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Numerical Simulation of the Model Ice Flexural Strength Based on Tsai-Wu Failure Criterion

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Abstract: In the past few decades, the navigation performance of ships and structures in ice-covered waters has not been fully studied, especially the influence of ice mechanical properties on icebreaking ability. Ice bending strength is a key ice parameter for predicting ship ice loads, and accurate ice bending strength is also the key to scaling model tests results to real ship. However, numerical simulation studies on model ice bending strength of ice tanks are often neglected. In this paper, an explicit finite element method model is used to simulate the ice cantilever beam test, and the failure load and bending strength of the ice are obtained. In this model, the Tsai-Wu failure criterion is used as the material constitutive model, and the required simulation parameters are obtained from the model ice test in ice tank. Parameter sensitivity analysis shows that the cantilever beam size of the model ice has a significant effect on the flexural strength. The results show that proper rounding at the root of the cantilever beam is beneficial to reduce stress concentration and obtain more accurate bending strength; the thickness, width and length of the cantilever beam should conform to a certain ratio, and consistent with the ITTC recommended reference. Therefore, the results of this study can promote model ice experiments and numerical studies and provide ice strength data support for ship design and polar ship maneuvering.

Key words: model ice; LS-DYNA; Tsai-Wu failure criterion; cantilever beam test

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0 Introduction

With the development of polar resource exploration and transportation, especially the opening of the Arctic waterway, the research on the interaction between polar ships, ocean structures and sea ice has become increasingly popular. The research on the basic mechanical properties of ice is the key to the development of resources and waterways, and related numerical simulations still need to be further studied.

The model scale test in ice tank is the state-of-the-art and most reliable prediction method of icebreaking performance. The suggestions and guidelines for basic mechanical performance test of model ice in ice tanks were given by ITTC Ice Committee, providing a reliable method for the test methods (ITTC, 2021^[1],

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Myland, et al 2020^[2]; Huang, et al 2022^[3]). Von Bock and Polach, et al (2013)^[4] combined experimental and numerical methods for the study of ice cantilever beam bending tests. And the combined method has been verified to be more effective in observing and understanding of the ice mechanics. Compared with actual sea ice, model ice has more stable mechanical properties, is easier to obtain, and provides a good support for numerical simulation. Bending damage is the main factor in the ship-ice interaction process because the flexural strength values of the inclined structure of the bow and ice are both lower than their compressive strength values. Therefore, the ice shows significant bending damage when it breaks (Enkvist, 1972^[5]; Lindqvist, 1989^[6] or Valanto, 2001^[7]). Fig. 1 illustrates that most of the ice is damaged by bending under the impact of the ship (Huang, 2010^[8]; Li et al., 2018^[9] and 2019^[10]; Guo et al., 2021^[11]), this clearly explains why flexural strength is often used in semi-empirical formulas to predict the resistance of ships in ice (Lindqvist, 1989^[6]; Suominen et al. 2021^[12], Huang et al., 2022^[3]). Local compression damage cannot be ignored, Varsta (1983)^[13] gave the state-of-the-art schematic diagram of ice damage when a ship impacts with level ice, as shown in Fig. 1. The work of this paper is to study the flexural failure of model ice based on the Tsai-Wu failure criterion using the finite element method (FEM). In general, it is very necessary to numerically simulate bending failure as the main failure form, which can provide a numerical model of ice material for numerical simulation of ship-ice interaction.

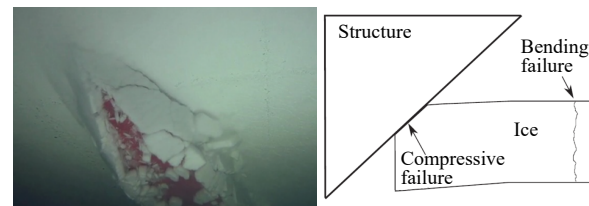


Fig.1 Inclined structure causing bending and compressive failure of ice (Guo et al, 2022^[11]; Varsta, 1983^[13])

The finite element method (FEM) is widely used in the numerical simulation of the interaction of ice structures. Firstly, the failure criterion for columnar-grained ice proposed (Varsta, 1983^[13]) is a non-linear dynamic finite element model to simulate the bending of the columnar sea ice wedge based on the Tsai-Wu failure criterion. With a unified model covering freshwater ice and sea ice proposed (Derradji-Aouat, 2000^[14]; Derradji-Aouat, 2003^[15]), Tsai-Wu yield function was more widely used as a criterion for the failure of ice materials (Liu et al., 2011^[16]; Gao et al., 2015^[17]; Xu et al., 2019^[18]; Li et al. 2022^[19]). Rossi et al. (2020)^[20] presented a study evaluating the validity of the Tsai-Wu constitutive material model laws in the LS-DYNA FE code. However, the study was aimed at composite materials under low-speed loading conditions, and further verification of the applicability of the model to ice materials is needed. In addition to the above discussion, the Tsai-Wu yield criterion has been widely used in the numerical simulation of ship-ice interaction, and good results have been obtained.

Although Tsai-Wu yield criterion is very effective, the determination of its parameters remains a difficult point. Because the mechanical properties of ice vary with many factors, such as salinity, temperature, loading rate, etc. (Chai et al. 2020^[21]; Riska, 2018^[22]; Li et al. 2019^[10]; Van Vliet and Metrikine 2019^[23]; Mikko, et al. 2019^[24]). In this simulation, the columnar model ice in the ice tank is selected, which further limits the environmental factors of the model ice. The columnar model ice is S2 type ice, and its structural characteristics are shown in Fig. 2 (Derradji-Aouat, 2003^[15]), Fig. 2(a) is the crystal structure of multi-year sea ice, and Fig. 2(b) is the crystal structure of first year sea ice. The brine and air pockets of multi-year ice are precipitated, and the columnar structure is more obvious. During the experiment in ice tank, the temperature was stabilized at -6°C , the temperature of the ice should be limited as the same temperature. According to the ITTC rules, the model ice should fail in the same mode as mostly brittle, at a higher test speed, but at the same time the speed must be slow enough to avoid significant hydrodynamic effects or specimen damage due to the high local impact of the test plunger. Therefore, in this study, the stress-strain

behavior of model ice is considered to be linearly elastic with a brittle mode of failure (Varsta, 1983^[13]; Derradji-Aouat, 2003^[15]). The parameters used in the Tsai-Wu yield criterion are selected by the above constraints. It can be seen from Fig. 3 that the ice exhibits obvious elasticity in the brittle stage, and finally fails after the yield stage. Of course, using Tsai-Wu failure criterion only is not enough to analyze the ice load. Therefore, it is necessary to define the constitutive equation describing the stress-strain in the elastic stage when performing numerical simulation. The type of anisotropy of S2-ice, called transversely isotropic material, and the relationship between stress and strain can be simplified into Eq. (1). Eq. (1) and Eq. (2) are empirical, and the constants involved must be determined experimentally. Gratz and Schulson (1994)^[25] and Gratz (1996)^[26] presented the results of a series of columnar saline ice tests using Laboratory Grown Saline Ice (LGSi). In the LS-DYNA FE code, the failure criterion for the tensile and compressive matrix mode is given as Eq. (3).

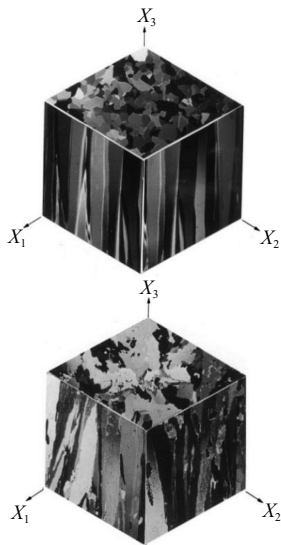


Fig.2 Structural characteristics of S2 type ice (Gratz, 1996^[26])

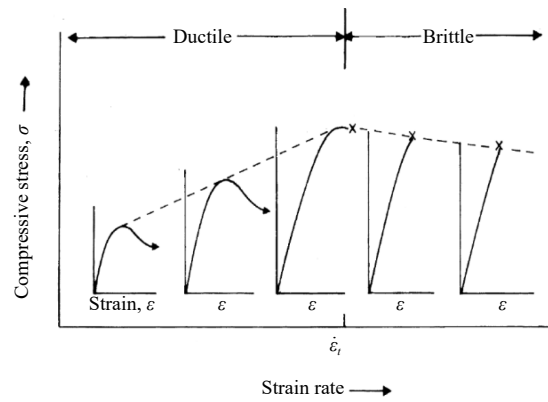


Fig.3 Schematic sketch of the effect of strain rate on the stress-strain behavior of ice (Schulson, 2001^[27])

1 Method

1.1 Mechanical properties of model ice

As described in the introduction, accurate modeling of the failure of ice materials requires a constitutive model and a failure criterion model. The relationship between stress $\{\sigma\}$ and strain $\{\epsilon\}$ for a transversely isotropic linear material can be defined with five independent constants in stress-strain matrix $[C]$. The engineering notations are adopted for stress and strain. The type of model ice described in this paper is the S2-type model ice, and the crystal structure grows into a columnar structure along the longitudinal direction. Affected by its crystal structure, this material is homogeneous in the 1–2 plane and heterogeneous in the 1–3 or 2–3 plane, and the coordinate system is defined as shown in Fig. 2. This means that the mechanical properties of the columnar ice material in the horizontal direction are the same, and the stress-strain constitutive model of the ice material can be simplified to the following form:

$$(c_{11} - c_{12})/2 = \frac{E_1}{2(1 + \nu_{12})} \tag{1}$$

where, c_{ij} are constants in stress-strain matrix $[C]$, $c_{11} = \frac{1 - \nu_{13}\nu_{31}}{E_1 E_3 \Delta}$; $c_{12} = \frac{\nu_{12} + \nu_{31}\nu_{13}}{E_1 E_3 \Delta}$; $c_{13} = \frac{\nu_{31} + \nu_{13}\nu_{31}}{E_1 E_3 \Delta}$;

$$c_{33} = \frac{1 - \nu_{12}\nu_{31}}{E_1^2 \Delta}; c_{44} = G_{13}; \Delta = \frac{1 - \nu_{12}\nu_{21} - 2\nu_{31}\nu_{13} - 2\nu_{21}\nu_{31}\nu_{13}}{E_1^2 E_3}$$

Consequently, there are five unknown elastic constants in Eq. (1). The numerical values of these constants are given by Evers (1993)^[27], Sazonov (2011)^[28] and ice tank test data, where the results of model ice test are presented in Tab. 1.

Tab.1 Parameters of model ice in Tsai-Wu theory at -6 °C

Parameters	E_d /MPa	E_b /MPa	E_c /MPa
Value	40.64	40.64	56.90
Parameters	ν_{ab}	ν_{ac}	ν_{bc}
Value	0.3	0.3	0.3
Parameters	G_{ac} /MPa	G_{bc} /MPa	G_{ab} /MPa
Value	14.08	14.08	15.63

The application of numerical methods to ice problems has created a need for a more sophisticated macroscopic failure criterion than the maximum principal stress criterion. Another drawback of the common criteria is their unsuitability for describing anisotropic materials.

The basic assumption of Tsai-Wu strength criterion is that there exists a failure surface in the stress-space in the following scalar form:

$$f(\sigma_{ij}) = \sum_{i,j} F_{ij} + \sigma_{ij} + \sum_{i,j,k,l} G_{ijkl} \sigma_{ij} \sigma_{kl} \tag{2}$$

where, F_{ij} and G_{ijkl} are intensity factors used to express the relationship between intensities. The components of the Tsai-Wu criterion F_{ij} and G_{ijkl} can be determined by substituting the strength values S and U in the following formulae:

$$\begin{aligned} F_{11} &= \frac{1}{S_{1T}} - \frac{1}{S_{1C}}; F_{33} = \frac{1}{S_{3T}} - \frac{1}{S_{3C}}; G_{1111} = \frac{1}{S_{1T}S_{1C}}; G_{3333} = \frac{1}{S_{3T}S_{3C}} \\ G_{1122} &= \frac{1}{2\nu_{12}(S_{1C}^{P2})^2} \left[1 + F_{11}(1 + \nu_{12})S_{1C}^{P2} - G_{1111}(1 + \nu_{12}^2)(S_{1C}^{P2})^2 \right] \\ G_{1133} &= \frac{1}{2\nu_{31}(S_{1C}^{P3})^2} \left[1 + (F_{11} + F_{33}\nu_{31})S_{1C}^{P3} - (G_{1111} + G_{3333}\nu_{31}^2)(S_{1C}^{P3})^2 \right] \\ G_{1212} &= \frac{1}{2}(G_{1111} - G_{1122}) \\ G_{1313} &= \frac{1}{U_c^2(45^\circ)} \left[1 + \frac{1}{2}(F_{11} + F_{33})U_c(45^\circ) \right] - \frac{1}{4}(G_{1111} + G_{3333} + 2G_{1133}) \end{aligned} \tag{3}$$

The constitutive model and a failure criterion model required in the numerical simulation model ice process are analyzed above. In this paper, the special material model MAT_55 in LS-DYNA code is used, which is based on Tsai-Wu failure criterion.

1.2 Numerical simulation in LS-DYNA

In the LS-DYNA progressive failure model of ice, the Tsai-Wu criterion is treated as the tensile and compressive modes and the element erosion method is combined. When the stresses go to 0, the integration point is deleted. The failure criterion for the tensile and compressive matrix mode is given as

$$e_{md}^2 = \frac{\sigma_{bb}^2}{Y_c Y_t} + \left(\frac{\sigma_{bb}}{S_c} \right)^2 + \frac{(Y_c - Y_t)\sigma_{bb}}{Y_c Y_t} - 1 \left\{ \begin{array}{l} \geq 0 \text{ failed} \\ < 0 \text{ elastic} \end{array} \right\} \tag{4}$$

where Y_c and Y_t are the compression stress and tensile stress respectively, and S_c is the shear stress.

The directional indices used in LS-DYNA should be taken into account when using material parameters, the a, b, c coordinate system in the program is the same as the x_1, x_2, x_3 coordinate system in Fig. 2. In

MAT_055, an integration point is deleted (all stresses go to zero) only if the tensile stress at that point reaches. Other strengths serve to cap stresses but do not delete the integration point. In an optional damage model for transverse shear strain, out-of-plane stiffness (GBC and GCA) can get linearly decreased to model interlaminar shear failure, as shown in Fig. 4. The GBC and GCA are shear modulus ab and bc space, which is equal to Eq. (2). When failure has occurred in all the through-thickness integration points, the shell element is deleted, as shown in Fig. 5.

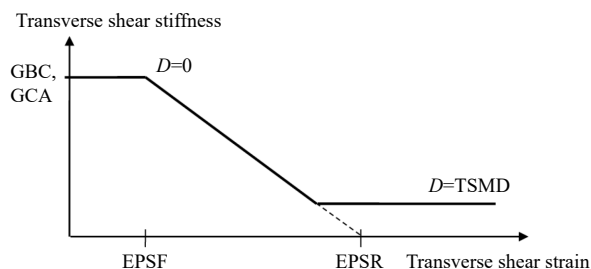


Fig.4 Linear damage for transverse shear behavior

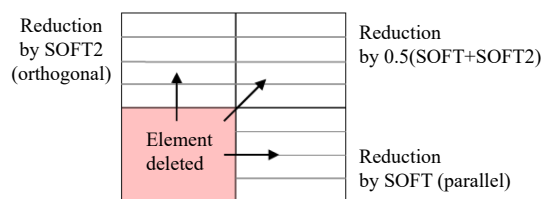


Fig.5 Diagram of element deleted and direction dependent softening

1.3 Simulation of cantilever beam test

Two different cantilever beams were built, with one rounded at the root and the other not rounded at the root. Fig. 6 shows the schematic diagram of beam dimensions. From ITTC (2021)^[1], the in-situ cantilever beam test is the most common and best-known method to determine the flexural strength of an ice sheet. The numerical model size $l \times b \times h$ is defined as $0.4 \text{ m} \times 0.16 \text{ m} \times 0.06 \text{ m}$. Compared with the experimental model, the numerical model is simplified on the basis of ensuring the simulation accuracy, so as to save disk storage space and calculation time. Only the part of the indenter that is actually in contact with the ice remains, and is pressed down at a constant rate $v = 5 \text{ mms}^{-1}$ until the model ice damage.

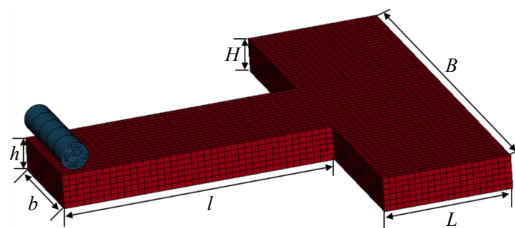


Fig.6 Mesh of model ice cantilever beam test

The impactor was modeled as a spherical rigid body using the 8 noded solid elements (ELFORM 1) with three degrees of freedom for each node and the MAT_RIGID material model used. The initial velocity $v = 5 \text{ mms}^{-1}$ was defined in the negative z -direction by using INITIAL_VELOCITY card. The cantilever beam model was free to bend, and the three sides of the ice sheet model were set to be rigidly fixed. A segment-based CONTACT_AUTOMATIC_SURFACE_TO_SURFACE was established between the impactor and the model ice with SOFT=2. The FE model is shown in Fig. 6.

The flexural strength simulation is affected by boundary conditions and numerical simplifications as shown in Eq. (4). Svec et al^[29] described the notch effects at the root, it is shown that the size of the radius between ice sheet and beam affects the flexural strength greatly. The rigid clamp-support was used around ice sheets, which was the same as that in Ref. [30]. The buoyancy effect was modeled with Motion_Nodes, where displacements caused a reaction force like springs.

2 Results and discussion

2.1 Effect of ice sheet size on flexure strength

ITTC recommends three commonly used bending strength test methods, which are also commonly used in numerical simulation. They are cantilever beam method, three-point bending method and four-point

bending method. In ice tank tests, the cantilever beam method is the only recommended method because of its convenience in specimen fabrication and minimized impact on ice sheet by in-situ testing (Von Bock and Polach^[30] and Huang, et al^[3]). The three-point bending test uses ice room or field ice. Rena et al (2019)^[31] simulated a three-point bending test using a fully lagrangian particle method. Ehler and Kujala (2014)^[32] used LS-DYNA to simulate bending failure of ice beams using plastic model and element deletion method. The different tensile and compressive properties of ice were considered in the material model (the material parameters obtained using a particle swarm optimization (PSO) algorithm), and different stress-strain relationships were adopted for tensile and compressive properties. Compared with the three-point and four-point bending tests, the cantilever beam test is essentially different regarding the influence of the surrounding test environment. Therefore, the size of the ice sheet around the cantilever beam will be affected by the results of the cantilever beam test, and this phenomenon is more significant in the numerical simulation.

When the cantilever beam specimen is loaded in the ice tank, the ice sheet can be regarded as an infinite elastic boundary instead of a fixed boundary. Therefore, the size of the ice sheet should be large enough to eliminate the influence of the fixed boundary. In this paper, different ice sheet sizes are selected to analyze the influence of the size of the ice sheet around the cantilever beam on the cantilever beam test. The size of the cantilever beam numerical model $l \times b \times h$ is $0.4 \text{ m} \times 0.16 \text{ m} \times 0.06 \text{ m}$ and the ice sheet $L \times B \times H$ is defined as shown in Fig. 6. The size $L \times B \times H$ of the ice sheet model is changed to verify the influence range of the elastic region of the model. The thickness of the ice sheet is consistent with that of the cantilever beam, and the length range L is 1, 1.5, 2, 2.5 and 3 times the length l of the cantilever beam respectively. The simulation results are shown in Tab. 2. It can be seen from the simulation results in Fig. 7 that the stress has a greater influence in the longitudinal direction, and the simulation should satisfy that the length of surrounding ice sheet must be at least three times the length of the cantilever beam.

Tab.2 Simulation results of the case $h=0.06 \text{ m}$, $v=5 \text{ mm/s}$

Ice sheet size/m	Max bending load/N	σ_f/kPa
0.4×0.32	7.92	33.02
0.6×0.32	7.87	32.80
0.8×0.48	7.84	32.68
1.0×0.48	7.71	32.12
1.2×0.48	7.28	30.33

Ice will produce different failure modes under different loading rates. ITTC recommends that model ice should meet the brittle failure of ice as much as possible when loading. The loading rate simulated in this paper exceeds the ductile-brittle conversion loading rate of the model ice by $10^{-4} \sim 10^{-3} \text{ s}^{-1}$, showing obvious brittle failure. This result is consistent with that of the test in the ice tank, as shown in Fig. 3 and Fig. 8. In the numerical simulation results, the simulated value is within a reasonable range compared with the experimental value.

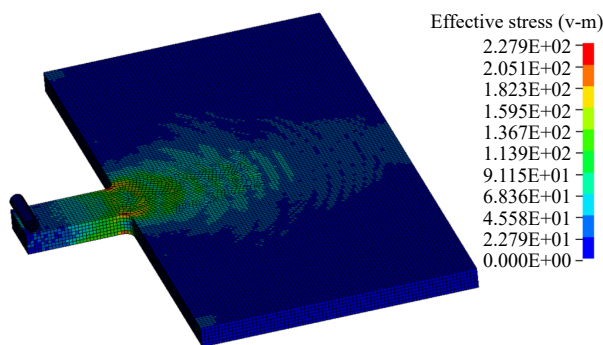


Fig.7 Effective stress of model ice cantilever beam test simulations

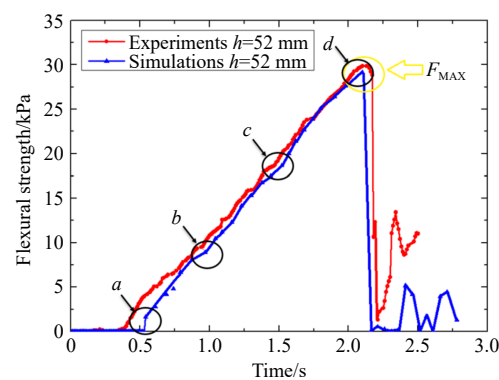


Fig.8 Flexural strength of experimental and numerical results

The flexural strength was calculated by the following equation:

$$\sigma_f = 6Fl/bh^2 \tag{5}$$

where F is the maximum loading force, l is length of beam, b is width of beam and h is ice thickness as shown in Fig. 6.

2.2 Stress concentration in the root of ice cantilever

When cantilever beams are used in ice tank, they are loaded to failure at rates sufficient to fail brittlely. Studies have shown that in-situ cantilever beam tests commonly used to determine the flexural strength of ice sheets are significantly affected by stress concentrations around the beam root. Stress concentrations can be eliminated by pre-drilling holes, as related research Frederking, et al., (1983)^[33] and Svec(1985)^[29] carried out field tests and finite element theoretical analysis. There are still stress field concentration problems near the connection between the cantilever beam and the surrounding ice plate remaining to be resolved, such as the size of the stress concentration factor and the influence of the root opening on the stress distribution.

The stress concentration factor at the end of the unchamfered cutout was determined using the numerical method described above and compared to the cantilever condition where the end of the cantilever beam is usually at right angles. The geometric model is shown in Fig. 9, and the simulation results are shown in Tab. 3. As the diameter of the stress relief hole increases, the peak stress decreases monotonically, which is consistent with the experimental results of Svec (1985)^[30]. Even a single chamfer can significantly reduce stress concentrations caused by unmodified right angles. For small-diameter holes ($d/h \ll 0.8$), the stress has double maxima, one occurring in the beam near the edge of the hole and the second at the back of the hole, as shown in Fig. 10 and Fig. 11. And Fig. 11(a)–(d) show the damage conditions of the ice cantilever beam at different time points, which correspond to the moments shown in Fig. 8(a)–(d).

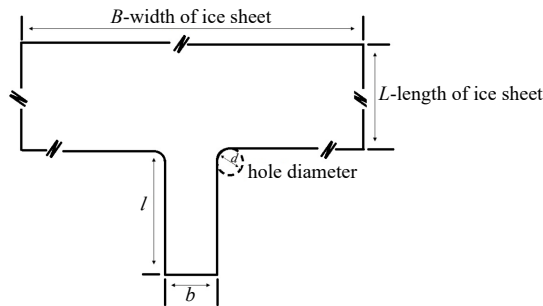


Fig.9 Geometric model of the ice

Tab.3 Stress concentrations

b/cm	Hole diameter/mm	Max σ_y /kPa
4	0.6	38.10
6	1.25	27.2
8	1.88	23.6
12	3.13	20.4
16	0.5	19.7

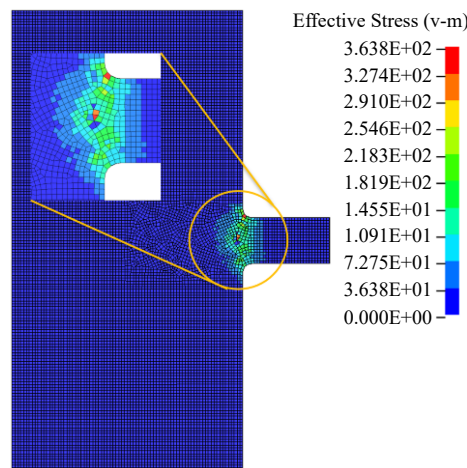


Fig.10 Bottom view of cantilever beam failure

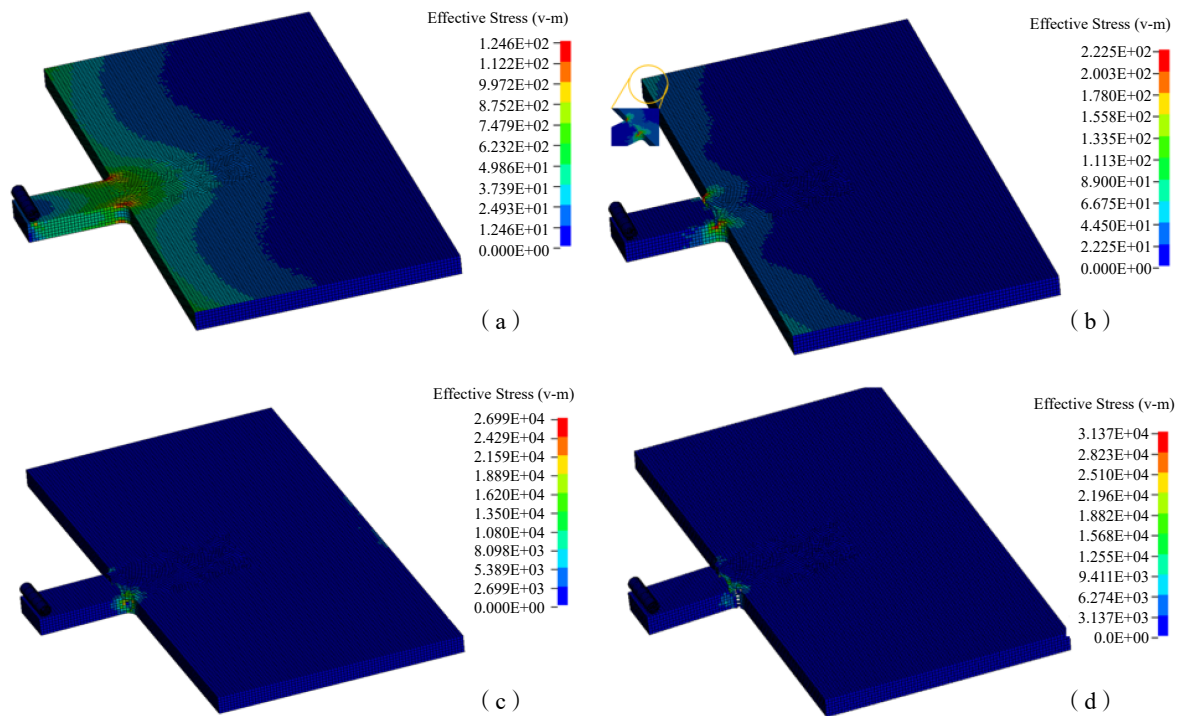


Fig.11 Failure process of cantilever beam

3 Conclusions

The FEM method of the LS-DYNA code is proposed to simulate the cantilever beam test on ice according to the Tsai-Wu criterion. The model ice material is modeled as a brittle material, with the elastic process and failure point governed by the material constitutive equation and failure criterion. The comparison between field tests and previous calculation results shows that the model used in this paper can be used for the simulation and prediction of ice material failure. Next, the model of the cantilever beam is simplified, and the numerical calculation of the bending failure of the cantilever beam with different sizes and root opening sizes is carried out.

It can be concluded that: (1) the model ice material in the ice tank should be regarded as a brittle material. Compared with the test data, LS-DYNA MAT_055 has sufficient accuracy in simulating the ice cantilever beam test; (2) in the numerical simulation of the cantilever beam model, the size of the surrounding ice sheet will obviously affect the simulation results, and its length should be at least 3 times the length of the cantilever beam to effectively reduce the boundary influence; (3) the use of chamfered openings has proven to be far superior to simply cutting cantilever beams, and it is found that the width and thickness ratio of the cantilever beam should exceed 1 to obtain accurate bending strength values.

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基于 Tsai-Wu 破坏准则的模型冰弯曲强度数值模拟

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摘要: 在过去的几十年里, 船舶和结构在冰区的航行性能尚未得到充分研究, 尤其是冰力学性质对破冰能力的影响。冰弯曲强度是预测船舶冰载荷的关键冰参数, 准确的冰弯曲强度也是将模型试验结果回归实船的关键参数。然而, 对模型冰弯曲强度的数值模拟研究往往被忽视。本文采用显式有限元法模型模拟冰悬臂梁试验, 得到冰的破坏载荷和弯曲强度。本模型采用 Tsai-Wu 失效准则作为材料本构模型, 通过冰水池中的模型冰试验获得所需的模拟参数。参数敏感性分析表明, 模型冰悬臂梁尺寸对弯曲强度有显著影响。结果表明: 在悬臂梁根部适当倒圆角有利于减小悬臂梁根部应力集中, 获得更为准确的弯曲强度; 悬臂梁的厚度、宽度和长度应符合一定的比例, 与 ITTC 推荐参考一致。本研究结果可促进模型冰的试验和数值研究, 为船舶设计和极地船舶操纵提供冰强度数据支持。

关键词: 模型冰; LS-DYNA; Tsai-Wu 失效准则; 悬臂梁试验。

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