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A Wake-Induced Electromagnetic Fields Calculation Model for Marine Moving Objects

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Abstract: To calculate the electromagnetic fields induced by the flow around moving objects in the ocean, Maxwell's equations for a moving conductive medium were derived based on the principle of relativity, and the Lorentz covariant problem of Ohm's law was addressed. By applying the Lorentz transformation to shift the problem into a reference frame moving with the object, the calculation of electromagnetic fields induced by the wake becomes a steady-state problem, significantly simplifying the process.Formulas for electromagnetic fields induced by point sources and dipole flow fields are provided, revealing that a vortex field is a necessary condition for generating induced charge. An approximate calculation model for the induced electromagnetic fields in the wake of a marine moving object is established, with which it is estimated that the induced magnetic intensity at a distance of 1 kilometer behind the object is approximately 0.0006 nT.

Key words: wake; induced electromagnetic fields; Lorentz transformation

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0 Introduction

Moving objects in the ocean generate large-scale and long-lasting wakes. Due to the conductivity of seawater, the motion of seawater in the wake across the Earth's magnetic field induces electromagnetic fields. These induced electromagnetic fields have become important subjects of study in marine physics[1].

In the 19th century, while formulating the law of electromagnetic induction, Michael Faraday also investigated the relationship between seawater motion and the geomagnetic field. From the 1960s and 1970s onward, research on electromagnetic fields induced by seawater flow has been conducted from an ocean physics perspective[2], focusing on ocean waves[3−5], tides[6], currents[7−8], and internal waves[9]. After Fraser[3] noted that the flow induced by ship motion can also disturb the geomagnetic field, studies on calculation models[10−12], simulation methods[13−16], experimental technologies[17−19], and the characteristics of wake-induced electromagnetic fields[20−23] have become key areas in the marine science and technology.

The calculation method for the magnetic fields induced by seawater motion is based on simplified Maxwell equations. For quasi-steady flows, a steady-state approximation is used, such as for ocean circulation[7]. For flows where time variation is not dominant, the displacement current term in Maxwell's equations is ignored (e.g., waves and swell[4−5]); For the wake of ships, Maxwell's equations in the ground frame[10−11,13] are solved, where the wake's time-dependent nature complicates the problem. Ohm's law is essential for these

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calculations[23−24]. For the current form of Maxwell's equations, Ohm's law does not satisfy the covariance of coordinate transformation, and it is necessary to solve the unsteady Maxwell's equations in the ground coordinate system or to use some approximations[12]. By adopting a frame moving with the ship, the wake field becomes time-invariant, simplifying the problem significantly.

This paper analyzes the covariance of Ohm's law and proposes an improved form of Maxwell's equations to ensure Lorentz covariance. Maxwell's equations are then presented in the ship's moving frame, along with solution methods for steady-state wake-induced electromagnetic fields.

1 The covariance of Ohm's law

According to the electromagnetic theory, the electric field strength $\boldsymbol{E}$ and magnetic induction strength $\boldsymbol{B}$ induced by seawater flow in the Earth's magnetic field satisfy Maxwell's equations. In the ground frame (x,y,z), there are

$$\begin{cases} \nabla\cdot\boldsymbol{E}=\dfrac{\rho_{\mathrm{e}}}{\varepsilon_0} \\ \nabla\times\boldsymbol{E}=-\dfrac{\partial\boldsymbol{B}}{\partial t} \\ \nabla\cdot\boldsymbol{B}=0 \\ \nabla\times\boldsymbol{B}=\mu_0\boldsymbol{J}+\mu_0\varepsilon_0\dfrac{\partial\boldsymbol{E}}{\partial t} \end{cases} \tag{1}$$

where $\boldsymbol{J}$ is the electric current density, ρ_{e} is the charge density, ε_0 and μ_0 are the permittivity and permeability of free space, respectively, t is time. For conductive seawater, the current density relates to the electric and magnetic fields as

$$\boldsymbol{J}=\sigma(\boldsymbol{E}+\boldsymbol{u}\times\boldsymbol{B}) \tag{2}$$

where, $\boldsymbol{u}$ is the velocity vector of seawater, and σ is the conductivity of seawater. Eq.(2) represents Ohm's law.

In special relativity, Maxwell's equations exhibit Lorentz covariance. The relationship between physical quantities in a moving frame with velocity $(v,0,0)$ and those in the ground frame is given by

$$\begin{aligned} \boldsymbol{E}' &= (E_x,\gamma(E_y-vB_z),\gamma(E_z+vB_y)) \\ \boldsymbol{B}' &= \left(B_x,\gamma\left(B_y+\frac{v}{c^2}E_z\right),\gamma\left(B_z-\frac{v}{c^2}E_y\right)\right) \end{aligned} \tag{3}$$

$$\begin{aligned} \rho_{\mathrm{e}}' &= \gamma\left(\rho_{\mathrm{e}}-\frac{vJ_x}{c^2}\right) \\ \boldsymbol{J}' &= (\gamma(J_x-v\rho_{\mathrm{e}}),J_y,J_z) \end{aligned} \tag{4}$$

$$\boldsymbol{u}'=\frac{1}{1-\dfrac{vu_x}{c^2}}(u_x-v,u_y/\gamma,u_z/\gamma) \tag{5}$$

where the superscript apostrophe represents the physical quantity in the moving frame, and $\gamma=1/\sqrt{1-v^2/c^2}$ is the Lorentz factor, c is the speed of light.

In the moving frame, according to Lorentz covariance, Ohm's law should also hold and have the same form as Ohm's law in the ground frame, that is

$$\boldsymbol{J}'=\sigma(\boldsymbol{E}'+\boldsymbol{u}'\times\boldsymbol{B}') \tag{6}$$

In addition, by substituting the inverse transformation of Eqs. (3) and (4) into Eq.(2) , the calculation formula for current density in the moving frame can also be obtained

$$\boldsymbol{J}' = \frac{\sigma}{\gamma\left(1+\dfrac{vu'_x}{c^2}\right)}\left(\boldsymbol{E}' + \boldsymbol{u}' \times \boldsymbol{B}' + \boldsymbol{i}'\frac{v}{c^2}\boldsymbol{u}' \cdot \boldsymbol{E}'\right) - \boldsymbol{i}' v\rho'_{\mathrm{e}} \tag{7}$$

where $\boldsymbol{i}'$ is the unit vector of the coordinate x components in the moving frame, which is consistent with the direction of the frame's motion velocity. Eq.(7) is significantly different from Eq.(6) , which means that Eq.(6) has no covariance and requires a new form to ensure the covariance of Ohm's law.

2 Maxwell's equations for moving conductor

The issue of Ohm's law covariance arises from the incomplete electromagnetic constitutive relations for moving conductive media. By introducing two auxiliary quantities, electric displacement vector $\boldsymbol{D}$ and magnetic field strength $\boldsymbol{H}$, the polarization and magnetization effects of electromagnetic medium can be solved[25]. For a moving medium, the influence of motion is reflected by reconstructing the relationships among the electric displacement vector $\boldsymbol{D}$, magnetic field strength $\boldsymbol{H}$, polarization strength $\boldsymbol{P}$, and magnetization strength $\boldsymbol{M}$[26−28]. For the case of moving conductors, there have also been studies[29−30], but there is no complete and concise form proposed, mainly low-speed approximation results[31−33]. In this section, Maxwell's equations for moving conductors will be presented through the Lorentz transformation between the frame that moves with the conductive medium and the ground frame.

For a conductive medium, assuming its polarization and magnetization are both zero, at a point in space, Maxwell's equations in the frame moving with the medium are

$$\begin{cases} \nabla'' \cdot \boldsymbol{E}'' = \dfrac{\rho_0}{\varepsilon_0} \\ \nabla'' \times \boldsymbol{E}'' = -\dfrac{\partial \boldsymbol{B}''}{\partial t''} \\ \nabla'' \cdot \boldsymbol{B}'' = 0 \\ \nabla'' \times \boldsymbol{B}'' = \mu_0\sigma\boldsymbol{E}'' + \mu_0\varepsilon_0\dfrac{\partial \boldsymbol{E}''}{\partial t''} \end{cases} \tag{8}$$

where the superscript with two strokes represents the physical quantity in the frame that moves with the medium, and ρ_0 is the charge density measured in a frame that is stationary relative to the medium. Convert the first equation of Eqs. (8) to the ground frame without converting the charge density, resulting in

$$\nabla \cdot \boldsymbol{E} + u\left(\frac{1}{c^2}\frac{\partial E_x}{\partial t} - (\nabla \times \boldsymbol{B})_x\right) = \frac{\rho_0}{\gamma_{\mathrm{u}}\varepsilon_0} \tag{9}$$

where $\gamma_{\mathrm{u}} = 1/\sqrt{1-u^2/c^2}$. Convert the fourth equation of Eq.(8) to the ground frame and write it as a component in the form of

$$\begin{cases} \dfrac{u}{c^2}\nabla \cdot \boldsymbol{E} + \left(\mu_0\varepsilon_0\dfrac{\partial E_x}{\partial t} - (\nabla \times \boldsymbol{B})_x\right) = -\dfrac{\mu_0\sigma}{\gamma_{\mathrm{u}}}E_x \\ \left(\mu_0\varepsilon_0\dfrac{\partial E_y}{\partial t} - (\nabla \times B)_y\right) = -\gamma_{\mathrm{u}}\mu_0\sigma\left(E_y - uB_z\right) \\ \left(\mu_0\varepsilon_0\dfrac{\partial E_z}{\partial t} - (\nabla \times \boldsymbol{B})_z\right) = -\gamma_{\mathrm{u}}\mu_0\sigma\left(E_z + u\boldsymbol{B}_y\right) \end{cases} \tag{10}$$

Multiply the first equation in Eqs. (10) by u and subtract it from Eq.(9) to obtain

$$\nabla \cdot \boldsymbol{E} = \gamma_{\mathrm{u}}\left(\frac{\rho_0}{\varepsilon_0} + \mu_0\sigma u E_x\right) \tag{11}$$

Multiply Eq.(8) by $\dfrac{u}{c^2}$ and subtract the first equation in Eqs. (10) to obtain

$$(\nabla \times \boldsymbol{B})_x = \gamma_u \mu_0 (\sigma u E_x + u\rho_0) + \mu_0 \varepsilon_0 \frac{\partial E_x}{\partial t} \tag{12}$$

The formula $c^2 = 1/\mu_0\varepsilon_0$ for calculating the speed of light is utilized in the above calculations. Considering $\boldsymbol{u}=(u,0,0)$, the following related statement extensions can be made as

$$\begin{aligned} &uE_x \to \boldsymbol{u}\cdot\boldsymbol{E} \\ &\rho_0 u \to \rho_0 \boldsymbol{u} \\ &-uB_z \to (\boldsymbol{u}\times\boldsymbol{B})_y \\ &uB_y \to (\boldsymbol{u}\times\boldsymbol{B})_z \end{aligned} \tag{13}$$

At the same time, similar transformations and calculation methods are adopted for the middle two equations of Eq.(8) , and Maxwell's equations for a moving conductive medium in the ground frame are obtained as follows

$$\begin{cases} \nabla\cdot\boldsymbol{E} = \gamma_u\left(\dfrac{\rho_0}{\varepsilon_0} + \mu_0\sigma\boldsymbol{u}\cdot\boldsymbol{E}\right) \\ \nabla\times\boldsymbol{E} = -\dfrac{\partial\boldsymbol{B}}{\partial t} \\ \nabla\cdot\boldsymbol{B} = 0 \\ \nabla\times\boldsymbol{B} = \gamma_u\mu_0\left[\sigma(\boldsymbol{E}+\boldsymbol{u}\times\boldsymbol{B}) + \rho_0\boldsymbol{u}\right] + \mu_0\varepsilon_0\dfrac{\partial\boldsymbol{E}}{\partial t} \end{cases} \tag{14}$$

Compared with Eq.(1) , we can obtain

$$\rho_e = \gamma_u\rho_0 + \gamma_u\mu_0\varepsilon_0\sigma\boldsymbol{u}\cdot\boldsymbol{E} \tag{15}$$

$$\boldsymbol{J} = \gamma_u\left[\sigma(\boldsymbol{E}+\boldsymbol{u}\times\boldsymbol{B}) + \rho_0\boldsymbol{u}\right] \tag{16}$$

Eq.(15) indicates that the charge of a moving conductive medium includes the net charge in a stationary state and the charge induced by the medium's motion. Eq.(16) is the form of Ohm's law, which includes the current generated by the movement of net charges and the expansion effect of medium motion. We substitute the expressions for charge density and current density into Eqs. (15) and (16) , and also substitute the inverse transformations of Eqs. (3) and (5) into Eq.(4), to recalculate the expressions for current density and charge density in the moving frame

$$\rho'_e = \gamma_{u'}\rho_0 + \gamma_{u'}\mu_0\varepsilon_0\sigma\boldsymbol{u}'\cdot\boldsymbol{E}' \tag{17}$$

$$\boldsymbol{J}' = \gamma_{u'}\left[\sigma(\boldsymbol{E}'+\boldsymbol{u}'\times\boldsymbol{B}') + \rho_0\boldsymbol{u}'\right] \tag{18}$$

The expressions of Eqs. (17) and (18) are exactly the same as those of Eqs. (15) and (16) , indicating that the newly defined charge density and Ohm's law are covariant under Lorentz transformation, thus ensuring that the expression of the equation system has an invariant form under various inertial frames, and thereby providing a consistent mathematical and physical description of the variation of the electromagnetic fields in the ship frame proposed earlier.

3 Induced electromagnetic fields of steady-state wave and flow

The induced electromagnetic fields by seawater motion, as the name suggests, originate from the movement of seawater.When the motion remains constant in an appropriate reference frame, the induced electromagnetic fields also remain constant, thereby transforming the problem into a steady-state problem. The induced electromagnetic fields from various steady marine motions—such as ship waves generated by vessels moving at constant speed, internal waves caused by submerged bodies (the so-called body effect), and slowly varying ocean currents—all exhibit this characteristic. These are collectively categorized as steady-state wave- and flow-induced electromagnetic problems. Based on Eq.(14) , the time derivative term can be

eliminated by considering the realistic condition where the flow velocity is significantly lower than the speed of light ($\gamma_u \cong 1$) . The governing equations for this class of electromagnetic fields therefore become

$$\begin{cases} \nabla \cdot \boldsymbol{E} = \dfrac{\rho_0}{\varepsilon_0} + \mu_0 \sigma \boldsymbol{u} \cdot \boldsymbol{E} \\ \nabla \times \boldsymbol{E} = 0 \\ \nabla \cdot \boldsymbol{B} = 0 \\ \nabla \times \boldsymbol{B} = \mu_0 \left[\sigma (\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B}) + \rho_0 \boldsymbol{u} \right] \end{cases} \tag{19}$$

Performing a divergence operation on the fourth equation in Eq.(19) leads to

$$\sigma \nabla \cdot \boldsymbol{E} + \sigma \nabla \cdot (\boldsymbol{u} \times \boldsymbol{B}) + \nabla \cdot (\rho_0 \boldsymbol{u}) = 0 \tag{20}$$

Applying the vector operation formula

$$\nabla \cdot (\boldsymbol{u} \times \boldsymbol{B}) = \boldsymbol{B} \cdot (\nabla \times \boldsymbol{u}) - \boldsymbol{u} \cdot (\nabla \times \boldsymbol{B}) \tag{21}$$

By substituting the fourth equation of Eq.(19) into Eq.(21), we can obtain

$$\nabla \cdot (\boldsymbol{u} \times \boldsymbol{B}) = \boldsymbol{B} \cdot (\nabla \times \boldsymbol{u}) - \mu_0 \left[\sigma \boldsymbol{u} \cdot \boldsymbol{E} + \rho_0 \boldsymbol{u} \cdot \boldsymbol{u} \right] \tag{22}$$

Substitute the first equation of Eqs. (19) and (22) into Eq.(20), and use $\gamma_u \cong 1$ to obtain

$$\frac{\rho_0}{\varepsilon_0} + \frac{1}{\sigma} \nabla \cdot (\rho_0 \boldsymbol{u}) = -\boldsymbol{B} \cdot (\nabla \times \boldsymbol{u}) \tag{23}$$

Due to $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m, $\sigma = 5$ s/m, and the $\boldsymbol{u}$ is generally on the order of m/s, the above equation can be approximately expressed as

$$\rho_0 \cong -\varepsilon_0 \boldsymbol{B} \cdot (\nabla \times \boldsymbol{u}) \tag{24}$$

As shown in Eq.(24) , the presence of vorticity in the oceanic velocity field generates induced charges, while irrotational motion yields zero charge density when measured in the medium's reference frame.

Furthermore, from Eq.(19) , it can be seen that the curl of the electric field $\boldsymbol{E}$ and the divergence of the magnetic field $\boldsymbol{B}$ are equal to 0, which can be expressed as

$$\boldsymbol{E} = \nabla \phi \tag{25}$$

$$\boldsymbol{B} = \nabla \times \boldsymbol{A} \tag{26}$$

where, ϕ is a scalar function, $\boldsymbol{A}$ is a vector function, and $\nabla \cdot \boldsymbol{A} = 0$, in this way

$$\begin{cases} \Delta \phi = \dfrac{\rho_0}{\varepsilon_0} + \mu_0 \sigma \boldsymbol{u} \cdot \boldsymbol{E} \\ \Delta \boldsymbol{A} = -\mu_0 \left[\sigma (\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B}) + \rho_0 \boldsymbol{u} \right] \end{cases} \tag{27}$$

The electric and magnetic fields in the right-hand expression of Eq.(27) can be decomposed into their background and induced components as follows

$$\boldsymbol{E} = \boldsymbol{E}_e + \boldsymbol{E}_u \tag{28}$$

$$\boldsymbol{B} = \boldsymbol{B}_e + \boldsymbol{B}_u \tag{29}$$

where, $\boldsymbol{E}_e$ and $\boldsymbol{B}_e$ are the electric field and magnetic induction intensity of the Earth in the moving frame, both being spatially uniform and temporally constant within the studied domain. $\boldsymbol{E}_u$ and $\boldsymbol{B}_u$ are the electric field and magnetic induction intensity induced by seawater motion, generally much smaller than the electric field and magnetic field of the Earth. Therefore, Eqs. (24) and (27) can be approximated as

$$\begin{cases} \rho_0 = -\varepsilon_0 \boldsymbol{B}_e \cdot (\nabla \times \boldsymbol{u}) \\ \Delta \phi = \dfrac{\rho_0}{\varepsilon_0} + \mu_0 \sigma \boldsymbol{u} \cdot \boldsymbol{E}_e \\ \Delta \boldsymbol{A} = -\mu_0 \left[\sigma (\boldsymbol{E}_e + \boldsymbol{u} \times \boldsymbol{B}_e) + \rho_0 \boldsymbol{u} \right] \end{cases} \tag{30}$$

Eq.(30) constitute an approximate governing system for steady-state wave- and flow-induced electromagnetic phenomena, forming a well-posed boundary value problem when supplemented with

appropriate electromagnetic boundary conditions. For infinite space, the solutions of the steady-state wave- and flow-induced electric field and magnetic induction intensity obtained using scalar and vector Poisson equations are

$$\begin{cases}\rho_0 = -\varepsilon_0 \boldsymbol{B}_{\mathrm{e}} \cdot (\nabla \times \boldsymbol{u}) \\ \boldsymbol{E}_{\mathrm{u}} = -\nabla \iiint_v \dfrac{(\rho_0/\varepsilon_0 + \mu_0 \sigma \boldsymbol{u} \cdot \boldsymbol{E}_{\mathrm{e}})}{4\pi r} \mathrm{d}V \\ \boldsymbol{B}_{\mathrm{u}} = \nabla \times \iiint_v \dfrac{\mu_0 [\sigma (\boldsymbol{E}_{\mathrm{e}} + \boldsymbol{u} \times \boldsymbol{B}_{\mathrm{e}}) + \rho_0 \boldsymbol{u}]}{4\pi r} \mathrm{d}V \end{cases} \tag{31}$$

The third equation in Eq.(31) is the Biot Savart formula, where $\boldsymbol{r}$ is the vector from the field point to the spatial integration point, and r is its magnitude.

4 Induced electromagnetic source and sink flow fields

4.1 Induced electromagnetic a source flow field

The velocity field of the unit point source located at $\boldsymbol{r}_0$ can be expressed as

$$\boldsymbol{u}(\boldsymbol{r}') = -\nabla \frac{1}{4\pi |\boldsymbol{r}' - \boldsymbol{r}_0|} \tag{32}$$

According to Eq.(31) , the induced electromagnetic source field is

$$\begin{cases}\rho_0 = 0 \\ \boldsymbol{E}_{\mathrm{u}} = \dfrac{\mu_0 \sigma}{16\pi^2} \nabla \left(\boldsymbol{E}_{\mathrm{e}} \cdot \nabla \iiint_v \dfrac{1}{|\boldsymbol{r}' - \boldsymbol{r}_0|} \dfrac{1}{|\boldsymbol{r} - \boldsymbol{r}'|} \mathrm{d}V' \right) \\ \boldsymbol{B}_{\mathrm{u}} = -\dfrac{\mu_0 \sigma}{16\pi^2} \nabla \times \left(\boldsymbol{B}_{\mathrm{e}} \times \nabla \iiint_v \dfrac{1}{|\boldsymbol{r}' - \boldsymbol{r}_0|} \dfrac{1}{|\boldsymbol{r} - \boldsymbol{r}'|} \mathrm{d}V' \right) \end{cases} \tag{33}$$

The kernel integral of Eq.(33) is

$$\iiint_v \frac{1}{|\boldsymbol{r}' - \boldsymbol{r}_0|} \frac{1}{|\boldsymbol{r} - \boldsymbol{r}'|} \mathrm{d}V' = \iiint_v \frac{1}{|\boldsymbol{r}'' + \boldsymbol{D}|} \frac{1}{|\boldsymbol{r}'' - \boldsymbol{D}|} \mathrm{d}V'' \tag{34}$$

where $\boldsymbol{D} = \dfrac{\boldsymbol{r} - \boldsymbol{r}_0}{2}$, $\boldsymbol{r}'' = \boldsymbol{r}' - \dfrac{\boldsymbol{r} + \boldsymbol{r}_0}{2}$, equivalent to translating the coordinate origin to $\dfrac{\boldsymbol{r} + \boldsymbol{r}_0}{2}$. Define the angle between $\boldsymbol{r}''$ and $\boldsymbol{D}$ as ϕ, then

$$\iiint_v \frac{1}{|\boldsymbol{r}'' + \boldsymbol{D}|} \frac{1}{|\boldsymbol{r}'' - \boldsymbol{D}|} \mathrm{d}V'' = \int_0^{\infty} \int_0^{\pi} \frac{-2\pi r''^2 \mathrm{d}r'' \mathrm{d}\cos\phi}{\sqrt{(r''^2 + D^2)^2 - (2r''D)^2 \cos^2\phi}} = D \int_0^{\infty} 2\pi\eta \sin^{-1} \frac{2\eta}{\eta^2 + 1} \mathrm{d}\eta \tag{35}$$

where $\eta = r''/D$. When $\eta \to \infty$, the integral of Eq.(35) tends towards infinity, which does not converge, can be written as a two-stage integral as follows

$$D \int_0^{\infty} 2\pi\eta \sin^{-1} \frac{2\eta}{\eta^2 + 1} \mathrm{d}\eta = D \int_0^{\Lambda} 2\pi\eta \sin^{-1} \frac{2\eta}{\eta^2 + 1} \mathrm{d}\eta + D \int_{\Lambda}^{\infty} 2\pi\eta \sin^{-1} \frac{2\eta}{\eta^2 + 1} \mathrm{d}\eta \tag{36}$$

where Λ is a large but finite value, and the second term on the right side of Eq.(36) is approximately

$$D \int_{\Lambda}^{\infty} 2\pi\eta \sin^{-1} \frac{2\eta}{\eta^2 + 1} \mathrm{d}\eta = 4\pi D \int_{\Lambda}^{\infty} \mathrm{d}\eta = 4\pi \int_{\Lambda D}^{\infty} \mathrm{d}r'' \tag{37}$$

let

$$\Pi = \int_0^{\Lambda} 2\pi\eta \sin^{-1} \frac{2\eta}{\eta^2 + 1} \mathrm{d}\eta \tag{38}$$

The value of Eq.(37) is infinite, but Π is a finite value. The integral of Eq.(34) is written as follows

$$\iiint_v \frac{1}{|\boldsymbol{r}'-\boldsymbol{r}_0|}\frac{1}{|\boldsymbol{r}-\boldsymbol{r}'|}\mathrm{d}V' = D\Pi + 4\pi\int_{\Lambda D}^{\infty}\mathrm{d}r'' \tag{39}$$

Substituting Eq.(39) into Eq.(33) yields the induced electromagnetic fields of the point source induced flow field, but its value is infinite and has no practical significance.

4.2 Induced electromagnetic dipole flow field

Arrange the source and sink of dipole at positions $\pm d$ on the x-axis, respectively. According to the calculation formula for the induced electromagnetic fields of a point source induced flow field in the previous section, a kernel integral similar to Eq.(33) can be obtained as follows

$$\iiint_v \left(\frac{1}{|\boldsymbol{r}'-\boldsymbol{r}_0^+|}\frac{1}{|\boldsymbol{r}-\boldsymbol{r}'|}-\frac{1}{|\boldsymbol{r}'-\boldsymbol{r}_0^-|}\frac{1}{|\boldsymbol{r}-\boldsymbol{r}'|}\right)\mathrm{d}V' = (D^+-D^-)\Pi - 4\pi\int_{\Lambda D^-}^{\Lambda D^+}\mathrm{d}\boldsymbol{r}'' = (D^+-D^-)(\Pi-4\pi\Lambda) \tag{40}$$

where $\boldsymbol{r}_0^+$ is the location $(-d,0,0)$ of point source, $\boldsymbol{r}_0^-$ is the location $(d,0,0)$ of point sink, and $D^{\pm}=\left|\frac{\boldsymbol{r}-\boldsymbol{r}_0^{\pm}}{2}\right|$. Using Eq.(38) , Eq.(40) can be expressed as

$$(D^+-D^-)(\Pi-4\pi\Lambda) = (D^+-D^-)\int_0^{\Lambda}2\pi\left(\eta\sin^{-1}\frac{2\eta}{\eta^2+1}-2\right)\mathrm{d}\eta \tag{41}$$

By using numerical integration, we can obtain

$$\int_0^{\Lambda}2\pi\left(\eta\sin^{-1}\frac{2\eta}{\eta^2+1}-2\right)\mathrm{d}\eta = -12.56 \tag{42}$$

This leads to

$$\iiint_v \left(\frac{1}{|\boldsymbol{r}'-\boldsymbol{r}_0^+|}\frac{1}{|\boldsymbol{r}-\boldsymbol{r}'|}-\frac{1}{|\boldsymbol{r}'-\boldsymbol{r}_0^-|}\frac{1}{|\boldsymbol{r}-\boldsymbol{r}'|}\right)\mathrm{d}V' = -12.56\times(D^+-D^-) \tag{43}$$

The induced electromagnetic fields of the flow field induced by the point source and point sink arranged at position $\pm d$ on the x-axis are as follows

$$\begin{cases}\boldsymbol{E}_{\mathrm{u}} = \dfrac{-12.56\mu_0\sigma}{16\pi^2}\nabla[\boldsymbol{E}_{\mathrm{e}}\cdot\nabla(D^+-D^-)]\\ \boldsymbol{B}_{\mathrm{u}} = \dfrac{12.56\mu_0\sigma}{16\pi^2}\nabla\times[\boldsymbol{B}_{\mathrm{e}}\times\nabla(D^+-D^-)]\end{cases} \tag{44}$$

4.3 The electromagnetic field induced by wake of a marine moving object

This section demonstrates a simple application of the derived theory: calculating the magnetic fields induced by the flow field of a moving vehicle with constant speed in an unbounded fluid domain. The flow field is expressed using potential of source and sink. A ground-fixed reference frame (x, y, z) is established with the positive x-axis pointing towards the east, the positive y-axis pointing towards the north, the positive z-axis pointing upwards. The length, diameter, and volume of the vehicle are $L = 100$ m, $D = 10$ m, and $V = 5000$ m^3, respectively. The speed is $U = 10$ m/s, and its direction is (−1, 0, 0), moving from east to west. The geomagnetic intensity is $\boldsymbol{B}_{\mathrm{e}} = 5\times10^{-5}$ T, which is parallel to the horizontal plane and points to the north (0, 1, 0) . These are shown in Fig. 1.

In the geodetic frame, the flow field induced by the moving vehicle varies with time, and the problem studied has unsteady characteristics, requiring the solution of the unsteady Maxwell's equations. If the reference frame is established on the moving vehicle, the flow field around it will not change over time, and the problem will become a steady-state problem, requiring only the solution of the stationary Maxwell's equations. The frame moving with the vehicle is (x', y', z'), whose coordinate axes are aligned with the geodetic frame, but the origin is located at the midpoint of the vehicle. In frame (x', y', z'), the vehicle is

fixed, but there is an incoming flow of $\boldsymbol{U}$= (10, 0, 0) from far ahead. According to the electromagnetic fields transformation relationship between reference frames, under the approximation of $\gamma_u \cong 1$, the Earth's magnetic field remains unchanged $\boldsymbol{B}'_e = \boldsymbol{B}_e$, and there is a uniform electric field with a direction z' in space, and its magnitude is UB_e, namely $(0, 0, UB_e)$.

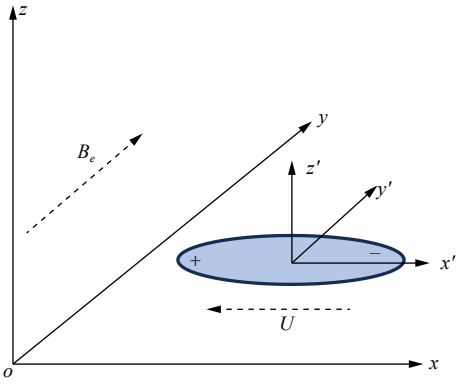


Fig.1 Definition of coordinate system and problems

For the simplification of the problem, without considering the viscous effect, the induced flow field of the vehicle is expressed using a source and a sink with a distance of L and an intensity of UV/L. The calculation formula for the velocity field is[23]

$$\boldsymbol{u}' = -\frac{UV}{4\pi L}\nabla'\left(\frac{1}{\sqrt{(x'+L/2)^2+y'^2+z'^2}} - \frac{1}{\sqrt{(x'-L/2)^2+y'^2+z'^2}}\right) \tag{45}$$

The superscripts in the above equation represent the physical quantities in the reference frame that move with the vehicle, $\boldsymbol{i}'$, $\boldsymbol{j}'$, $\boldsymbol{k}'$ are the unit directions vectors of x', y', z' respectively. To simplify the calculation of the electromagnetic field strength at the far rear of the vehicle, only the points on the axis are considered, namely $y' = z' = 0$. Using Eq.(44) , we obtain

$$\rho_0 = 0 \tag{46}$$

$$\boldsymbol{E}'_u = \frac{12.56\mu_0\sigma U^2 V B_e}{32\pi^2(x'^2 - L^2/4)}\boldsymbol{k}' \tag{47}$$

$$\boldsymbol{B}'_u = \frac{-12.56\mu_0\sigma U V B_e}{32\pi^2(x'^2 - L^2/4)}\boldsymbol{j}' \tag{48}$$

Using the inverse transformation of Eq.(3) , considering that $U << c$, transform Eqs. (47) - (48) into the ground frame, we have

$$\boldsymbol{E}_u = 0 \tag{49}$$

$$\boldsymbol{B}_u = \frac{-12.56\mu_0\sigma U V B_e}{32\pi^2\left((x-Ut)^2 - L^2/4\right)}\boldsymbol{j} \tag{50}$$

From Eq.(50) , the variation of magnetic induction intensity with distance in the wake of a sailing vehicle is shown in Fig. 2. At a distance of 1 kilometer behind the vehicle, the magnetic induction intensity is about 0.0006 nT, which can be detected by the most sensitive magnetic detector currently available.

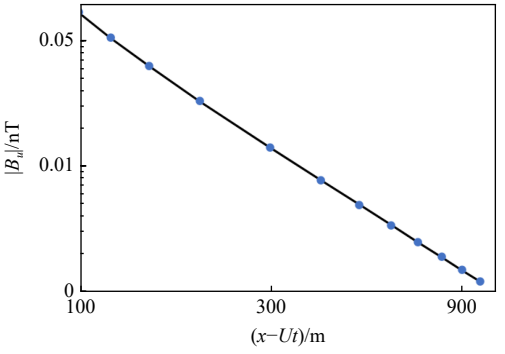


Fig.2 Magnetic induction intensity in the wake of a vehicle

5 Concluding remarks

The electromagnetic induction calculation and characteristics of marine moving object's wake are complex interdisciplinary problems of electromagnetics and fluid mechanics. In this article, the Lorentz covariance of Ohm's law is analyzed and presents Maxwell's equations in moving conductors through the principle of relativity, thereby solving the Lorentz covariance problem of Ohm's law. By using Lorentz transformation, the problem of the induced magnetic fields in the wake of a vehicle is transformed into a

reference frame that follows the vehicle's motion and the unsteady problem is transformed into a steady-state problem, greatly simplifying the solving process. At the same time, an approximate calculation method for the induced magnetic fields of vehicle’s wake is provided.

However, the rationality of several issues in the theoretical analysis of electromagnetic fields in moving conductors still needs further research: 1) the relationship between Ohm's law and polarization and magnetization is unclear, and whether Ohm's law includes polarization and magnetization needs further analysis; 2) the relationship between the evolution equation of charge and the velocity of conductor motion is unclear. The use of moving coordinates seems to indicate that the evolution of charge is independent of the velocity of the medium. However, further research is needed to investigate what happens when there is a spatial gradient in the velocity of the medium.

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海洋运动物体尾流感应电磁场计算模型

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摘要: 为计算海洋运动物体绕流场感应的电磁场, 本研究基于相对性原理推导了运动导电介质中的麦克斯韦方程组, 并解决了欧姆定律的洛伦兹协变性问题。通过洛伦兹变换将问题转换至随体坐标系, 尾流感应电磁场的计算简化为稳态问题。同时, 本研究给出了点源与偶极子流场感应电磁场的解析表达式, 证明涡量场是产生感应电荷的必要条件。建立的航行体尾流感应电磁场近似计算模型表明, 在物体后方 1 公里处感应磁感应强度约为 0.0006 纳特斯拉。

关键词: 尾流; 感应电磁场; 洛伦兹变化
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