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Deformation and Perforation of Simply-Supported Circular Plates Struck Normally by Flat-Faced Projectiles*

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Abstract: A previously proposed approximate quasi-static theoretical method is employed herein to examine the deformation and perforation of a simply-supported circular plate struck by a flat-faced projectile at normal incidence. Based on the principle of virtual work, static load-displacement relationships are first obtained and then used to predict the energy-absorbing capacity of the plate subjected to relatively low speed impact. It is shown that good agreement is obtained between the theoretical predictions and the available experimental data, provided that adiabatic shear plugging failure does not occur, but the influence of strain rate sensitivity is taken into account.

Key words: deformation; perforation; simply-supported circular plate; projectile; ballistic limit

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1 Introduction

The deformation and failure of structures impacted by projectiles is very important for designing structural systems in engineering such as nuclear power plant and defense installation. When a projectile strikes a target, complex phenomena may occur which involves elastic, plastic deformation, crack initiation and propagation, strain rate effects, thermal softening, projectile breakup or even explosion depending on impact velocity, geometry and material properties of both the projectile and the target. In order to understand the deformation and failure mechanisms and to design the containment and protective structures against projectiles or dropped objects, it is necessary to carry out extensive experimental programs to examine the effects of the various factors cited above. However, there have been relatively few available experimental data that can be used to guide the engineering design.

Extensive reviews on the ballistic impact have been published in recent years[1~4]. In these publications, Ref. [1] and Ref. [2] focused on the analytical procedures other than experimental investigations, whilst, Ref. [3] and Ref. [4] documented theoretical analyses together with experiments. As can be seen in these reviews, the theoretical solutions are usually obtained only based upon idealistic simplifications to the real complicated events. For example, when the projectile impact velocities are appreciably higher than the ballistic limit, the global deformation of target can be neglected to achieve the reasonable approximation of the energy absorbed by the plate in the perforation process. When a relative large mass strikes the target with low velocities, the global deformation may dominate the response. The combination of global response and local failure has been demonstrated to be difficult to analyse[5].

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It is expected that a quasi-static procedure should be suitable for the analysis of metal plates struck by projectiles traveling with relatively low speed where both global and local effects contribute significantly to the overall target response and perforation. The accuracy of the method has been studied by some authors. Ref. [6] explained the theoretical basis of the quasi-static method and employed an error analysis to examine the accuracy of this procedure, and suggested that the dynamic plastic response of a structure struck by a heavy mass traveling at a low speed could be estimated reasonably by the procedure mentioned above. Ref. [7] also investigated the accuracy of the method for predicting the inelastic behavior of structures under impact and comparisons were made between the quasi-static theoretical predictions and the experimental results and the same conclusion was drawn as above.

Wen and Jones[8] developed an approximate quasi-static theoretical procedure to study the behavior of a fully clamped circular metal plate struck by a mass at low velocity. Based on the principle of virtual work, the load-deflection relationship and energy-absorbing capability of the plate were presented. It is shown that the theoretical predictions are in good agreement with the available experimental data.

The objective of this paper is to examine the deformation and perforation of axially restrained simply-supported circular pates struck normally by flat-nosed projectiles by closely following the method suggested by Wen and Jones[8]. Analytical results are presented and comparisons are made with the experimental data reported in Ref. [5] and discussed.

2 Approximate Quasi-Static Theoretical Analysis

2.1 Basic Equations

Consider a simply-supported circular plate having an outer radius R and thickness H which is loaded at the center by a circular flat-nosed punch with radius a, as shown in Fig. 1. If the material of the plate obeys von Mises yield condition, the transverse displacement field for the plate loaded by a flat-nosed punch at the centre has the following form[9]

$$W = W_0 \quad \text{for} \quad r \leqslant a \tag{1}$$

and

$$W = \frac{W_0}{c}\left[1-\left(\frac{r}{R}\right)^{\frac{1}{2}}\right] \quad \text{for} \quad a \leqslant r \leqslant R \tag{2}$$

where $c = 1-(a/R)^{1/2}$ is a constant for a specific impact problem, W is the transverse displacement of the plate at a radius r and W_0 is the transverse displacement at the periphery of the punch-plate interface.

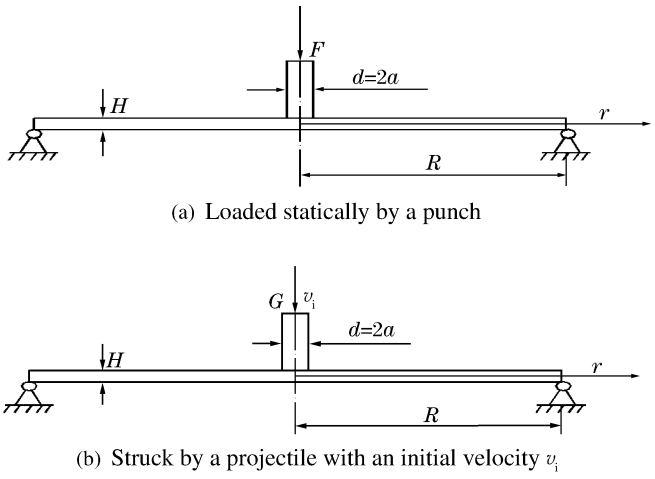


Fig. 1 A simply-supported circular plate

From the von Mises yield criterion for infinitesimal displacements as shown in Fig. 2(a) and the associated flow rule, using Eq. (1), we can obtain $M_r = 0$, $M_\theta = M_0$ everywhere in the plate except in the punch region where $M_r = M_\theta = M_0$, see Ref. [9]. M_r, M_θ and M_0 are the radial bending moment per unit length, circumferential bending moment per unit length and the fully plastic bending moment per unit length, respectively.

It is evident form the foregoing statement and, also, from Fig. 2(a), that this generalized stress field is not continuous at $r = a$, but it is used as an approximation to the actual one. As mentioned in

introduction, the difficulties of predicting the impact perforation of plates often lead to the development of empirical equations and the approximate theoretical solutions which have a potential for a greater range of validity compared to the empirical formulae.

It has been demonstrated in the previous works[10,11] that the indentation occurs in a beam when loaded statically or dynamically and is due mainly to the shearing action at the periphery of the punch on the upper surface of a beam. This behavior has also been observed in a circular plate loaded dynamically in Ref. [12].

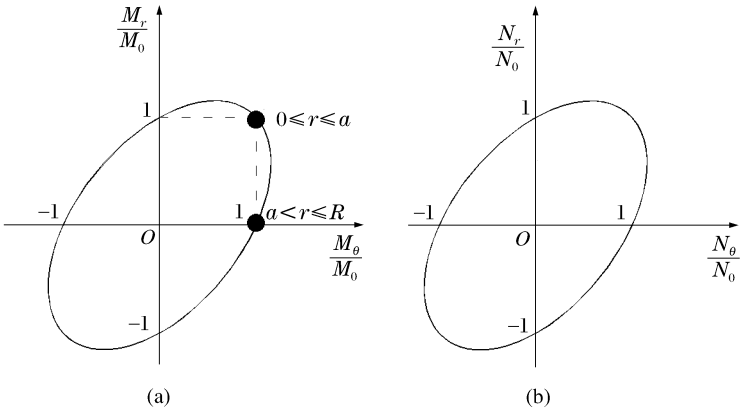


Fig. 2　Limited interaction yield conditions

According to the Eq. (11) of the Ref. [12], if the magnitude of the indentation is expressed as Δ and the shear strain γ can be calculated as follows

$$\gamma = \Delta / e \tag{3}$$

where e is the half-width of the shear zone, as shown in Fig. 3. By closely following the study in Ref. [8], it can be shown that

$$e = \sqrt{3}\, H/8 \tag{4}$$

Combining (4) and (3) yields

$$\Delta = \sqrt{3}\, \gamma H/8 \tag{5}$$

Thus, a transverse shear (plugging) failure occurs at the loading point for a critical indentation Δ_c, or

$$\Delta_c = \sqrt{3}\, \gamma_c H/8 \tag{6}$$

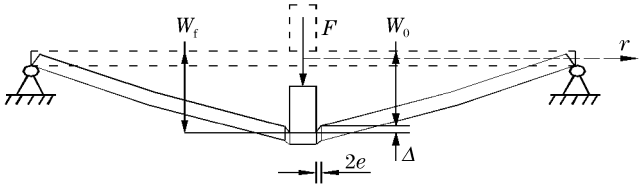


Fig. 3　Definition of W_f, W_0, Δ and e for a simply-supported circular plate

Where, γ_c is the critical shear strain, which also corresponds to the critical transverse shear force (F_u).

The transverse shear stress, τ, at the punch-plate interface, maybe written in the following form

$$\tau = \tau_u (\gamma/\gamma_c)^n \tag{7}$$

or, equivalently

$$F_s = F_u (\Delta/\Delta_c)^n \tag{8}$$

where n is work-hardening index for the plate material; F_s is transverse shear load and τ_u is critical shear stress. For steel plates may be taken as[8]

$$\tau_u = \lambda \sigma_y [0.41(H/d) + 0.42] \tag{9}$$

where $\lambda = \sigma_u/\sigma_y$ with σ_u, σ_y being the UTS and the yield stress of the plate material, respectively.

2.2　Load-Displacement Relationship

With the basic equations derived above, it is now possible to determine an approximate static load-displacement relationship for a simply-supported circular plate struck by a flat-faced circular punch at the center with $a \ll R$. According to the principle of virtual work and by closely following the method presented in Ref. [8], it is easy to show that

$$F = K_m W_0 + F_c \tag{10}$$

for the global deformation and

$$F = F_s = F_u(\Delta/\Delta_c)^n \tag{11}$$

for the local (shear) response. $K_m = \pi N_0(R-a)/2Rc^2$, $F_c = (2-\sqrt{a/R})\pi M_0/c$ are the membrane stiffness and the static collapse load of the plate for infinitesimal displacements, respectively. It is observed that $F_c \to 2\pi M_0$ when $a \to 0$, which agrees with Ref. [7] when the influence of transverse shear deformations and finite displacements are neglected.

Rearranging Eqs. (10) and (11) and using $W_t = W_0 + \Delta$ yields

$$W_t = (F - F_c)/K_m + (F/F_u)^{1/n}\Delta_c \tag{12}$$

Eq. (12) is an approximate static load-displacement relationship for a simply-supported rigid-plastic circular plate loaded at the center by a flat-ended circular punch having $a \ll R$.

2.3 Energy-Absorbing Capability of a Plate

The energy absorbed by a similar rigid-plastic plate loaded statically, as shown in Fig. 1(a), is

$$E_t = E_{bm} + E_s \tag{13}$$

where E_{bm} and E_s are the energies dissipated by bending deformations and membrane stretching related to W_0 and by transverse shear effects related to Δ, respectively, and are evaluated as follows[8]

$$E_{bm} = \int_0^{W_0}(K_m W_0 + F_c)\mathrm{d}W_0 = \frac{K_m}{2}W_0^2 + F_c W_0 \tag{14a}$$

and

$$E_s = \int_0^{\Delta} F_s \mathrm{d}\Delta = E_{cs}[(K_m W_0 + F_c)/F_u]^{(n+1)/n} \tag{14b}$$

when using Eqs. (10) and (11) and where $E_{cs} = F_u\Delta_c/(1+n)$ is the critical shear work done at the punch-plate interface when a shear (plugging) failure occurs. The permanent maximum transverse displacement can be written in the following non-dimensional form

$$\delta = \frac{W_p}{H} = \left\{ -J + \left[J^2 + \frac{16c^2\psi^3(\chi_t - \chi_s)}{\beta\pi}\right]^{1/2}\right\} \cdot \frac{\beta}{2} + \left\{\frac{[J^2 + 16c^2\psi^3(\chi_t - \chi_s)/\beta\pi]^{1/2}}{4\lambda c^2(0.41 + 0.42\psi)}\right\}^{1/n} \cdot \frac{\sqrt{3}}{8}\gamma_c \tag{15}$$

where W_p is the maximum permanent transverse displacement of the plate including the indentation at the loading point and $\chi_t = E_t/\sigma_y d^3$, $\chi_s = E_s/\sigma_y d^3$, $J = c + c^2$, $\beta = R/(R-a)$ and $\psi = d/H$.

It is assumed that the critical transverse force (F_u) is reached when a transverse shear (plugging) failure occurs. Substituting $F = F_u$ into Eq. (10) gives

$$W_{os} = (F_u - F_c)/K_m \tag{16a}$$

or

$$W_{os} = \frac{4Ra\lambda c^2}{R-a}[0.41(H/d) + 0.42] - \frac{HRc}{2(R-a)}(2 - \sqrt{a/R}) \tag{16b}$$

after using Eq. (9) and $K_m = \pi N_0(R-a)/2Rc^2$, $F_c = (2-\sqrt{a/R})\pi M_0/c$. Where W_{os} is the critical transverse displacement at which a transverse shear (plugging) failure occurs. Substituting W_{os} for W_0 in Eq. (14) and using Eq. (13) yields

$$E_t = \frac{K_m}{2}W_{os}^2 + F_c W_{os} + E_{cs} \tag{17}$$

which, using Eq. (16a), becomes

$$E_t = \frac{F_u^2 - F_c^2}{2K_m} + \frac{F_u\Delta_c}{1+n} \tag{18a}$$

or

$$\frac{E_t}{\sigma_y d^3} = A\left(\frac{H}{d}\right) + B\left(\frac{H}{d}\right)^2 + C\left(\frac{H}{d}\right)^3 \tag{18b}$$

where $E_t = E_p$ is the perforation energy of the plate and A, B, C are defined by the following equations

$$A = 0.554\,\frac{\lambda^2 Rc^2}{R-a} \tag{19a}$$

$$B=0.286\frac{\lambda\gamma_c}{1+n}+1.082\frac{\lambda^2Rc^2}{R-a} \tag{19b}$$

$$C=0.279\frac{\lambda\gamma_c}{1+n}+0.528\frac{\lambda^2Rc^2}{R-a}-\frac{\pi}{16}\cdot\frac{R}{R-a}\left(2-\sqrt{\frac{a}{R}}\right)^2 \tag{19c}$$

3 Comparison with Experimental Data and Discussion

The quasi-static procedure developed above is compared with the experimental results for the simply-supported plates loaded quasi-statically and dynamically, which are reported in the Ref. [5].

It is well known that the mild steel is strongly sensitive to the rate of loading and the dynamic yield stress (σ_d) can be determined by the well-known Cowper-Symonds empirical formula[18,15], viz,

$$\sigma_d=\sigma_y[1+(\dot{\varepsilon}_m/D)^{1/q}] \tag{20}$$

where $D=40.4\ s^{-1}$ and $q=5$ are empirical constants chosen to describe the strain rate sensitive behavior of mild steel. According to the approximate methods suggested by Perrone and Bhadra[19,20], the mean strain rate ($\dot{\varepsilon}_m$) in Eq. (20) may be expressed as

$$\dot{\varepsilon}_m=\frac{W_0v_i\ln(R/a)}{6\sqrt{2}R(R-a)c^2} \tag{21}$$

where v_i is the initial impact velocity. When the plate fails in shear (plugging) mode, then Eq. (21) can be rewritten as

$$\dot{\varepsilon}_m=\frac{W_{0s}v_c\ln(R/a)}{6\sqrt{2}R(R-a)c^2} \tag{22}$$

where v_c is the critical impact velocity or the ballistic limit and W_{0s} is the critical transverse displacement of the plate when a transverse shear (plugging) failure occurs and is determined from Eq. (16). The mean strain rate predicted using Eq. (22) is generally less than $10\ s^{-1}$ and is of the same order of magnitude as the experimental observations on the circular mild steel plates investigated in the Refs. [5,22]. Therefore, Eq. (18b) can be recast into the following form

$$\frac{E_p}{\sigma_y d^3}=\left[1+\left(\frac{W_{0s}v_c\ln(R/a)}{6\sqrt{2}DR(R-a)c^2}\right)^{1/q}\right]\cdot\left[A\left(\frac{H}{d}\right)+B\left(\frac{H}{d}\right)^2+C\left(\frac{H}{d}\right)^3\right] \tag{23}$$

which, substituting $E_p=Gv_c^2/2$ into Eq. (23) and rearranging, yields an approximate value for the ballistic limit

$$v_c=\sqrt{\frac{2\sigma_y d^3}{G}}\left[1+\left(\frac{W_{0s}v_c\ln(R/a)}{6\sqrt{2}DR(R-a)c^2}\right)^{1/q}\right]^{1/2}\cdot\left[A\left(\frac{H}{d}\right)+B\left(\frac{H}{d}\right)^2+C\left(\frac{H}{d}\right)^3\right]^{1/2} \tag{24}$$

where G is the projectile mass.

Fig. 4 shows variation of the coefficient A, B and C with the ratio of plate thickness to striker diameter (H/d) for the simply-supported circular mild steel plates investigated in Ref. [5], where the parameters, n and γ_c, are taken to be the typical value of 0.25 and 0.8 for mild steel, respectively, according to the Refs. [5,8,21]. The plates, details of which are given in Table 1, were simply-supported at a diameter of 248 mm and each plate was loaded by means of a flat-ended cylindrical punch with 12.7 mm in diameter in the quasi-static test, or a flat-nosed projectile with the same diameter,

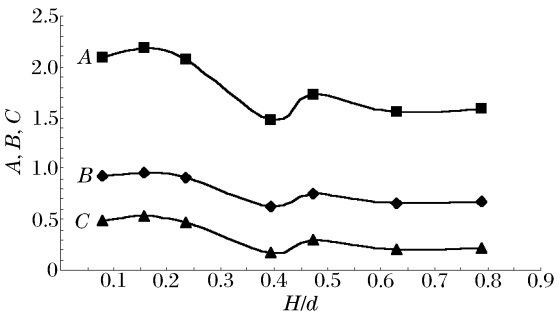


Fig. 4　Variation of the coefficients of A, B and C in Eq. (25) with the ratio to plate thickness to striker diameter (H/d) for the simple supported circular mild steel plate examined in Ref. [5]

launched by a compressed air-gun in the dynamic investigation. It appears that the value of A, B and C decrease with increase in H/d except for the plate that the nominal plate thickness is 5 mm, as shown in Fig. 4. However, if the plates were made from the same material in terms of the mechanical properties such as λ, n and γ_c, then the coefficients A, B and C should remain constant according to Eqs. (19a)～(19c).

Table 1 Some of the mechanical properties of the materials examined in Ref. [5]

Nominal plate thickness H/(mm)	R /(mm)	Yield stress σ_y /(MPa)	U. T. S. σ_u /(MPa)	Young's modulus /(MPa)
1 (In rolling direction)	124	191	310	240
2 (Transverse to the rolling direction)	124	181	300	230
3	124	273	440	228
5	124	352	472	220
6	124	305	446	235
8	124	342	472	211
10	124	351	490	230

Note: Support diameter $2R=248$ mm; plate diameter $D_0=300$ mm; punch diameter $d=12.7$ mm.

Fig. 5 shows variation of E_{cs}/E_p and E_{bm}/E_p with the ratio of the plate thickness to striker diameter (H/d) for the same plates as above. It is clear, as expected, that E_{cs}/E_p is a monotonically increasing function of H/d, while E_{bm}/E_p monotonically decreases with an increase in H/d with the range of parameters studied.

Fig. 6 shows comparison between ballistic limit velocities predicted by Eq. (24) and the experimental data for simply-supported circular mild steel plates reported in Ref. [5]. It is seen from Fig. 6 that good agreement is found between the predictions and the corresponding experimental data up to some critical value of H/d. This is due, possibly, to a change in the energy-absorbing mechanism in the plates from a global structural response combined with a shear failure at the projectile-target interface to a predominant plugging failure for larger values of H/d.

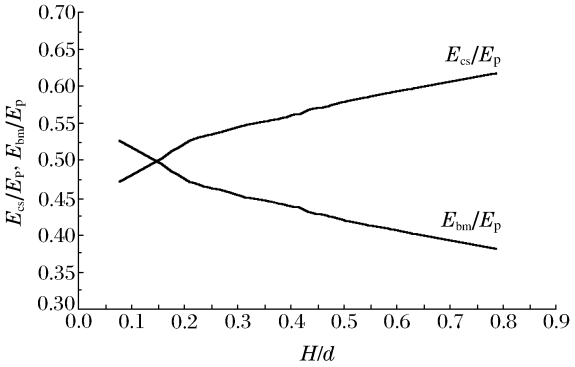


Fig. 5 Variation of E_{cs}/E_p and E_{bm}/E_p with the ratio of plate thickness to striker diameter (H/d) for the simple supported circular mild steel plate investigated in Ref. [5]

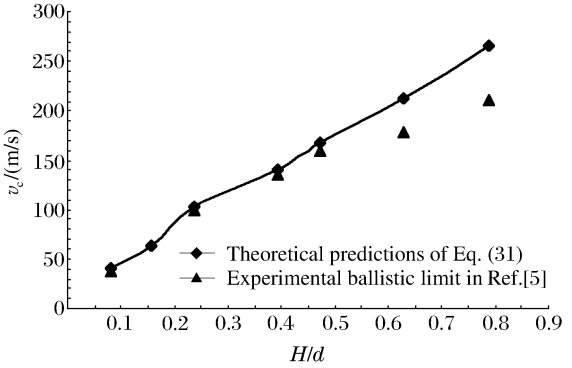


Fig. 6 Comparison of the theoretical predictions with the experimental results for the simple supported circular mild steel plate struck by a projectile at the center

4 Conclusions

An approximate quasi-static theoretical analysis has been given in this paper to examine the deformation and failure of axially restrained simply-supported circular plates struck by flat-ended projectiles by closely following the study in Ref. [8]. Based on the principle of virtual work, static load-displacement relationships are established and then employed to predict the energy-absorbing capacities of plates, which were perforated at relative low speed.

It is shown that the theoretical predictions agree well with the experimental observations in terms of the ballistic limits when the influence of material strain rate sensitivity is taken into account and provided that the perforation does not occur by adiabatic shear plugging. It is also shown that the quasi-static procedure employed is suitable for analysing the response of a simply-supported circular plate struck by a flat-faced projectile at relatively low velocity.

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简支圆板在平头弹丸撞击下的变形与穿透

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摘要： 采用准静态近似理论方法分析简支圆板在平头弹丸正撞下的变形和穿透。首先基于虚功原理得到了静态荷载位移关系，并将其用于预测低速冲击下圆板的能量吸收能力。当考虑了材料的应变率敏感性，并且不发生绝热剪切冲塞破坏的前提下，理论预测和实验结果吻合良好。

关键词： 变形；穿透；简支圆板；弹丸；弹道极限

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