

Synchronization Stability of Hybrid Power Systems Integrated With Grid-Forming Inverters and Grid-Following Inverters

Dian LU, Jingrong YU, Xiawei LU, and Jiaqi YU

Abstract—This paper investigates the synchronization stability of hybrid power systems integrated with grid-forming (GFM) inverters and grid-following (GFL) inverters. In hybrid power systems, the interactions between GFM and GFL inverters bring about challenges for the synchronization stability analysis. To address this issue, a fourth-order synchronization model considering controller interactions is established. Then, the influence of interactions on the stable equilibrium point (SEP) and the synchronization process is fully clarified. It is found that interactions are detrimental to the SEP of GFM inverters but beneficial to the SEP of GFL inverters. For synchronization processes, the instability and stabilization caused by controller interactions are presented, indicating the important effect on the synchronization process. In addition, suggestions for controller design to improve synchronization dynamics through controller interactions are provided. Simulation results validate these findings.

Index Terms—Grid-following (GFL) inverter, grid-forming (GFM) inverter, interaction, synchronization stability.

I. INTRODUCTION

GRID-CONNECTED inverters are commonly used to connect renewable energy sources (RESs) and the power grid [1]–[2]. Currently, the majority of inverters are based on grid-following (GFL) control, which could efficiently control the amplitude and angle of the current injected into the grid [3]–[4]. However, with the growing penetration of RESs, grid operators put forward higher requirements in terms of grid strength, voltage and frequency support, and black-start capacity [5]–[6]. Grid-forming (GFM) inverters featuring voltage source characteristics have recently received a lot of attention as a promising solution for power grids with a high penetration of RESs [7]–[8]. The co-integration of GFL and

GFM inverters is expected in the future power system due to their different features and superiority [9]–[10].

Unlike traditional power systems, the synchronization stability analysis of hybrid power systems is challenged by different synchronization controllers and their interactive behaviors. In hybrid power systems, GFL and GFM inverters are synchronized via the phase-locked loop (PLL) and the active power controller (APC), respectively [11]. The coexistence of different synchronization controllers complicates the synchronization mechanism of hybrid power systems. Moreover, the coupling between APC dynamics and PLL dynamics is another critical issue that should be considered. Recently, GFM inverters have been designed to have fast active power response to maintain frequency stability in the low-inertia power system [12]–[13], which causes the power control loop to overlap in timescale with PLL. Controller interactions between APC and PLL pose challenges to characterizing dynamic synchronization behaviors. Several different types of interactions can occur in hybrid power systems, and their impact on system stability can vary under different operating conditions. These interactions include:

- 1) Frequency and Phase Interaction. In normal conditions, the GFM inverter controls system frequency, while the GFL inverter's PLL follows and corrects small phase deviations. In low-inertia systems, APC and PLL responses may overlap, causing potential instability.
- 2) Power and Voltage Interaction. During grid disturbances like voltage dips, the GFM inverter maintains voltage stability, while the GFL inverter supports by injecting reactive power. The GFM inverter's power control becomes critical in sustaining grid voltage.
- 3) Current Injection Interaction. During grid faults, the GFL inverter's current injection is influenced by PLL, with reactive current prioritized for low voltage ride through (LVRT). This interaction with the GFM inverter's power control plays a crucial role in maintaining system stability.

Different synchronization controllers and their interactions create a new bottleneck, hindering the modeling and interaction analysis, which are essential to the synchronization stability of hybrid power systems.

Regarding the modeling of hybrid power systems, few theoretical studies considering controller interactions have been developed. The model of the paralleled system consisting of GFM and GFL inverters is established in [14] and [15], but

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controller interactions are not concerned. The model of power systems containing both synchronous generators (SGs) and GFL inverters is developed in [16], where PLL is modeled by an algebraic equation. The timescale of this research focuses on the rotor motion dynamics. Compared to SGs, GFM inverters with the frequency-supporting feature have faster synchronization dynamics, creating an overlap with PLL dynamics. In this scenario, it is indispensable to consider controller interactions between APC and PLL in the modeling of synchronization stability analysis.

The influence of interactions on synchronization stability can be seen in both the existence of stable equilibrium points (SEPs) and the synchronization process [17]. It is observed in [18] that the SEPs of GFL and GFM inverters are partly altered by interaction terms. The specific equilibrium conditions of hybrid power systems remain to be systematically elucidated. Interactions also have a great impact on the synchronization process. In [14], the optimal current injection design of GFL inverters to obtain the best synchronization performance of GFM inverters is explored, which does not consider the controller dynamics of GFL inverters. In [15], the stable region of a hybrid power system is determined without the involvement of controller parameters. The maximum power angle of SGs limited by PLL synchronization is found in [16], where PLL dynamics and controller interactions are overlooked. In [19], the role of PLL in the synchronization process is elaborated, which focuses on the path analysis and less on the stability evaluation. The above studies have not given full consideration to controller interactions in the synchronization process analysis.

This paper aims to fill this gap, and its contributions are summarized as follows:

- 1) Taking controller interactions into account, a fourth-order synchronization model for the hybrid power system is proposed, which paves the way for interaction analysis.
- 2) The equilibrium conditions of the hybrid power system are fully illustrated, considering the instability of the GFM or GFL inverter. It is found that interactions are detrimental to the SEP of GFM inverters but beneficial to the SEP of GFL inverters.
- 3) The instability and stabilization caused by controller interactions are presented. The influence of the control parameters of one inverter on the synchronization process of another inverter through control interactions is revealed.

The current limitation of GFM inverters is crucial in practical systems, especially during grid faults. It prevents excessive current output, protecting both the inverter and system components. When triggered, this limitation may reduce the inverter's current injection, potentially impacting transient stability by affecting voltage, frequency, and synchronization.

While this study does not focus on the impact of current limitations, we acknowledge their importance in real-world operations. Future research will explore the dynamic behavior of current limitations and their effects on system transient response and fault recovery, ensuring a more comprehensive understanding under complex conditions.

The rest of this paper is organized as follows. The fourth-

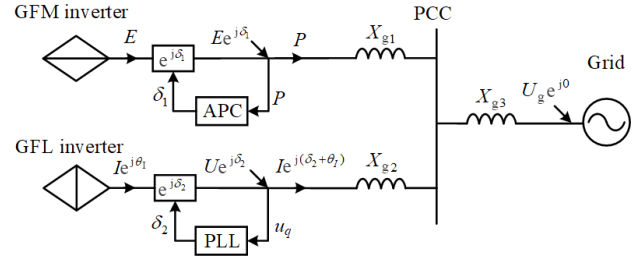


Fig. 1. Simplified circuit of the hybrid power system.

order synchronization model for hybrid power systems is established in Section II. Then, the effects of interactions on synchronization stability are discussed in Section III. After that, the simulation verifications are given in Section IV. Finally, Section V concludes this paper. Due to resource limitations, our study primarily focuses on simulations. Future research will involve experimental validation based on real inverter prototypes.

II. SYSTEM DESCRIPTION AND MODELING

A. System Description

The simplified circuit of a hybrid power system is shown in Fig. 1, including a GFM inverter, a GFL inverter, and the grid. X_{g1} , X_{g2} , and X_{g3} are the line impedances. E , U , and U_g represent the voltage amplitudes of the GFM inverter, the GFL inverter, and the grid, respectively. I and P represent the current and power injected into the grid by the GFL inverter and the GFM inverter, respectively. θ_1 is the current injection angle of the GFL inverter. In this paper, if θ_1 is not specified, it defaults to 0, implying that only active current is injected. The GFL inverter's current injection phase angle was assumed to be fixed for simplicity. However, we acknowledge that under LVRT conditions, the phase angle dynamically changes as the inverter injects reactive current proportional to the grid voltage drop. Future work will incorporate these LVRT requirements to evaluate their impact on the transient stability and synchronization of the hybrid system. We do not utilize the equal area criterion (EAC) in this study.

The phase difference between the GFM inverter voltage and grid voltage is denoted by δ_1 , and the phase difference between the GFL inverter voltage and grid voltage is denoted by δ_2 .

The GFL inverter synchronizes with the grid through PLL, and the GFM inverter synchronizes with the grid through APC. The dynamics of the voltage control loop and current control loop are usually designed to be much faster than those of APC and PLL [20], which can be disregarded in the analysis of synchronization stability. Thus, the GFM inverter is deemed to be a power-synchronized voltage source, and the GFL inverter is regarded as a PLL-synchronized current source [6].

When both the GFM inverter and the GFL inverter are connected to the PCC at the same time, the interactions between the GFL inverter and the GFM inverter emerge, which brings about different SEPs from the single inverter

system. It is noted that the timescales of synchronization dynamics are governed by synchronization controllers. Since the dynamics of APC and PLL overlap in timescale, the controller interactions between PLL and APC cannot be ignored in the synchronization process analysis.

B. Model of the Hybrid Power System

Controller interactions between the GFM inverter's APC and the GFL inverter's PLL are key to understanding the dynamic synchronization behavior. The interaction terms in this model are derived from the dynamic responses of both APC and PLL. These terms capture the coupling between the GFM inverter's active power regulation and the GFL inverter's synchronization process. Specifically, when the GFM inverter adjusts voltage and frequency, it influences the GFL inverter's current injection, which in turn affects the overall system's synchronization stability.

The interaction terms are calculated to reflect how the control dynamics of one inverter can influence the other. For instance, rapid changes in the GFM inverter's active power output may lead to phase oscillations in the GFL inverter's PLL, potentially destabilizing the system if not properly accounted for. This model incorporates these effects to ensure accurate prediction of both stable and unstable operating regions based on the selected control parameters.

The virtual synchronous generator (VSG) is adopted in APC, which is designed to emulate the output characteristics of SGs [21]. Then, APC of the GFM inverter can be modeled as

$$\begin{cases} \frac{d\delta_1}{dt} = \omega_1 - \omega_g = \Delta\omega_1 \\ J \frac{d\omega_1}{dt} = P_{\text{ref}} - P - D(\omega_1 - \omega_g) \end{cases} \quad (1)$$

where P_{ref} is the active power reference, J denotes the virtual moment, and D is the damping coefficient. In this paper, the reactive power controller of the GFM inverter adopts constant voltage control.

According to the simplified circuit, the active power output by the GFM inverter can be written as

$$P = \frac{EU_g}{X_1} \sin \delta_1 - K_1 EI \cos(\delta_{12} - \theta_1) \quad (2)$$

where $X_1 = X_{g1} + X_{g3}$, $K_1 = X_{g3}/(X_{g1} + X_{g3})$, and $\delta_{12} = \delta_1 - \delta_2$.

The dynamics of PLL can be expressed as follows

$$\begin{cases} \frac{d\delta_2}{dt} = \omega_2 - \omega_g = \Delta\omega_2 \\ \frac{d\omega_2}{dt} = k_i u_q + k_p \frac{du_q}{dt} \end{cases} \quad (3)$$

where k_p and k_i represent the proportional and integral gains.

The voltage of the GFL inverter is obtained as

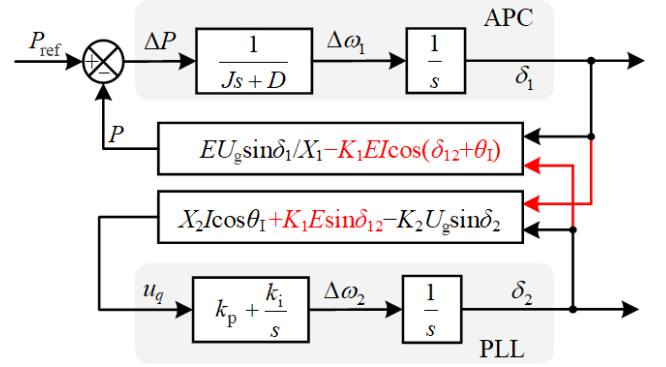


Fig. 2. Equivalent model of the hybrid power system (interaction terms are marked in red).

$$Ue^{j\delta_2} = K_1 E e^{j\delta_1} + K_2 U_g e^{j0} + jX_2 I e^{j(\delta_2 + \theta_1)} \quad (4)$$

where $K_2 = X_{g1}/(X_{g1} + X_{g3})$ and $X_2 = X_{g2} + X_{g1}/X_{g3}$.

Then, the q -axis component of the GFL inverter voltage can be derived as

$$u_q = X_2 I \cos \theta_1 + K_1 E \sin \delta_{12} - K_2 U_g \sin \delta_2 \quad (5)$$

To visually interpret the effect of interactions between different inverters on synchronization stability, a hybrid power system model is established in Fig. 2. The interaction terms $-K_1 EI \cos(\delta_{12} - \theta_1)$ and $K_1 E \sin \delta_{12}$ are emphasized in Fig. 2, which will affect SEPs of the hybrid power system.

As shown in Fig. 2, it is found that there are controller interactions between PLL and APC, which complicate the dynamics of synchronization. The synchronization process of the hybrid power system is jointly dominated by the dynamics of APC and PLL due to controller interactions, which should be described by a fourth-order model.

Define the system states:

$$\begin{cases} x_1 = \delta_1 \\ x_2 = \omega_1 - \omega_g \\ x_3 = \delta_2 \\ x_4 = \omega_2 - \omega_g \end{cases} \quad (6)$$

Combining (1) and (3)–(5), a fourth-order synchronization model of the hybrid power system is formulated as follows

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = [\Delta P - D x_2]/J \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = [u_q - D_{21}(x_4 - x_2) - D_2 x_4]/J_2 \end{cases} \quad (7)$$

where

$$\begin{cases} \Delta P = P_{\text{ref}} - EU_g \sin x_1 / X_1 + K_1 EI \cos(x_1 - x_3 - \theta_1) \\ u_q = X_2 I \cos \theta_1 + K_1 E \sin(x_1 - x_3) - K_2 U_g \sin x_3 \\ D_{21} = k_p K_1 E \cos(x_1 - x_3) / k_i \\ D_2 = k_p (K_2 U_g \cos x_3 - L_2 I \cos \theta_1) / k_i \\ J_2 = (1 - k_p L_2 I \cos \theta_1) / k_i \end{cases} \quad (8)$$

The proposed model takes controller interactions into account, which makes it possible to analyze the effects of interactions on synchronization stability.

III. INTERACTION ANALYSIS

This section will investigate the interactions between the GFM inverter and the GFL inverter on synchronization stability. Synchronization stability of the hybrid power system entails SEPs and good enough synchronization behaviors. The system is modeled with a single inverter and a hybrid system containing both GFM and GFL inverters to highlight the complex interactions between different inverter control strategies. The comparison with a system containing two identical inverters, while valuable, is beyond the scope of this study. The interaction analysis will be conducted from two aspects: the impact on the SEP and the synchronization process.

A. Effect on the Equilibrium Point

The existence of the SEP is a precondition for synchronization stability. The SEP of a hybrid power system is determined by $\Delta P = 0$ and $u_q = 0$.

The active power of the GFM inverter can be re-expressed as

$$P = P_m \sin(\delta_1 + \varphi_1) \quad (9)$$

where

$$P_m = \sqrt{\left(\frac{EU_g}{X_1}\right)^2 + (K_1 EI)^2 - 2\frac{EU_g}{X_1} K_1 EI \sin(\delta_2 + \theta_1)} \quad (10)$$

$$\varphi_1 = \arctan \frac{-K_1 EI \cos(\delta_2 + \theta_1)}{EU_g / X_1 - K_1 EI \sin(\delta_2 + \theta_1)}$$

When $P_m > P_{ref}$, the GFM inverter has a SEP. According to (10), the active current injection ($\theta_1 = 0$) of the GFL inverter generally reduces the maximum transmission power P_m , while the reactive current injection ($\theta_1 = -\pi/2$) increases P_m .

It can be obtained from (10) that interactions have a great effect on the output maximum active power P_m , which yields $|EU_g/X_1 - K_1 EI| \leq P_m \leq |EU_g/X_1 + K_1 EI|$. Even if the GFL inverter loses stability, the GFM inverter will possess a SEP due to $|EU_g/X_1 - K_1 EI| > P_{ref}$. The unstable GFL inverter will cause δ_1 to fluctuate in a bounded manner around the SEP δ_{1s} , which can be obtained from

$$\delta_{1s} = \arcsin \frac{P_{ref}}{EU_g / X_1} \quad (11)$$

According to whether it is related to δ_1 and δ_2 , u_q of the GFL inverter can be decomposed into U_v and U_{vref} rewritten as

$$u_q = U_{vref} - U_v = X_2 I \cos \theta_1 - U_m \sin(\delta_2 + \varphi_2) \quad (12)$$

where

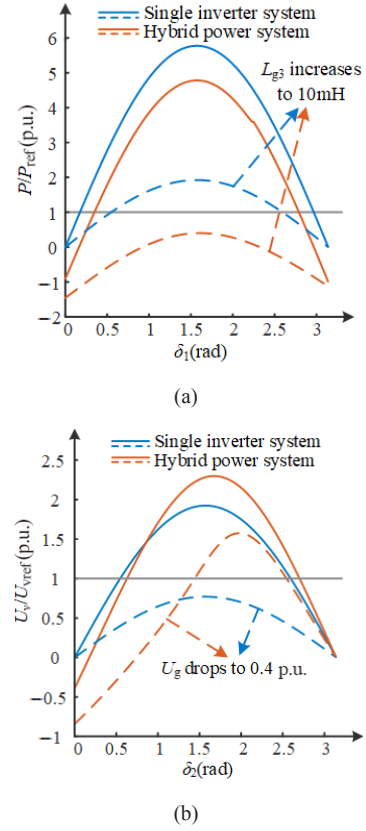


Fig. 3. Comparison of the single inverter system and the hybrid inverter system under Case 1. (a) P - δ_1 curves of GFM inverters. (b) U_v - δ_2 curves of GFL inverters.

$$U_m = \sqrt{(K_1 E)^2 + (K_2 U_g)^2 + 2K_1 K_2 E U_g \cos \delta_1} \quad (13)$$

$$\varphi_2 = \arctan \frac{-K_1 E \sin \delta_1}{K_1 E \cos \delta_1 + K_2 U_g}$$

When $U_m > U_{vref}$, the SEP of the GFL inverter exists. From (13), it is found that $|K_1 E - K_2 U_g| \leq U_m \leq |K_1 E + K_2 U_g|$. When $|K_1 E - K_2 U_g| > U_{vref}$, the GFL inverter still has a SEP even if the GFM inverter loses synchronization under the fault. The unstable GFM inverter will cause the δ_2 to oscillate near δ_{2s} , which can be obtained by

$$\delta_{2s} = \arcsin \frac{X_2 I \cos \theta_1}{K_2 U_g} \quad (14)$$

According to the above analysis, interactions will affect the existence and position of the SEP. In addition, the SEPs of a hybrid power system after the destabilization of the GFM or GFL inverter are obtained.

To further reveal the effect of interactions on the SEP, the P - δ_1 and U_v - δ_2 curves of a single inverter system and a hybrid power system are plotted in Fig. 3. It can be found that the SEPs of the hybrid power system are different from those of the single inverter system due to interactions. In Fig. 3(a), the GFM inverter that originally had a SEP will lose its SEP due to the connection of the GFL inverter under the same fault. It is noted that the well-designed GFM inverter is prone to losing the

TABLE I
COMPARISON OF THE SINGLE INVERTER SYSTEM AND THE HYBRID POWER SYSTEM UNDER DIFFERENT FAULT CONDITIONS

The fault types	The GFM inverter P_m (p.u.)		The GFL inverter U_m (p.u.)	
	Single inverter system	Hybrid power system	Single inverter system	Hybrid power system
U_g drops to 0.4 p.u.	2.309	1.585	0.770	1.575
Z_{g1} increases to 6 mH	2.886	2.481	1.924	2.066
Z_{g2} increases to 6 mH	5.773	4.803	1.443	1.642
Z_{g3} increases to 6 mH	2.886	1.466	1.155	2.069
No fault occurs	5.773	4.781	2.886	2.299

SEP due to interactions. On the other hand, a GFL inverter that has no SEPs will gain a SEP due to the presence of the GFM inverter in Fig. 3(b). The synchronization stability of the GFL inverter can be enhanced by placing a GFM inverter near the GFL inverter.

According to Fig. 3, a larger P_m and U_m are beneficial to the SEP of GFM and GFL inverters. Table I shows the P_m and U_m of the single inverter system and the hybrid power system under different fault conditions. In Table I, the P_m of the single inverter system is always larger than that of the hybrid inverter system, while the U_m of the single inverter system is always smaller than that of the hybrid inverter system. This illustrates that interactions are detrimental to the GFM inverter having a SEP but beneficial to the GFL inverter having a SEP.

It is noted that the power reference of the GFM inverter and the current reference of the GFL inverter in this paper did not change during the fault.

B. Effect on the Synchronization Process

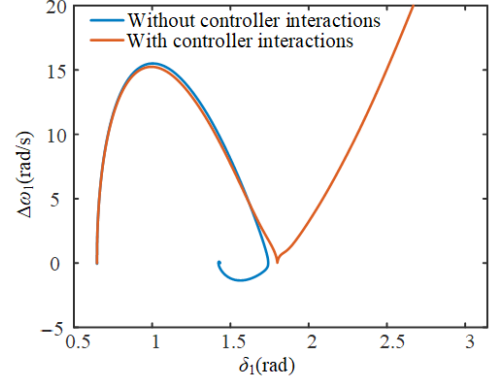
Even if the SEPs exist, the system may still be destabilized due to poor dynamics of the synchronization controllers themselves or their interactions [22]. In previous studies, controller interactions were not considered in the synchronization process analysis, which is the focus of this part.

When PLL dynamics and controller interactions are overlooked, the synchronization process of the GFM inverter can be described by

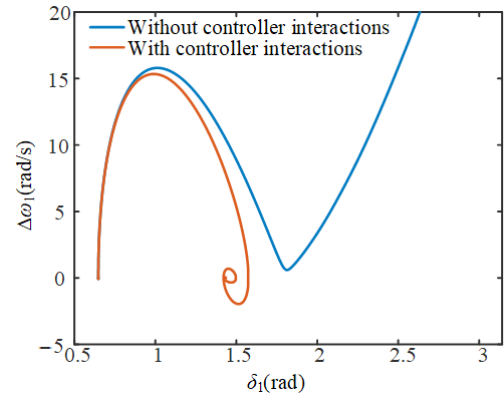
$$\begin{cases} \frac{d\delta_1}{dt} = \omega_1 - \omega_g = \Delta\omega_1 \\ J \frac{d\omega_1}{dt} = P_{ref} - P - D(\omega_1 - \omega_g) \\ u_q = X_2 I \cos \theta_1 + K_1 E \sin \delta_{12} - K_2 U_g \sin \delta_2 = 0 \end{cases} \quad (15)$$

Then, the synchronization process of the GFM inverter can be described by a two-order derivative equation plus a quasi-steady equation. In (15), δ_2 is treated as a parameter variable rather than a state variable and is only determined by δ_1 and grid parameters.

However, when the dynamics of APC and PLL overlap in timescale, PLL dynamics cannot be ignored and controller



(a)



(b)

Fig. 4. The effect of controller interactions on the synchronization stability of the GFM inverter when U_g drops to 0.6 p.u.. (a) In Case 3, the phase trajectory without considering controller interactions converges, and the phase trajectory considering controller interactions diverges. (b) In Case 4, the phase trajectory without considering controller interactions diverges, and the phase trajectory considering controller interactions converges.

interactions cannot be decoupled, which has a great impact on the synchronization process. Taking controller interactions into account, the synchronization behavior of the GFM inverter is also related to PLL control parameters, as described in (7).

Fig. 4 shows the effect of controller interactions on the synchronization stability of a GFM inverter. The phase trajectories of the GFM inverter with and without considering controller interactions are plotted. The instability and stabilization of the GFM inverter caused by controller interactions can be found in Fig. 4(a) and 4(b), respectively. This indicates that controller interactions do affect the synchronization process. The neglect of controller interactions can lead to incorrect synchronization stability analysis results.

Fig. 5 plots the phase trajectories of GFM inverters with different PLL parameters in Case 3. It is found that the different control parameters adopted by PLL will change the synchronization dynamics of the GFM inverter through controller interactions. In the Laplace domain, the transfer function of PLL's PI controller is given by:

$$G_{PLL}(s) = k_p + \frac{k_i}{s} \quad (16)$$

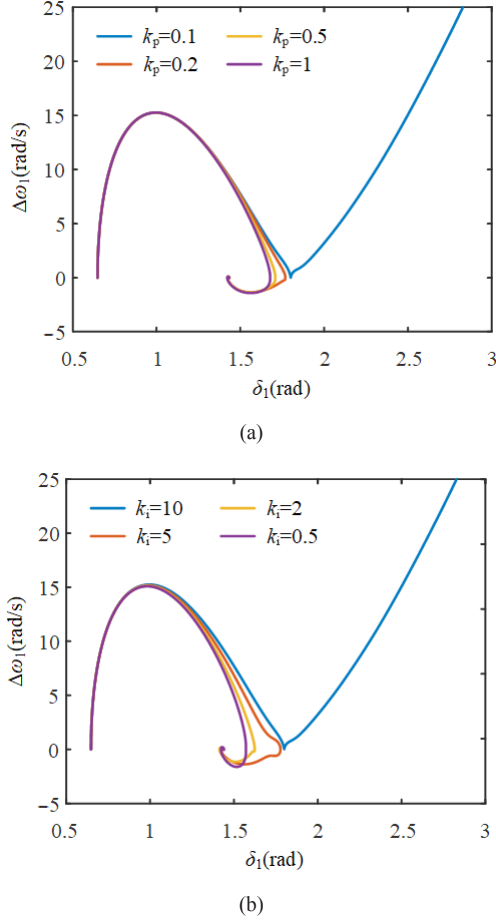


Fig. 5. The synchronization process of the GFM inverter under different control parameters of PLL when U_g drops to 0.6 p.u.. (a) Different k_p in PLL. (b) Different k_i in PLL.

where k_p is the proportional gain and k_i is the integral gain. The input is the phase error, and the output is the frequency adjustment. When considering the inverter dynamics, the overall closed-loop transfer function of PLL system can be expressed as:

$$\Theta_{inv}(s) = \frac{k_p s + k_i}{s^2 + k_p s + k_i} \Theta_{grid}(s) \quad (17)$$

where $\Theta_{grid}(s)$ and $\Theta_{inv}(s)$ represent the grid phase and inverter phase in the Laplace domain, respectively.

This transfer function describes how the inverter's output phase responds to the grid phase, with the key control parameters k_p and k_i having a significant effect on system dynamics and stability. The proportional gain k_p controls the system's response speed to phase errors, while the integral gain k_i impacts the correction of long-term steady-state errors.

By analyzing the transfer function, it is evident that inappropriate settings for k_p and k_i may result in system overreaction or delayed response, which could negatively affect system synchronization stability and transient performance.

In Fig. 5(a) and 5(b), the increase of k_p and the reduction of k_i can drive the divergent phase trajectory to converge and reduce the overshoot of δ_1 . It is indicated that increasing k_p and

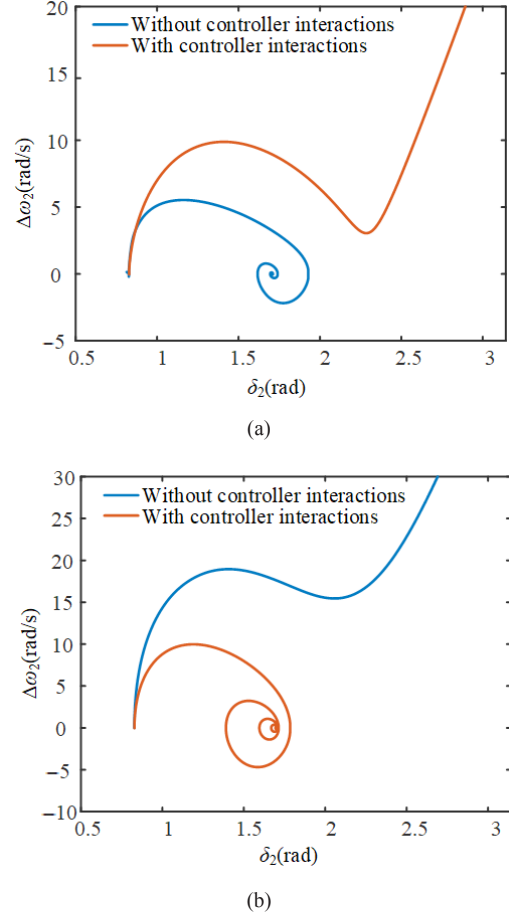


Fig. 6. The effect of controller interactions on the synchronization stability of the GFL inverter when U_g drops to 0.4 p.u.. (a) In Case 5, the phase trajectory without considering controller interactions converges, and the phase trajectory considering controller interactions diverges. (b) In Case 6, the phase trajectory without considering controller interactions diverges, and the phase trajectory considering controller interactions converges.

reducing k_i can not only improve the synchronization stability of the GFL inverter but also contribute to the synchronization stability of the GFM inverter.

Ignoring APC dynamics and controller interactions, the synchronization dynamic of the GFL inverter can be described by

$$\begin{cases} \frac{d\delta_2}{dt} = \omega_2 - \omega_g = \Delta\omega_2 \\ \frac{d\omega_2}{dt} = k_i u_q + k_p \frac{du_q}{dt} \\ P_{ref} = P = \frac{EU_g}{X_1} \sin \delta_1 - K_1 EI \cos(\delta_{12} + \theta_1) \end{cases} \quad (18)$$

where δ_1 is only determined by δ_2 and grid parameters.

However, the synchronization dynamics of the GFM also depend on APC control parameters. APC is involved in shaping the synchronization dynamics of the GFL inverter, as shown in (7).

In Fig. 6, the phase trajectories of GFL inverters with and without controller interactions are provided. Fig. 6(a) and 6(b)

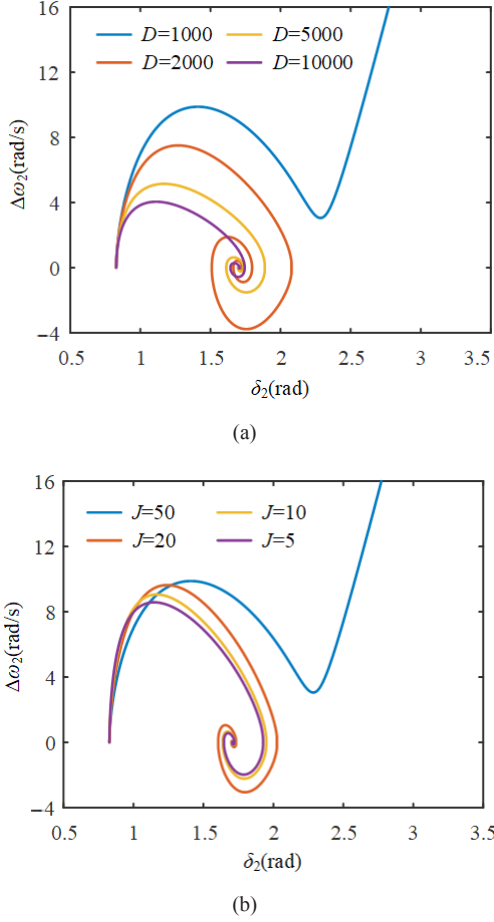


Fig. 7. The synchronization process of the GFL inverter under different control parameters of APC when U_g drops to 0.4 p.u.. (a) Different D in APC. (b) Different J in APC.

show the instability and stabilization of the GFL inverter caused by controller interactions, respectively. This illustrates the important influence of control interactions on the evaluation of synchronization stability, which cannot be ignored.

Fig. 7 analyzes the influence of different APC parameters on the synchronization process of GFL inverters in Case 5. It can be found that both increasing D and decreasing J can avoid the synchronization instability of the GFL inverter, as shown in Fig. 7(a) and 7(b). And the larger the D , the smaller the J , the better the synchronization process of the GFL inverter. It fully demonstrates that a good parameter design of APC can also improve the synchronization stability of the GFL inverter through controller interactions.

IV. VERIFICATIONS

To verify the correctness of the theoretical analysis, a hybrid power system in Fig. 1 is built in MATLAB/Simulink. The corresponding parameters are listed in the appendix.

A. Verification of the Proposed Model

It is necessary to verify the correctness of the proposed model in Section II. The dynamic responses of the proposed

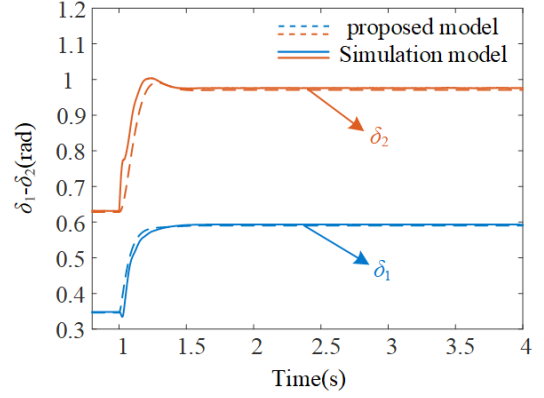


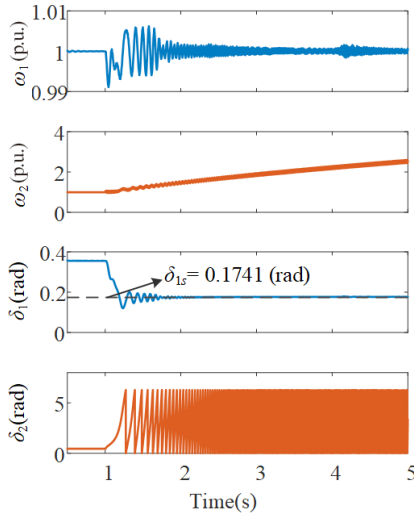
Fig. 8. Validations of the proposed model under Case 1. The grid voltage drops to 0.6 p.u. at $t = 1.0$ s.

model and the simulation model under large disturbances are shown in Fig. 8. Comparing the two response curves of the GFM inverter in Fig. 8, it is calculated that the average error of the proposed model during synchronization is about 2.45% and the steady-state error is about 0.50%. For the GFL inverter, the average error is about 3.85%, and the steady-state error is about 0.56%. It is obtained that the proposed model can capture the synchronization behaviors of the hybrid power system, and the errors caused by ignoring the voltage and current controller dynamics can be tolerated.

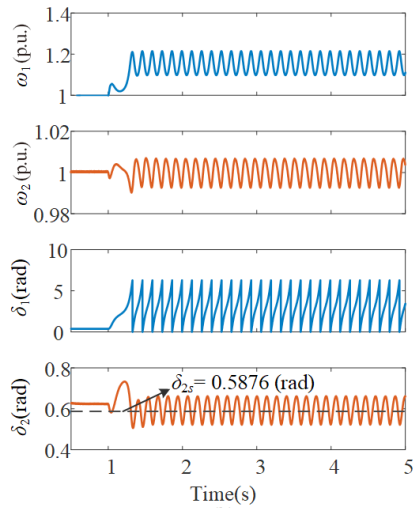
B. Verification of the Equilibrium Point Analysis

Fig. 9 verifies the SEPs of the hybrid power system after the GFL or GFM inverter loses stability. In Fig. 9(a), the GFM inverter still has a SEP because of $|EU_g/X_1 - K_1EI| > P_{\text{ref}}$, even if the GFL inverter is unstable. It is observed that the power angle of the GFM inverter oscillates near the SEP, which is caused by the unstable GFL inverter. The SEP of the GFM inverter is 0.1741 rad, which can be obtained by (11). Due to $|K_1E - K_2U_g| > U_{\text{verf}}$, the GFL inverter is allowed to have a SEP after connecting an unstable GFM inverter in Fig. 9(b). The SEP of the GFL inverter is 0.5876 rad, which is calculated by (14). There is also the power angle fluctuation of the GFL inverter near the SEP.

To verify the influence of interactions on the SEP of the GFM inverter and the GFL inverter, the simulation results of single inverter systems and hybrid power systems are shown in Fig. 10. The parameters of the single inverter system are the same as those of the hybrid power system, and the parameters in Case 1 are adopted. The $P-\delta_1$ and $U_v-\delta_2$ curves of hybrid power systems and single inverter systems in Case 1 are shown in Fig. 3. In Fig. 10(a), the GFM inverter in the hybrid power system is unstable due to the lack of a SEP, while the single GFM inverter has a SEP. In Fig. 10(b), the GFL inverter in the hybrid power system has a SEP, while the single GFM inverter loses stability due to the lack of a SEP. The simulation results are the same as those obtained from Fig. 3. It is verified that interactions are beneficial for the GFL inverter to have a SEP but harmful for the GFM inverter to have a SEP.



(a)



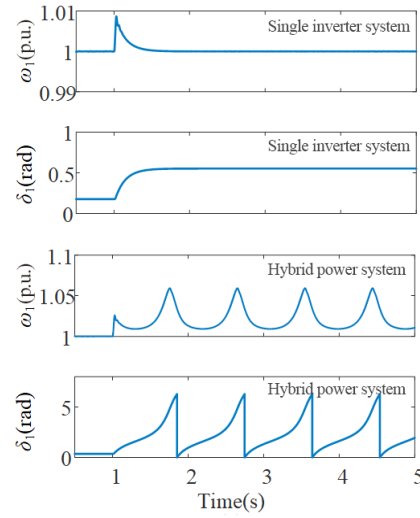
(b)

Fig. 9. Simulation results. (a) In Case 1, the GFM inverter still has a SEP when L_{g2} increases to 12 mH. (b) In Case 2, the GFL inverter still has a SEP when L_{g1} increases to 20 mH.

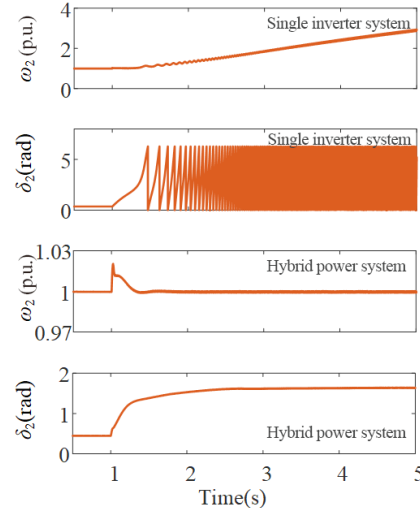
C. Verification of the Synchronization Process Analysis

To verify the influence of controller interactions on the synchronization process of the GFM inverter, the simulation results are given in Fig. 11. The phase trajectories of GFM inverters in Cases 3 and 4 are shown in Fig. 4. Synchronization stability analysis ignoring controller interactions believes that the GFM inverter will become stable in Case 3 and unstable in Case 4. However, the actual conclusion is the opposite due to the existence of controller interactions. The GFM inverter is unstable in Case 3 but stable in Case 4, as shown in Fig. 11(a) and 11(b). The instability and stabilization of the GFM inverter are caused by controller interactions, as presented in Fig. 11.

The simulation results in Fig. 12 are provided to verify the influence of controller interactions on the synchronization process of the GFL inverter. The phase trajectories of GFL inverters in Cases 5 and 6 are shown in Fig. 6. Without considering the controller interaction, the stability assessment



(a)



(b)

Fig. 10. Simulation results. (a) When L_{g3} increases to 10 mH, the single GFM inverter does not lose stability, the GFM inverter in the hybrid power system does. (b) When U_g drops to 0.4 p.u., the single GFL inverter loses stability, and the GFL inverter in the hybrid power system does not.

considers that the GFL inverter remains stable in Case 5 and loses stability in Case 6. However, due to controller interactions, the simulation results in Fig. 12(a) and 12(b) show that the GFL inverter is unstable in Case 5 and stable in Case 6. The instability and stabilization of the GFL inverter are caused by controller interactions, as shown in Fig. 12.

According to Figs. 11 and 12, controller interactions do have an impact on the synchronization process of hybrid power systems. Ignoring controller interactions may result in incorrect stability evaluation conclusions.

Fig. 13 illustrates the effect of different control parameters on the synchronization process of the hybrid power system. In Fig. 13(a), with $k_p = 0.1$ and $k_i = 10$, the GFM inverter loses stability, causing the GFL inverter to oscillate around the SEP. When k_p increases to 1 or k_i decreases to 0.5, the GFM inverter does not lose synchronization with the grid. This shows that adjusting the control parameters of PLL can improve

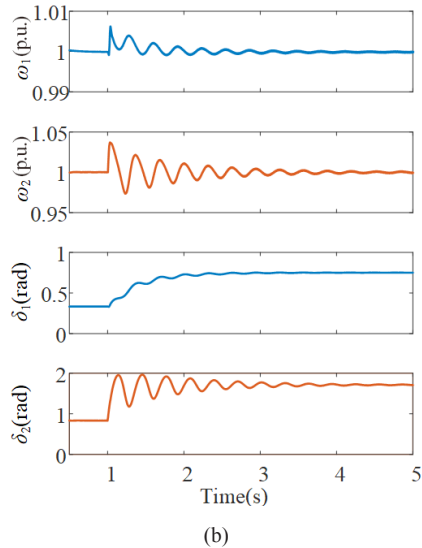
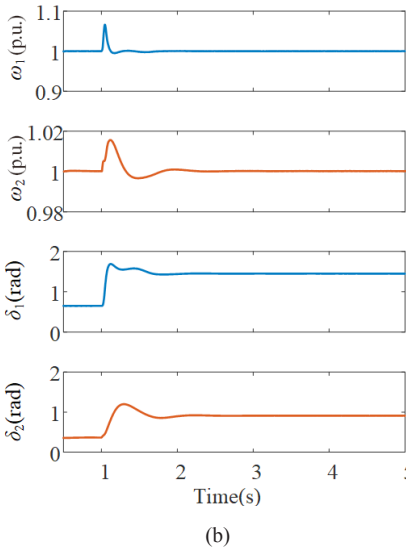
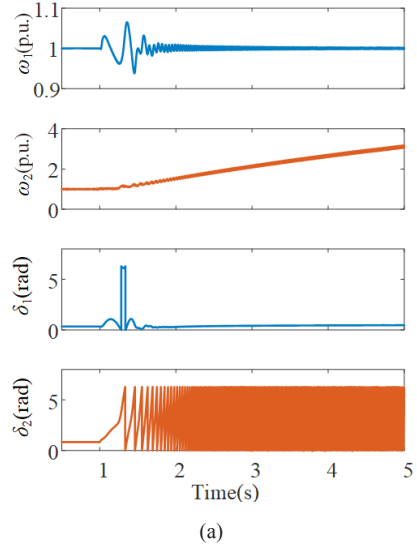
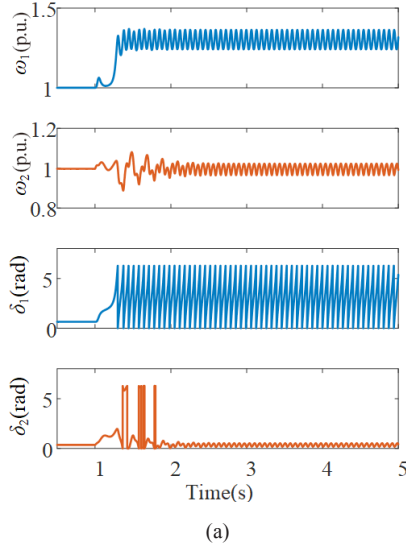


Fig. 11. Simulation results when U_g drops to 0.6 p.u.. (a) In Case 3, the GFM inverter loses stability, which is considered stable when controller interactions are ignored. (b) In Case 4, the GFM inverter does not lose stability, which is considered unstable when controller interactions are ignored.

Fig. 12. Simulation results when U_g drops to 0.4 p.u.. (a) In Case 5, the GFL inverter loses stability, which is considered stable when controller interactions are ignored. (b) In Case 6, the GFL inverter does not lose stability, which is considered unstable when controller interactions are ignored.

the synchronization dynamics of the GFM inverter through controller interactions.

In Fig. 13(b), when $J = 50$ and $D = 1000$, the GFL inverter loses stability and the GFM inverter maintains synchronization with the grid. As J decreases and D increases, the instability of the GFL inverter is removed. This indicates that designing a small J and large D for APC can benefit the synchronization stability of GFL inverters via controller interactions.

The study's results indicate that the control parameters of GFM and GFL inverter significantly affect their synchronization stability and dynamic responses. Key recommendations include:

- 1) Optimization of the APC Parameters for GFM Inverters
Higher J values improve stability during frequency fluctuations but slow response times. In low-inertia grids, reducing J can enhance responsiveness. Increasing the damping coefficient D effectively reduces oscillations, especially during significant frequency variations.

- 2) Adjustment of PLL Parameters for GFL Inverters
Increasing k_p improves frequency tracking and phase synchronization, while reducing k_i minimizes phase angle overshoot, enhancing stability under disturbances.
- 3) Optimization of Interaction Effects
Adjusting the GFM inverter's damping coefficient and the GFL inverter's PLL parameters improves synchronization. For instance, increasing k_p in the GFL inverter enhances both its own and the GFM inverter's synchronization stability through interaction effects.

V. CONCLUSION

This paper investigates the synchronization stability of a hybrid power system integrated with the GFM inverter and the GFL inverter. It is revealed that the interactions between the GFM inverter and the GFL inverter are conducive to the

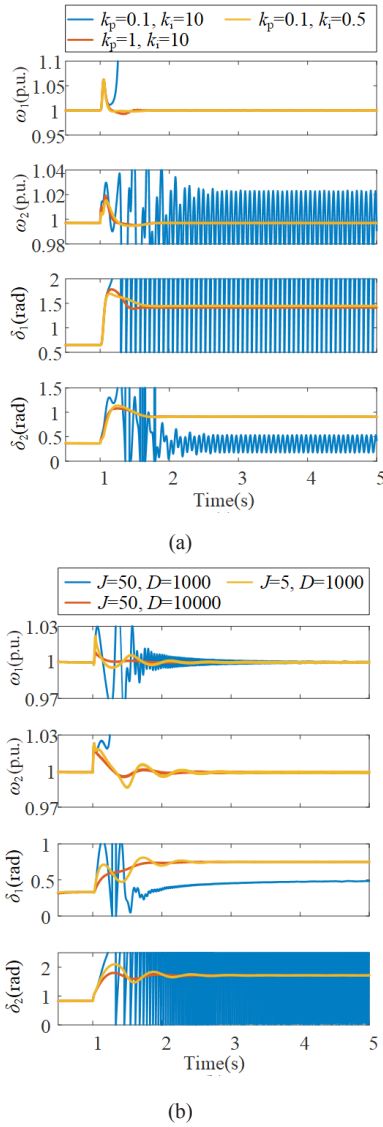


Fig. 13. Simulation results. (a) The synchronization process under different control parameters of PLL when U_g drops to 0.6 p.u.. (b) The synchronization process under different control parameters of APC when U_g drops to 0.4 p.u..

existence of SEPs for the GFL inverter but harmful to the existence of SEPs for the GFM inverter. Meanwhile, the instability and stabilization of the GFM and GFL inverter caused by controller interactions are found in this paper. It fully demonstrates the influence of controller interactions on their synchronization process. In addition, the influence of controller parameters of one inverter on the synchronization stability of another inverter in the hybrid power system is also discussed. It can provide parameter design suggestions for improving synchronization dynamics through controller interactions.

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APPENDIX

Common parameters

$U_g(\text{pk})=311\text{ V}(1.0\text{ p.u.})$, $\omega_g=100\pi\text{ (rad/s)}(1.0\text{ p.u.})$, $P_{\text{ref}}=20\text{ kW}(1.0\text{ p.u.})$, $I=2.0\text{ p.u.}$, $\theta_1=0$, $L_{g1}-L_{g3}=2,4,2\text{ mH}$.

Case 1

$J=50, D=5000, k_p=0.1, k_i=1$.

Case 2

$J=20, D=400, k_p=0.1, k_i=1$.

Common parameters

$U_g(\text{pk})=1.0\text{ p.u.}$, $\omega_g=1.0\text{ p.u.}$, $P_{\text{ref}}=2.0\text{ p.u.}$, $I=1.0\text{ p.u.}$, $\theta_1=0$, $L_{g1}-L_{g3}=4,2,2\text{ mH}$.

Case 3

$J=20, D=475, k_p=0.1, k_i=10, k_{i,\text{dec}}=0.5$.

Case 4

$J=20, D=450, k_p=0.05, k_i=0.5$.

Common parameters

$U_g(\text{pk})=1.0\text{ p.u.}$, $\omega_g=1.0\text{ p.u.}$, $P_{\text{ref}}=1.0\text{ p.u.}$, $I=2.0\text{ p.u.}$, $\theta_1=0$, $L_{g1}-L_{g3}=2,6,2\text{ mH}$.

Case 5

$J=50, D=1000, D_{\text{inc}}=10000, k_p=0.1, k_i=1$.

Case 6

$J=50, D=10000, k_p=0.1, k_i=5$.

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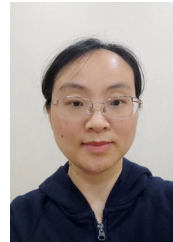
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