[0.20,0.62]	[0.09,0.28]	[0.24,0.54]
q_2	[0.46,0.81]	[0.34,0.31]	[0.48,0.74]	[0.45,0.43]	[0.40,0.37]	[0.34,0.65]	[0.39,0.44]	[0.37,0.47]	[0.39,0.75]	[0.36,0.67]

Continued

q	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}
q_3	[0.59,0.54]	[0.05,0.50]	[0.79,0.54]	[0.27,0.61]	[0.52,0.48]	[0.29,0.58]	[0.36,0.28]	[0.08,0.41]	[0.39,0.54]	[0.45,0.62]
q_4	[0.53,0.49]	[0.51,0.55]	[0.34,0.95]	[0.71,0.39]	[0.25,0.41]	[0.17,0.70]	[0.67,0.47]	[0.36,0.48]	[0.29,0.45]	[0.15,0.40]
q_5	[0.40,0.76]	[0.46,0.70]	[0.20,0.56]	[0.36,0.56]	[0.56,0.72]	[0.39,0.65]	[0.33,0.57]	[0.11,0.47]	[0.35,0.52]	[0.46,0.70]

Table 2 Required effectiveness values of historical combat tasks

q	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}
q_1	[0.06,0.33]	[0.73,0.92]	[0.42,0.61]	[0.37,0.59]	[0.23,0.40]	[0.45,0.69]	[0.38,0.56]	[0.35,0.54]	[0.07,0.21]	[0.31,0.54]
q_2	[0.54,0.79]	[0.08,0.27]	[0.53,0.76]	[0.20,0.35]	[0.21,0.43]	[0.45,0.65]	[0.23,0.47]	[0.29,0.49]	[0.58,0.71]	[0.51,0.67]
q_3	[0.36,0.55]	[0.30,0.47]	[0.32,0.60]	[0.41,0.70]	[0.24,0.49]	[0.37,0.58]	[0.03,0.27]	[0.15,0.43]	[0.36,0.58]	[0.35,0.62]
q_4	[0.27,0.49]	[0.35,0.58]	[0.74,0.90]	[0.17,0.34]	[0.23,0.46]	[0.59,0.79]	[0.32,0.49]	[0.28,0.44]	[0.30,0.51]	[0.17,0.38]
q_5	[0.59,0.80]	[0.53,0.74]	[0.41,0.53]	[0.39,0.56]	[0.55,0.79]	[0.46,0.74]	[0.41,0.56]	[0.24,0.43]	[0.28,0.46]	[0.50,0.66]

Table 3 Initial interval suitability matrix of weapon portfolio to combat tasks

t	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}
t_1	[0.23,0.38]	0	0	0	0	0	0	0	0	[0.42,0.57]
t_2	[0.21,0.47]	[0.42,0.61]	[0.74,0.92]	[0.38,0.61]	[0.36,0.61]	[0.21,0.41]	0	0	0	0
t_3	[0.74,0.97]	0	0	[0.70,0.95]	0	0	[0.75,0.92]	0	[0.09,0.23]	0
t_4	0	0	0	0	[0.59,0.79]	[0.59,0.75]	0	[0.06,0.25]	0	0
t_5	0	0	[0.76,0.94]	0	[0.20,0.45]	0	[0.18,0.43]	0	0	0
t_6	[0.04,0.28]	[0.05,0.21]	0	[0.74,0.93]	0	[0.26,0.40]	[0.41,0.57]	[0.53,0.77]	0	0
t_7	[0.35,0.64]	0	[0.78,1.00]	0	[0.56,0.80]	0	0	0	[0.08,0.23]	0
t_8	[0.18,0.39]	0	[0.40,0.65]	0	[0.75,0.93]	[0.23,0.38]	[0.38,0.58]	0	0	[0.57,0.77]
t_9	0	0	0	0	0	0	0	0	[0.26,0.47]	0
t_{10}	[0.00,0.22]	[0.25,0.45]	0	0	0	0	0	0	[0.43,0.61]	[0.60,0.73]

5.1 Example calculation

Based on the data in Tables 1–3, the indicator weights are first calculated by the interval entropy weighting method. Then, the missing suitability is completed by using the interval collaborative filtering method. Next, the suitability of each portfolio to the new task is estimated by using the clustering method. Finally, the optimal portfolio is determined.

Step 1 Determination of indicator weights

First, the decision matrix of the historical tasks is normalized according to (7) to obtain the normalized decision matrix:

$$\bar{\boldsymbol{P}}^{\mathrm{T}}=\begin{bmatrix}[0.02,\ 0.04] & [0.06,\ 0.11] & [0.04,\ 0.08] & [0.04,\ 0.08] & [0.06,\ 0.10]\\ [0.07,\ 0.12] & [0.02,\ 0.04] & [0.03,\ 0.07] & [0.04,\ 0.09] & [0.05,\ 0.09]\\ [0.05,\ 0.08] & [0.06,\ 0.10] & [0.04,\ 0.08] & [0.08,\ 0.14] & [0.04,\ 0.06]\\ [0.04,\ 0.08] & [0.02,\ 0.05] & [0.05,\ 0.10] & [0.02,\ 0.05] & [0.04,\ 0.07]\\ [0.03,\ 0.05] & [0.03,\ 0.06] & [0.03,\ 0.07] & [0.03,\ 0.07] & [0.06,\ 0.10]\\ [0.05,\ 0.09] & [0.05,\ 0.09] & [0.04,\ 0.08] & [0.07,\ 0.13] & [0.05,\ 0.09]\\ [0.04,\ 0.08] & [0.03,\ 0.06] & [0.01,\ 0.04] & [0.04,\ 0.08] & [0.04,\ 0.07]\\ [0.04,\ 0.07] & [0.03,\ 0.07] & [0.02,\ 0.06] & [0.03,\ 0.07] & [0.03,\ 0.05]\\ [0.01,\ 0.03] & [0.06,\ 0.10] & [0.04,\ 0.08] & [0.04,\ 0.08] & [0.03,\ 0.06]\\ [0.04,\ 0.07] & [0.05,\ 0.09] & [0.04,\ 0.09] & [0.03,\ 0.06] & [0.05,\ 0.08]\end{bmatrix}.$$

Then, the optimization model is solved according to (12) and (13) to obtain the entropy value:

$$\boldsymbol{H}^{\mathrm{T}}=[[-0.99,0.93],[-0.99,0.94],[-0.99,0.93],\\ [-0.99,0.91],[-0.99,0.94]].$$

Finally, according to (17), the range of each indicator weight is

$$\bar{\omega}^{\mathrm{T}} = [[0.20,0.20],[0.20,0.21],[0.17,0.20],[0.20,0.24],[0.18,0.20]].$$

To facilitate the subsequent missing suitability filling and portfolio ranking, the mean value of the indicator weight interval is chosen as the final weight vector:

$$\omega^{\mathrm{T}} = [0.203,0.216,0.165,0.236,0.180].$$

Step 2 Calculation of portfolio suitability

According to the initial suitability matrix in Table 3, the missing suitabilities are filled with the interval collaborative filtering algorithm. The similarity between weapon portfolios and combat tasks is first calculated according to (18) and (19) by setting the significance adjustment parameter as $\gamma = \delta = 3$, respectively. Then the improved similarity is calculated according to (20) and (21). Since the calculation process is relatively simple and takes up a large space, it is not shown here.

Next, the similar nearest neighbors are determined based on the similarity between portfolios and combat tasks, at which point we set a similarity threshold of $\eta = \theta = 0.7$.

Finally, based on the determined similar nearest neighbors, all missing suitabilities are calculated according to (24)–(26).

Step 3 Weapon portfolio selection

For the new task, the interval fuzzy set clustering algorithm is first performed, and the estimated values of the suitability of each portfolio to the new task are calculated based on the cluster that is the most similar to the new task.

In the initial state of clustering, each historical task t_i is represented as a cluster s_i. The similarity between clusters is calculated according to (28). The two clusters with the greatest similarity are merged into one cluster. The center of the merged new cluster is calculated according to (27), and the distance matrix is updated. The above steps of merging clusters, calculating cluster centers, and updating the distance matrix are repeated until the number of tasks in the cluster most similar to the new task reaches the threshold value ϑ. In this example, the threshold value $\vartheta = 3$ is set. The composition of clusters when the clustering termination condition is reached is shown in Table 4.

Table 4 Composition of the cluster when the clustering termination condition is reached

Cluster	Task
s_1	t_5, t_8
s_2	t_2, t_6
s_3	$t_1, t_3, t_4, t_7, t_9, t_{10}$

At this point, the cluster with the highest similarity to the new task is s_3. The indicator values of the tasks in the clusters are integrated according to (32) to obtain the estimated suitability of each portfolio to the new task, as shown in Table 5.

Table 5 Estimates of the suitability

e	Value
e_1	[0.46, 0.60]
e_2	[0.35, 0.54]
e_3	[0.31, 0.56]
e_4	[0.35, 0.52]
e_5	[0.32, 0.53]
e_6	[0.29, 0.49]
e_7	[0.32, 0.44]
e_8	[0.26, 0.44]
e_9	[0.26, 0.40]
e_{10}	[0.24, 0.40]

Finally, a two-by-two comparison of the suitability of different portfolios is performed according to (2). The final ranking of the portfolios is $e_4 \succ e_3 \succ e_5 \succ e_7 \succ e_2 \succ e_{10} \succ e_8 \succ e_6 \succ e_1 \succ e_9$.

5.2 Discussion and analysis of results

In the above example research, the parameter settings impact the decision effect of the portfolio in three main ways. First, they affect the sparsity of the filled suitability matrix. Too high sparsity leads to no results, and too low sparsity leads to poor personalization of the decision results. Second, they influence the degree of differentiation of the different portfolios, i.e., the degree to which the optimal portfolio is distinguished from the others. Third, they affect the final ranking of the portfolios.

To analyze the impact of the values of similarity thresholds η and θ on the sparsity of the suitability matrix, 1 000 original suitability matrices with a sparsity of 0.3 are randomly generated, and the average sparsity of the suitability matrix after filling is calculated under different parameter settings. The variation of the sparsity with the parameter values is shown in Fig. 3.

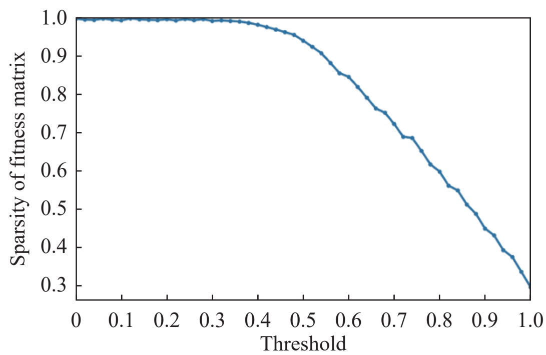

Fig. 3 Sparsity as a function of threshold parameter

Fig. 3 shows that as the threshold parameter increases from 0 to 0.4, the change in matrix sparsity is not yet obvious. Then, the sparsity starts to decrease when the threshold is greater than 0.4. Therefore, a threshold parameter between 0.4 and 0.5 is reasonable when considering the sparsity of the filled suitability matrix, which can achieve a higher sparsity while maintaining the specificity of the combat tasks.

Next, we analyze the effect of the threshold parameter on the differentiation of each portfolio. First, the differentiation is derived from the variance of the mean of the final suitability of each portfolio. Then, the differentiation of each portfolio under different parameter settings is calculated, and the results are shown in Fig. 4.

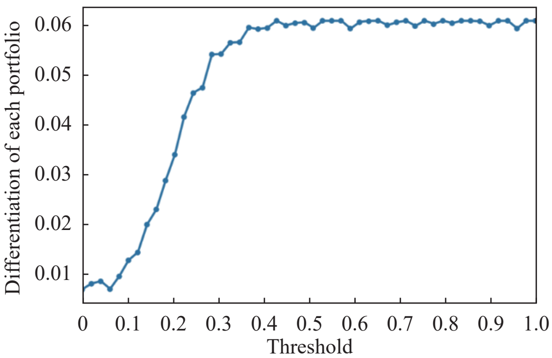

Fig. 4 Variation of discrimination with threshold parameter

As the threshold parameter increases from 0 to 0.3, the differentiation of each portfolio increases; the differentiation remains stable when the threshold is greater than 0.3. Therefore, a threshold parameter of about 0.3 is reasonable when considering the differentiation of each portfolio.

To analyze the influence of threshold parameters on the ranking of alternatives, the portfolio ranking is calculated with different parameter settings. To clearly demonstrate the changes in ranking, the top six portfolios are selected for analysis, and the results are shown in Fig. 5.

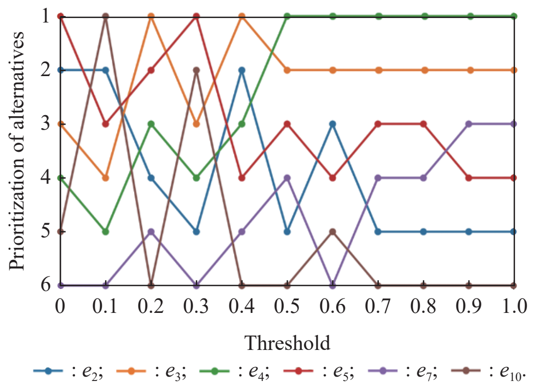

Fig. 5 Prioritization of alternatives as a function of threshold parameter

Once the threshold parameter exceeds 0.7, the priorities of the portfolios are more stable. Thus, a threshold parameter around 0.7 is reasonable when considering the final ranking of the portfolios, which can integrate the final ranking results with appropriate historical experience.

The optimal value of the threshold parameter varies from different perspectives. Therefore, it needs to be problem-specific and set in the context of the weapon portfolio selection problem and the DM's preference.

5.3 Comparative analysis

The proposed weapon portfolio selection method has two major improvements over the traditional method. First, the suitability of the weapon portfolio to the combat task is incorporated into the input of the method, and second, the interval value is used to describe the indicator values while the corresponding interval fuzzy set is used to rank the portfolios. The proposed method in this paper is compared with the traditional method in the following paragraphs to illustrate its progressiveness.

First, the method proposed in this paper is compared with three traditional decision-making methods based on interval hesitant fuzzy sets: technique for order preference by similarity to ideal solution (TOPSIS), vlsekriterijumska optimizacija I kompromisno resenje (VIKOR), and ELECTRE. Using the data in Table 1, the ranking results of the schemes calculated by each method are shown in Table 6.

Table 6 Portfolio ranking for different methods

Method	Portfolio ranking
Our method	$e_4 > e_3 > e_5 > e_7 > e_2 > e_{10} > e_8 > e_6 > e_1 > e_9$
TOPSIS	$e_3 > e_1 > e_2 > e_4 > e_6 > e_5 \sim e_{10} > e_7 > e_9 > e_8$
VIKOR	$e_4 > e_1 \sim e_3 > e_2 > e_6 > e_5 > e_7 > e_{10} > e_8 > e_9$
ELECTRE	$e_4 > e_3 > e_2 > e_1 > e_6 > e_5 > e_{10} \sim e_9 > e_8 > e_7$

Based on the above results, traditional methods cannot provide a strict ranking of all schemes. The TOPSIS method is unable to distinguish between portfolios e_5 and e_{10}; the VIKOR method fails to distinguish between portfolios e_1 and e_3; the ELECTRE method cannot distinguish between portfolios e_9 and e_{10}. Moreover, all these methods consider portfolio e_1 to be ranked higher. Still, according to the initial applicability of equipment portfolios to combat tasks in Table 3, portfolio e_1 generally has lower applicability to the task. Therefore, the final ranking obtained by the method proposed in this paper is more objective and reasonable. The two main reasons for this are as follows. First, the method proposed in this paper integrates more decision-making information, and second, traditional methods cannot compare the size of interval fuzzy sets in some cases, resulting in the inability to obtain a strict ranking of all portfolios.

Next, based on the estimated applicability values of each scheme to a new task calculated in Table 5, the ranking method using interval fuzzy sets proposed in this paper is compared with traditional minimum safeguard, maximum benefit, and average utility principles, as shown in Table 7.

Table 7 Portfolio sorting of different comparison methods

Ranking method	Portfolio ranking
Our method	$e_4 \succ e_3 \succ e_5 \succ e_7 \succ e_2 \succ e_{10} \succ e_8 \succ e_6 \succ e_1 \succ e_9$
Minimum safeguard principle	$e_1 \succ e_4 \sim e_2 \succ e_7 \sim e_5 \succ e_3 \succ e_6 \succ e_9 \succ e_8 \succ e_{10}$
Maximum benefit principle	$e_1 \succ e_3 \succ e_2 \succ e_5 \succ e_4 \succ e_6 \succ e_8 \sim e_7 \succ e_{10} \sim e_9$
Average utility principle	$e_1 \succ e_2 \succ e_4 \sim e_3 \succ e_5 \succ e_6 \succ e_7 \succ e_8 \succ e_9 \succ e_{10}$

Traditional principles cannot provide a strict ranking of the portfolios. Specifically, the optimistic principle cannot distinguish between portfolios e_2 and e_4, e_5 and e_7; the pessimistic principle cannot distinguish between portfolios e_7 and e_8, e_9 and e_{10}; the average principle cannot distinguish between portfolios e_3 and e_4. The ranking method using interval fuzzy sets proposed in this paper considers the expected utility and uncertainty of the portfolios comprehensively.

In summary, through comparative experiments, the weapon portfolio selection method proposed in this paper exhibits certain advantages with a more advanced approach compared to traditional methods.

6. Conclusions

Taking the combat task utility requirements as the weapon portfolio selection guide, this paper proposes a weapon portfolio selection method. The following contributions are made:

(i) The use of collaborative filtering to select weapon portfolios based on combat task utility requirements solves the problems of traditional methods losing specific capability requirements and insufficient support for specific combat tasks, making weapon portfolios more applicable to the corresponding combat tasks.

(ii) Interval-valued hesitant fuzzy theory is introduced and incorporated into the collaborative filtering method and the clustering method approach, which retains the fuzziness and hesitation contained in the decision information effectively, considering the complexity of weapons and the cognitive uncertainty of DMs. Moreover, the portfolio selection results are more applicable to the needs of combat tasks.

However, the selection method proposed in this paper requires more expert knowledge. Specifically, expert knowledge includes, at least, the performance characteristics of equipment, the interaction between pieces of equipment, and the changing combat environment. This knowledge can be applied to collaborative filtering and clustering methods to guide the selection of appropriate methods, parameters, and algorithms, thus making the selected equipment combination schemes more applicable to the needs of combat missions and achieving the optimal configuration of equipment and combat missions.

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