



Simple yet extraordinary: Super-polyhedra-built 3D chalcogenide framework of Cs₅Ga₉S₁₆ with excellent infrared nonlinear optical performance

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ABSTRACT

Non-centrosymmetric chalcogenides are attracting considerable attention as highly promising infrared nonlinear optical (IR-NLO) candidates, but it is challenging to simultaneously achieve sufficient second-harmonic-generation coefficient ($d_{\text{eff}} > 0.5 \times \text{AgGaS}_2$) and large energy gap ($E_g > 3.5 \text{ eV}$). In this work, a novel ternary chalcogenide, Cs₅Ga₉S₁₆ with an ultra-wide E_g of 4.05 eV, has been successfully obtained. This sulfide belongs to the monoclinic space group Pn (No. 7) with a novel 3D anionic [Ga₉S₁₆]¹⁵⁻ framework that is formed by super-polyhedral [Ga₉S₂₃] units through corner-sharing S atoms. Such a unique crystal structure displays desirable characteristics which indicate a promising IR-NLO candidate: favourable phase-matching feature, sufficient d_{eff} ($0.7 \times \text{AgGaS}_2$), ultrahigh laser-induced damage threshold ($31.6 \times \text{AgGaS}_2$) and broad transparent region (0.27–14.96 μm). In addition, systematic theoretical studies and structural analysis suggest that the desirable IR-NLO performances can be attributed to the super-polyhedral building blocks. This finding may provide useful insight into the understanding and designing other high-performance IR-NLO candidates with super-polyhedral-built structures.

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As the core parts in laser frequency conversion technology, nonlinear optical (NLO) materials have attracted considerable attention because of their capabilities of producing new coherent and tuneable laser source in various ranges [1–5]. To date, the commercially available NLO crystals in IR region are merely ternary chalcopyrite-type AgGaS₂ [7], AgGaSe₂ [6] and ZnGeP₂ [8]. All of them possess sufficient second-harmonic-generation (SHG) coefficient (d_{eff}) but exhibit several intrinsic problems, such as unfavorable laser-induced damage threshold (LIDT) or unexpected multi-photon absorption caused by small energy gaps (E_g), which seriously hindering their further application in the high-power laser bands. Among all the key conditions for an ideal IR-NLO candidate, achieving concurrently strong d_{eff} and wide E_g is the most challenging due to the plausible incompatibility [9,10]. Consequently, developing new IR-NLO crystals with improved comprehensive performance are critically needed and of great significance.

Metal chalcogenides with non-centrosymmetric (NCS) crystal structure is known to be the richest source of promising IR-NLO candidates [11–26]. A large number of new IR-NLO materials have been obtained in recent years, but only a few phase-matching (PM) metal chalcogenides that are able to display strong d_{eff} ($> 0.5 \times \text{AgGaS}_2$) and large E_g ($> 3.5 \text{ eV}$) simultaneously, such as [Li₂Cs₂Cl][Ga₃S₆] ($d_{\text{eff}} = 0.7 \times \text{AgGaS}_2$, $E_g = 4.17 \text{ eV}$) [27], Li₄MgGe₂S₇ ($d_{\text{eff}} = 0.7 \times \text{AgGaS}_2$, $E_g = 4.12 \text{ eV}$) [28], α-Li₂ZnGeS₄ ($d_{\text{eff}} = 4.7 \times \text{AgGaS}_2$, $E_g = 4.07 \text{ eV}$) [29], Cs₂Cd₂Ga₈S₁₅ ($d_{\text{eff}} = 0.5 \times \text{AgGaS}_2$, $E_g = 3.98 \text{ eV}$) [30], [Ba₄Cl₂][ZnGa₄S₁₀] ($d_{\text{eff}} = 1.1 \times \text{AgGaS}_2$, $E_g = 3.85 \text{ eV}$) [31], Li₂CdSiS₄ ($d_{\text{eff}} = 1.0 \times \text{AgGaS}_2$, $E_g = 3.76 \text{ eV}$) [32], BaLi₂GeS₄ ($d_{\text{eff}} = 0.5 \times \text{AgGaS}_2$, $E_g = 3.66 \text{ eV}$) [33], SrZnGeS₄ ($d_{\text{eff}} = 0.9 \times \text{AgGaS}_2$, $E_g = 3.63 \text{ eV}$) [34], Sr₂CdGe₂OS₆ ($d_{\text{eff}} = 0.8 \times \text{AgGaS}_2$, $E_g = 3.62 \text{ eV}$) [35]. However, the aforementioned compounds are concentrated in complex quaternary or more systems, while simple systems such as ternary compounds are very rare. As far as we know, the existing example is limited to only BaGa₄S₇ ($d_{\text{eff}} = 0.9 \times \text{AgGaS}_2$, $E_g = 3.54 \text{ eV}$) [36].

In this work, we focus on the ternary A(AE)/M/S (A = alkali metals; AE = alkaline-earth metals; M = group IIIA metal Ga, In or

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group IVA metal Si, Ge) system. The $[MS_4]$ tetrahedra are the most common NLO-active units in excellent IR-NLO chalcogenides. Meanwhile, the incorporation of highly electropositive A/AE metals in this system may have the additional advantage of increasing the E_g , which may help to enlarge the LIDT once a NCS material is found. Our systematic exploratory efforts have led to the discovery of a new member with 3D framework structure in this family [37–41], namely, $Cs_5Ga_9S_{16}$. Notably, it shows impressive IR-NLO properties, such as remarkable d_{eff} ($0.7 \times \text{AgGaS}_2$), and the largest E_g (4.05 eV) among known ternary NCS chalcogenides. These findings are quite encouraging for an excellent IR-NLO candidate ($d_{\text{eff}} > 0.5 \times \text{AgGaS}_2$ and $E_g > 3.5$ eV).

The sulfide $Cs_5Ga_9S_{16}$ is a new ternary phase found within the A–M–Q (A = alkali metals; M = group IIIA metal; Q = S, Se, Te) systems. It was obtained through high-temperature solid-phase method with CsCl as flux (detailed synthesis method can be found in Supporting information). The pure phase of the polycrystalline sample was verified based on the powder X-ray diffraction (XRD) results (Fig. S1 in Supporting information). Moreover, TG-DSC studies reveal that $Cs_5Ga_9S_{16}$ can be stable up to 1273 K without any obvious endo- or exo-thermal peaks and weight loss (Fig. S2 in Supporting information). Single-crystal XRD suggests that $Cs_5Ga_9S_{16}$ belongs to the monoclinic space group Pn (No. 7), with $a = 9.467(3)$ Å, $b = 9.719(3)$ Å, $c = 18.835(6)$ Å, $V = 1704.3(9)$ Å³, $Z = 2$ (Table S1), and features a 3D $[Ga_9S_{16}]^{5-}$ framework with the charge balancing Cs^+ ions residing between these voids (Fig. 1). In the structure, 9 crystallographically unique Ga atoms exhibit one type of coordination environment, i.e., the $[GaS_4]$ tetrahedral (Table S2 in Supporting information). The fundamental building block (FBB) is super-polyhedral $[Ga_9S_{23}]$ cluster which is composed of the vertex-sharing S atoms (Fig. 1a). These FBBs are further interlinked with each other *via* sharing S atoms (i.e., S2, S5, S6 and S12) along the three axial positions to form a 3D framework with Cs^+ cations located in the cavities (Fig. 1b). As listed in Table S3 (Supporting information), the Ga atoms exhibit normal Ga–S bond length, ranging from 2.219(4) Å to 2.483(5) Å [42–45]. There are 5 crystallographically unique Cs atomic positions with four various coordination geometries, i.e., bi-capped trigonal prism (CN = 8 for Cs1 and Cs2, $d_{(Cs-S)} = 3.415(4)–4.038(4)$ Å), distorted quadrangular

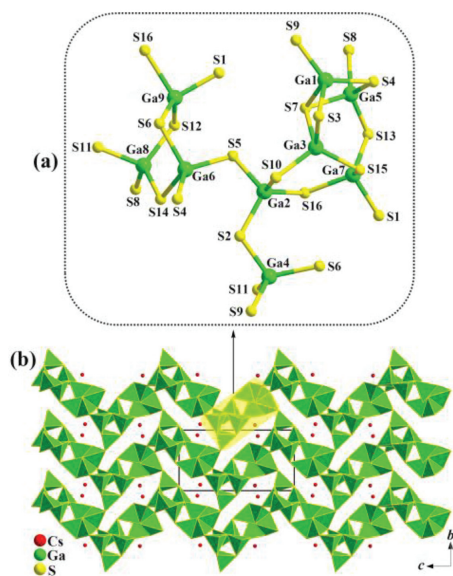


Fig. 1. Illustration of the structure of $Cs_5Ga_9S_{16}$: (a) Coordination geometry of super-polyhedral $[Ga_9S_{23}]$ FBB with the atom number outlined. (b) 3D framework structure of $Cs_5Ga_9S_{16}$ viewed along the a direction with the unit cell marked (super-polyhedral $[Ga_9S_{23}]$ cluster is outlined by a dashed area).

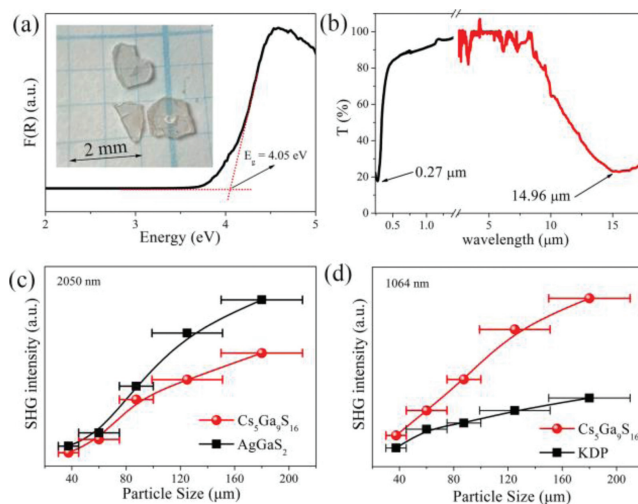


Fig. 2. Experimental results of $Cs_5Ga_9S_{16}$: (a) UV–visible–NIR diffuse reflectance spectrum (inset: polished single crystal wafers). (b) Optical transmission from UV–vis to IR band. (c, d) SHG signals versus particle size under 2050 nm and 1064 nm Q-switch laser, respectively.

(CN = 10 for Cs3, $d_{(Cs-S)} = 3.398(4)–3.998(4)$ Å), mono-capped trigonal prism (CN = 7 for Cs4, $d_{(Cs-S)} = 3.562(5)–3.860(4)$ Å), and tri-capped trigonal prism (CN = 9 for Cs5, $d_{(Cs-S)} = 3.495(4)–3.952(5)$ Å) (Fig. S3 in Supporting information). To the best of our knowledge, such diverse coordination modes of Cs atoms have been co-existed in $Cs_5Ga_9S_{16}$ together is unprecedented in Cs-based chalcogenides.

The UV–visible–NIR diffuse reflectance spectrum of $Cs_5Ga_9S_{16}$ displays one of the widest E_g (i.e., 4.05 eV) among all IR-NLO chalcogenides (Fig. 2a). Such ultra-wide E_g can effectively avoid multi-photon absorptions at the different laser wavelengths, which can obtain a higher LIDT. Moreover, single crystal of $Cs_5Ga_9S_{16}$ with a typical size of 1.5×1.0 mm polished down to ~ 0.1 mm thickness for the optical transmittance spectra. As given in Fig. 2b, $Cs_5Ga_9S_{16}$ displays a wide optical transmission from the UV–vis (cut-off wavelength: 0.27 μm) to far-IR band (cut-off wavelength: 14.96 μm). The SHG capability of $Cs_5Ga_9S_{16}$ was evaluated through the Kurtz-Perry method [46] using 2050 and 1064 nm laser irradiation, respectively, with the famous IR-NLO crystal AgGaS_2 and UV-NLO crystal KDP used for reference. As shown in Figs. 2c and d, the SHG signal tends to attain saturation with the increase of crystal size of sample, which indicates that $Cs_5Ga_9S_{16}$ possesses PM features and exhibits sufficient d_{eff} of about $0.7 \times \text{AgGaS}_2$ at 2050 nm and $2.2 \times \text{KDP}$ at 1064 nm, respectively. Furthermore, the powder LIDT as another significant parameter for NLO materials was evaluated by single pulse test method at 1064 nm [47]. Consequently, the experimental results of $Cs_5Ga_9S_{16}$ (ca. 89.1 MW/cm^2) is about 31.6 time that of reference AgGaS_2 (ca. 2.82 MW/cm^2) [48–50] and about 2.2 time that of reference KDP (ca. 39.8 MW/cm^2) [51–53], respectively.

To further investigate the microscopic mechanism of the linear optical and NLO properties of $Cs_5Ga_9S_{16}$, first-principles calculations were studied. As displayed in Fig. 3a, the calculated band structure reveals that $Cs_5Ga_9S_{16}$ is an indirect E_g compound with the top of the valence band (VB) and the bottom of the conduction band (CB) occupied the different k-point of the G and F, respectively. As anticipated, the calculated value ($E_g = 2.7$ eV) is underestimated compared to the experimental value ($E_g = 4.05$ eV), due to the own defects of the exchange-correlation function [54–56]. To obtain an accurate optical E_g , the calculated E_g *via* the HSE06 method (4.01 eV) was performed [57,58], which is very close to

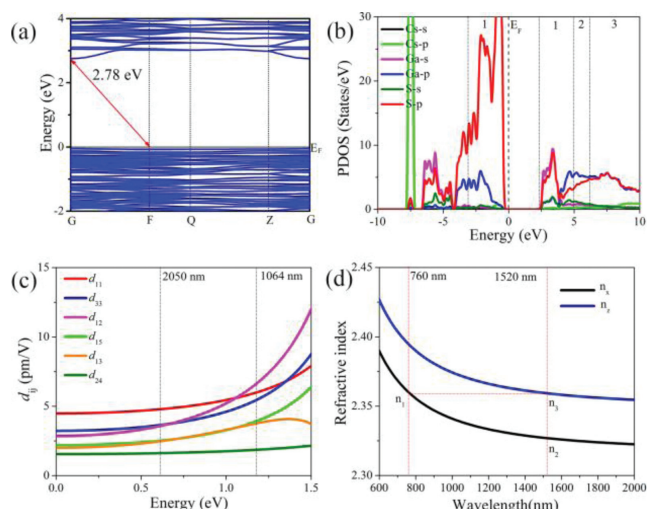


Fig. 3. Theoretical calculation results of $\text{Cs}_5\text{Ga}_9\text{S}_{16}$: (a) Electronic band structure; (b) PDOS (the Fermi level (E_F) is set at 0.0 eV); (c) Frequency-dependent SHG coefficients (d_{ij}); (d) The calculated dispersion of the refractive indices and the shortest type-I PM cut-off wavelength.

the experimental result. In addition, the first Brillouin zone with high symmetry points of $\text{Cs}_5\text{Ga}_9\text{S}_{16}$ is shown in Fig. S5 (Supporting information). The calculated partial densities of states (PDOS) reveal that the Ga-4p and S-3p, states make the main contributions to the lowest of the CB, while the highest of the VB primarily originates from Ga-4s and S-3p states, indicating that the optical E_g of $\text{Cs}_5\text{Ga}_9\text{S}_{16}$ is mainly due to the 3D $[\text{Ga}_9\text{S}_{16}]^{5-}$ framework (Fig. 3b). In addition, the SHG coefficients (d_{ij}) of $\text{Cs}_5\text{Ga}_9\text{S}_{16}$ have been also calculated. Based upon the crystal spatial symmetry and the Kleinman restriction [59], there are 6 independent nonzero d_{ij} , i.e., d_{11} , d_{12} , d_{13} , d_{15} , d_{24} and d_{36} , respectively (Fig. 3c). Among them, the effective SHG coefficient d_{eff} at 2050 nm and 1064 nm are about $0.3 \times \text{AgGaS}_2$ and $3.9 \times \text{KDP}$, respectively, which is comparable with the experimental results (Figs. 2c and d). Moreover, the minimum phase matching cut-off wavelength were calculated using the equation $n_x(2\omega) = n_z(\omega)$. The calculated PM range of the SHG response for $\text{Cs}_5\text{Ga}_9\text{S}_{16}$ is 760 nm (Fig. 3d) based on the dispersion curves of refractive indices.

The contributions to the principal source of the SHG response from the constituent units were also investigated based on the length-gauge formalism technique [60,61]. As displayed in Fig. 4a, it is clear that in the energy regions of VB-1 (-3.0 – 0.0 eV), CB-1 (3.0 – 5.0 eV) and CB-3 (6.2 – 10.0 eV), the d_{11} values increased most notably as the cut-off energy increased, which contribute mainly to the SHG effect. In other words, the states in the VB-1 section (i.e., the Ga-4p and S-3p states) and those in the CB-1 and CB-3 sections (i.e., the Ga-3s, Ga-4p and S-3p states together with minor S-3s) are mainly responsible for the d_{11} (Fig. 4b). These results mean that super-polyhedral $[\text{Ga}_9\text{S}_{23}]$ cluster in 3D $[\text{Ga}_9\text{S}_{16}]^{5-}$ framework, contribute to the SHG effect.

In summary, a novel ternary sulfide, $\text{Cs}_5\text{Ga}_9\text{S}_{16}$, is successfully prepared by a facile solid-state method. It adopts a NCS structure featuring a 3D anionic framework formed by vertex-sharing $[\text{Ga}_9\text{S}_{23}]$ super-polyhedral FBBs, with the intervening spaces filled by charge-balanced Cs^+ cations. Significantly, $\text{Cs}_5\text{Ga}_9\text{S}_{16}$ exhibits excellent IR-NLO performances, such as a favorable PM d_{eff} (0.7-fold that of the benchmark AgGaS_2), a large LIDTs (31.6-fold that of AgGaS_2), a wide transparent regions (0.27 – $14.96 \mu\text{m}$) and the highest E_g (4.05 eV) at known ternary NCS chalcogenides. Moreover, systematic theoretical analysis indicates that the IR-NLO properties of $\text{Cs}_5\text{Ga}_9\text{S}_{16}$ mainly originate from the contributions of 3D $[\text{Ga}_9\text{S}_{16}]^{5-}$ framework composed of super-polyhedral $[\text{Ga}_9\text{S}_{23}]$

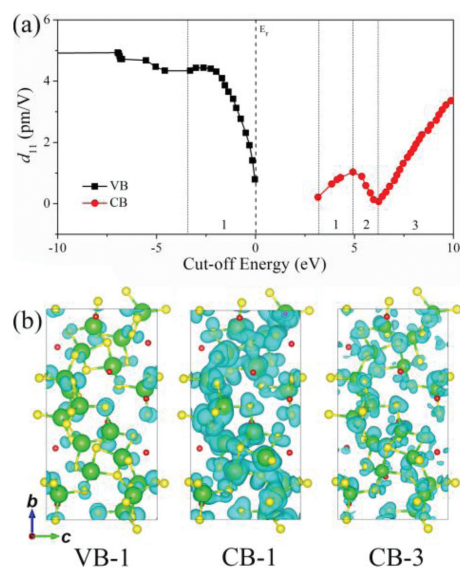


Fig. 4. Theoretical analysis of the NLO source: (a) Cut-off-energy-dependent variation of static coefficient d_{11} ; (b) Charge density maps in the VB-1, CB-1 and CB-3 regions along the bc plane.

groups. This finding further extends the study of promising IR-NLO candidates into the simple ternary system and will promote the development of new NCS chalcogenides with wide energy gaps.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.ccllet.2022.107838.

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