第 29 卷第 6 期 船舶力学 Vol.29 No.6
2025 年 6 月 Journal of Ship Mechanics Jun. 2025

Article ID：1007−7294(2025)06−0912−12

A New Approach for Melnikov Analysis of the Stability of a Ship with Water on Deck

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Abstract: To study the rolling motion of a ship in the presence of water on its deck, a linear-plus-quadratic damping term was incorporated into its equation of motion. Ship model tests indicates that the key dynamics of the physical system are preserved in the ship rolling equation with the linear-plus-quadratic type damping term. To take into account the presence of randomness in the excitation and the response, a new method was developed and a Melnikov criterion was obtained to provide an upper bound on the domain of the potential chaotic rolling motion (erratic rocking). Additionally, the Melnikov criterion proposed in this study was verified by the utilization of phase plane diagrams and Poincare maps. Furthermore, this research has made the initial endeavor to systematically modify the system parameters in the rolling equation of motion for ship stability analysis.

Key words: ship stability; Melnikov criterion; erratic rocking; Poincare section

CLC number：O352 **Document code**：A **doi:** 10.3969/j.issn.1007−7294.2025.06.006

0 Introduction

Presently, there is still a lack of comprehensive understanding regarding the large magnitude or strongly non-linear motions of ships, particularly in relation to non-linear roll motions. The significance of comprehending large roll motions cannot be emphasized; nonetheless, its intricate nature can be vexing. The challenges essentially arise from two domains: the determination of hydrodynamic forces and the derivation of solutions to the consequent nonlinear equations of motion for rigid bodies.

Given the challenges associated with precisely estimating the comprehensive hydrodynamic forces, it is a common practice to employ a roll model that effectively separates the six degrees of freedom (Wang[1−2]). Regrettably, a closed form solution for a 1-DOF nonlinear ordinary differential equation cannot be found in general, even when the excitation is sinusoidal. The equation under consideration demonstrates a wide range of qualitative characteristics in its dynamics, including harmonic, sub-harmonic, super-harmonic, periodic, and aperiodic behaviors. Additionally, the system may undergo state transitions, exhibit bifurcations, or display chaotic behaviors.

Current research endeavors in the field of nonlinear rolling motion have primarily focused on the derivation of approximate solutions for the one-degree-of-freedom (1-DOF) roll equation, as shown by Eq. (2). Numerous methodologies have been devised, each grounded in distinct assumptions and mathematical ideas.

Received date：2024−12−22

Foundation item：Supported by the National Natural Science Foundation of China (51979165)

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In their seminal work, Wright and Marshfield[3] introduced three distinct approaches for solving Eq. (2) when confronted with a sinusoidal $F(\tau)$, while considering both biased and unbiased scenarios. These methods include the harmonic balancing method, the perturbation method and the method of averaging. Francescutto and Contento[4], and other researchers, also examined comparable methodologies. Nayfeh and Khdeir[5−6] presented the multiple scales approach as an extension of the perturbation method, building upon the earlier work by Nayfeh[7]. The harmonic balancing method is an iterative technique that involves substituting an assumed harmonic solution, represented by a shortened Fourier series, into the roll equation. This substitution leads to the formulation of a set of nonlinear algebraic equations, which can be solved using numerical methods. To explore a more contemporary implementation of this approach, please refer to the work of Senjanovic[8]. The perturbation method is a widely-used technique utilized in the solution of differential equations that exhibit weak nonlinearity. The approach employed by Wright and Marshfield[3] involves assuming the damping force to be $\left(B'_{44}\dot{\phi} + B_3\dot{\phi}^3\right)$, as opposed to $\left(B'_{44}\dot{\phi} + B_{44q}\dot{\phi}\left|\dot{\phi}\right|\right)$. Ship model tests suggest that the key dynamics of the physical system are not well preserved in the ship rolling equation with the linear-plus-cubic type damping term. Moreover, it is important to note that this assumption also implies that the nonlinear restoring moment is negligible, which is not the case during instances of excessive rolling. In a theoretical context, this method is limited to identifying harmonic solutions that are in close proximity to the linear natural frequency ω_n. Nayfeh and Khdeir[5−6] conducted the multiple scales method, which is a perturbation method, up to second order. Their approach involved utilizing modeling assumptions that were comparable to those adopted by Wright and Marshfield[3]. The method of averaging is a frequently-employed technique in the investigation of nonlinear dynamics. The method described involves the simplification and solution of a weakly nonlinear system of equations through the introduction of variable modifications, as discussed by Nayfeh[7] and Wiggins[9] .

All of the aforementioned techniques were employed in an effort to acquire approximate solutions for the governing equation pertaining to steady-state solutions linked to periodic excitation. The stability study pertaining to these deterministic approaches is conducted by Floquet analysis, as described by Seydel[10] and Nayfeh and Khdeir[5−6], or through a similar approach. The precise impact of chaos on the phenomenon of capsizing remains undetermined by Floquet analysis at present.

Thompson[11] conducted a study that examined the occurrence of ship capsizing in regular seas through the utilization of numerical simulation. In contrast to the preceding paragraphs' discussion on steady-state analysis, this approach elucidates the influence of early conditions on the movements that ultimately result in capsizing.

The utilization of geometric approaches has been widely embraced by mathematicians for the analysis of nonlinear dynamics, particularly in cases where chaotic behavior is observed. One of the notable analytical outcomes derived from the employment of the geometric method is the identification of the Melnikov function, which serves as a predictive tool for the manifestation of chaotic behavior within a specific category of systems. The study conducted by Jeffery, Steven and Armin[12] offered several concepts, theories, and methodologies pertaining to the analysis of nonlinear ship motions. An analytical determination was made regarding the criteria that must be met for the initial conditions within the safe basin to be transferred beyond the safety boundaries.

The objective of this work is to employ and expand upon geometric methodologies in analyzing advanced ship motion models. The aim is to examine the impact of different assumptions made during the development of the 1-DOF roll model with water on deck on the risk of ship capsizing. Next, we proceed to derive the subharmonic Melnikov function in order to assess the presence of periodic motion within the safe basin of the perturbed system. In the third step, the Melnikov function is employed to examine the phenomenon of solutions escaping from the safe basin as disturbances, specifically excitation and damping, are intensified.

1 Modelling of the physical system

Recent cases of small vessels becoming capsized while operating in coastal waters have prompted researchers to investigate the behavior of vessels of this size while operating in a seaway. It is believed that one of the causes of such capsizals is the instabilities that are brought on by an excessive amount of deck water[13−15]. Despite the fact that the effect of hull flooding on static stability is a basic and well-known subject, the influence of a limited amount of deck water on the dynamic behavior of the vessel in a seaway has not yet been well explored analytically.

Typically, a small vessel of the type that we are interested in has a large flat afterdeck with a bulwark built around, largely for the purpose of ensuring the safety of the crew people. The bulwark is typically about 1 m high and is generally solid. It has freeing ports positioned at deck level to allow water to run off it. However, when compared to the total deck area, the freeing ports are very small. As a result, it is possible for water to pool on the deck to a significant depth. These vessels are known to operate for extended periods of time with the aft deck entirely submerged in water while operating in rough seas.

The challenge that we are concerned with in this instance is figuring out how the motions of the vessel are affected by the presence of such deck water. Calculating the static effect can be done using the same procedures that are often used to evaluate the stability of damaged ships. It is possible to verify that the static weight of the deck water alone is not sufficient to produce a capsizal for the vessel being considered.

Due to the fact that the flow of the deck water is dependent on the motions of the vessel, which in turn depend on the forces exerted on the vessel by the deck water, thus making the dynamic effect a significantly serious challenge. Therefore, it is necessary to find solutions to two problems coupled together. The first problem is to provide a description of the motions of the vessel in light of the forces that are acting on the vessel. The second problem involves determining the motion of the water on the deck and the forces excerted on the vessel by the deck water, based on the vessel's motion.

We are solely interested in waves with a wavelength much greater than the depth of the sea. The shallow-water theory can then be employed to formulate the deck water flow problem as a nonlinear hyperbolic system of equations.

For the purpose of solving nonlinear hyperbolic systems of equations, the random-choice method can be utilized. This method, which is not a difference scheme, enables us to compute the velocity and depth of the deck water on a grid. The values on the grid are determined by "randomly" sampling an explicit wave solution at each time step. Through the utilization of this strategy, it is possible for us to acquire a time-domain solution that is approximately equivalent to the flow of water on the deck. The only thing that is required of us is to couple this with an acceptable time domain solution for the motion of the ship. An assumption of the smallness of the incident waves and ship motion is typically used in the process of computing the response of a ship to a seaway. This makes it possible to linearize the equations of motion about some average position of the ship, which in turn makes the calculations significantly easier to understand. For the purpose of computing hydrodynamic coefficients, we make use of the linear strip theory, which currently is most frequently accepted. Due to the fact that the added-mass and damping coefficients caused by the free-surface waves formed by the ship's motion are frequency-dependent, it is not possible for us to simply integrate the equations of motion of the ship numerically. Employing the impulse response technique could allow us to account for the frequency-dependence of the ship's response. The solution we acquire is derived from a synthesis of these two techniques: the impulse response technique for calculating ship motions and the random-choice method for solving the shallow-water wave equations on the deck.

Throughout the entire study, we will make the assumption that the ship motions and wave heights are small. This will allow us to formulate the equations of motion in accordance with the linear theory. In addition, we make the assumptions that the fluid external to the ship is inviscid and that the flow is irrotational. This provides us with the opportunity to formulate the problem using potential theory. Furthermore, we assume that the ship hull is sufficiently slender to enable us to utilize the strip-theory approximation.

The flow of water on the deck will be treated as a strictly two-dimensional problem, and we will limit our attention to the case of beam seas. We are also going to make the assumption that the ship has an adequate fore-and-aft symmetry, which will allow us to ignore pitch and yaw when moving in beam seas. One equation is all that is left for heave, and we have two equations that are coupled together for sway and roll. With regard to the heave equation, we will not solve it explicitly; rather, we will assume that for practical purposes, the ship can be considered to follow the free surface exactly in heave. This is because direct observations of model tests indicate that the ship appears to follow the free surface closely in heave when the waves are sufficiently long. For the sole purpose of calculating the flow of deck water and the forces that deck water exerts on the ship, the heave acceleration that is computed in accordance with this assumption will be utilized. The equations of motion can be reduced to the two equations listed below for sway and roll (Jiang Changben[16]), according to our assumptions:

$$\begin{bmatrix} M+A_{22} & A_{24}-Mz_c \\ A_{42}-Mz_c & I_4+A_{44} \end{bmatrix}\begin{bmatrix} \ddot{\eta} \\ \ddot{\phi} \end{bmatrix} + \begin{bmatrix} B_{22} & B_{24} \\ B_{42} & B_{44} \end{bmatrix}\begin{bmatrix} \dot{\eta} \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & C_{44} \end{bmatrix}\begin{bmatrix} \eta \\ \phi \end{bmatrix} = \begin{bmatrix} F_2(t) \\ F_4(t) \end{bmatrix} \tag{1}$$

In the given context, η represents sway displacement and ϕ represents roll angle. The variables $A's$ and $B's$ denote added mass and damping coefficients, respectively. These coefficients can be determined by the utilization of a linear hydrodynamic software. The variable "M" represents the mass of the ship. The symbol "I_4" represents the moment of inertia of the ship about the roll axis, as measured in air. The vertical center of gravity of the ship, denoted as z_c, is measured from the base line. The roll-restoring moment coefficient, denoted as C_{44}, is a parameter used to quantify the ability of a system to restore its roll stability. The variables $F_2(t)$ and $F_4(t)$ represent the external wave excitation force and moment, respectively. Additionally, the symbol $(\dot{\ })$ is used to represent differentiation with regard to time. At this point in time, the deck is seen as a two-dimensional tank that contains water and is permitted to freely flow back and forth in response to the motions of the ship. We have made the assumption that the static roll restoring force coefficient C_{44} is a constant through the process of constructing Eq. (1). Up to around 30 degrees of roll angle, the C_{44} of a typical small boat remains constant. The exciting forces that are on the right side of Eq. (1) (that is, $F_2(t)$ and $F_4(t)$) must include the forces exerted on the vessel by the deck water. These forces are nonlinear functions of the ship's motion. We are able to compute the deck water forces in the time domain by utilizing the random-choice method; hence, we are looking for a solution that is in the time domain for the ship's motions. Due to the fact that Eq. (1) is a linear equation, the principle of superposition will make it possible for us to construct solutions for forcing functions that fluctuate arbitrarily in time. In point of fact, this equation has the precise form that is required to implement the impulse response technique.

The method of random choice is employed to calculate the velocity and the depth at points on a grid across the ship deck. We may employ basic hydrostatics to determine the total force and moment acting on the deck and bulwarks by the accumulated water on deck. The sway force acting on the port bulwark per unit of deck length by the deck water can be calculated. The sway force acting on the starboard bulwark per unit of deck length by the deck water can also be calculated. The moment per unit of deck length by the deck water can also be found based on the pressure in the fluid and the known functions of position in the tank computed on a grid across the deck width using the random-choice method.

In the general scenario, the decoupling of the two modes of motion (sway and roll) is not feasible due to the presence of damping. In the case of particular scenarios, specifically undamped or proportionately damped systems, it can be demonstrated that the ship undergoes rolling motion around a roll center resembling that of a pendulum. Furthermore, it is possible to separate the roll motion from the sway motion. Assuming the presence of general damping, the hypothesis of a pseudo roll center yields the subsequent equation for roll with a single degree of freedom, incorporating quadratic roll damping.

$$[I'_{44}+A'_{44}]\,\ddot{\phi}+B'_{44}\dot{\phi}+B_{44q}\dot{\phi}\left|\dot{\phi}\right|+\Delta GZ_m(\phi)=F(t) \tag{2}$$

in which

$$I'_{44}=I_4+A_{42}R_c,\ A'_{44}=A_{44}-Mz_cR_c,\ B'_{44}=B_{44}+B_{24}R_C,\ R_C=-\frac{A_{42}-Mz_c}{M+A_{22}},$$

$$F(t)=F_4(t)+F_2(t)\,R_c,\ C'_{44}\phi=\Delta GZ_m(\phi)$$

where, the variable "Δ" represents the displacement of the ship, while the variable "$GZ_m(\phi)$" denotes a modified polynomial approximation that accounts for the non-linear roll-restoring arm. According to Caglayan[15], it is proposed that the dominant dynamics of the water-on-deck issue can be estimated by employing a constant weight in order to attain an equivalent pseudo-static heel angle (loll angle) for the vessel. Hence, the "$GZ_m(\phi)$" curve can be derived through the adjustment of the original roll-restoring arm curve $GZ(\phi)$, taking into account the influence of weight and the moment caused by water-on-deck. In accordance with the research conducted by Jeffery[17], we employ a third-order polynomial function to estimate the curve of the righting arm in the loll condition, specifically focusing on the segment preceding the loll angle.

$$GZ_m(\phi)=-C_1\phi+C_3\phi^3 \tag{3}$$

in which the constants C_1 and C_3 can be determined through the utilization of a ship hydrostatic stability program, such as NAPA, in conjunction with an appropriate interpolation method.

The aforementioned Eq. (2), despite its simplicity, captures the fundamental dynamics of the physical system, as shown by model experiments (Spyrou, et al. [18]; Francescutto and Contento[4]). We can figure out the value of B_{44q} in Eq. (2) by making use of various empirical formulas. In the case of harmonic excitation occurring in seas with regular waves, the value of $F(t)$ in Eq. (2) can be determined as follows:

$$F(t)=AF_{\mathrm{roll}}(\omega)\cos(\omega t) \tag{4}$$

where, the variable "A" represents the amplitude of the wave, whereas "$F_{\mathrm{roll}}(\omega)$" denotes the moment amplitude per unit wave amplitude at a given frequency "ω".

In order to investigate the dynamic stability of a ship, a commonly-employed approach involves conducting numerical integration on the nonlinear differential equation of motion (Eq. (2)) using an appropriate numerical technique, such as the fourth-order Runge Kutta method. Nevertheless, a novice practitioner in numerical analysis may spend a considerable amount of time attempting to integrate Eq. (2)

without uncovering its fundamental or crucial characteristics.

The utilization of the Melnikov method is a novel approach for naval architects to assess the dynamic stability of a ship. This method is rooted in the contemporary field of nonlinear dynamics theory. The Melnikov criterion is a significant outcome derived from the analysis of a ship's dynamic stability using the Melnikov method. In certain instances, it is possible to establish a direct correlation between the ship design characteristics and the wave characteristic parameters by a straightforward mathematical formula. This will significantly improve the effectiveness of systematically analyzing the dynamic stability of a ship. Although it may not always be possible to derive the analytical formulation of the Melnikov function in certain instances, it is nevertheless feasible to numerically integrate the Melnikov integral in a direct and uncomplicated manner. In the subsequent part, we shall establish a Melnikov criterion to investigate the nonlinear phenomenon of ship rolling with water-on-deck.

2 A new approach of Melnikov analysis

Calculating the distance between stable and unstable manifolds and locating transverse homoclinic intersections are both tasks that can be accomplished through the application of the Melnikov method (for reasons why these statements are not detached from the ship stability issues and for some words to bridge the two, readers may refer to Chapter 5 of Reference [2]). The Melnikov technique calculates the distance between these two manifolds by first starting with a dynamical system in which the homoclinic solutions are already known, and then slightly perturbing this system in order to find out what happens to these manifolds as a result of the perturbation. This method is based on the fact that the homoclinic solutions are already known. After going through this process, we will have a formula or criterion for determining the distance between the stable and unstable manifolds based on the parameters of the system and the phase.

To facilitate the application of the Melnikov method for the analysis of ship rolling with water-on-deck, it is necessary to transform Eq. (2) into a dimensionless form.

$$\ddot{x}(\tau)+\varepsilon\delta\dot{x}(\tau)+\varepsilon\delta_3\dot{x}(\tau)\,|\dot{x}(\tau)|-\ x(\tau)+kx^3(\tau)\ =\varepsilon\gamma\cos(\Omega\tau) \tag{5}$$

where

$$x\ =\phi,\ \ \tau\ =\omega_n t,\ \ \omega_n=\sqrt{\frac{C_1\varDelta}{I'_{44}+A'_{44}}},\ \ \Omega=\frac{\omega}{\omega_n},$$

$$\varepsilon\delta=\frac{B'_{44}\omega_n}{C_1\varDelta},\ \ \varepsilon\delta_3=\frac{B_{44q}}{I'_{44}+A'_{44}},\ \ k=\frac{C_3}{C_1},\ \ \varepsilon\gamma=\frac{AF_{\text{roll}}(\omega)}{C_1\varDelta}$$

The aforementioned derivation strategy closely adheres to the approach presented by Wang and Tan[19] in their analysis of dynamic stability of a barge utilizing the Melnikov method. In Eq. (5), the variable representing time is normalized by the linear natural frequency denoted as ω_n. The differentiation in Eq. (5) is performed with regard to the independent variable, namely time (τ). The terms denoted by the symbol "ε" are assumed to have small values and will be considered as perturbations in the subsequent analysis. The efficacy of this treatment has been substantiated through the examination of coefficient values for actual ships in several studies conducted by Wang[19], Hsieh[20] , and Jiang, et al. [21].

The first-order form of Eq. (5) is as follows:

$$\dot{x}(t)=y(t) \tag{6}$$

$$\dot{y}(t)=x(\tau)-kx^3(\tau)+\varepsilon\left(\gamma\cos(\Omega\tau)-\delta y(t)-\delta_3 y(t)\,|y(t)|\right) \tag{7}$$

Eqs. (6) and (7) denote a perturbation of a Hamiltonian system that is integrable. The system in an undisturbed state (ϵ=0) is as follows:

$$\dot{x}(t) = y(t) \tag{8}$$

$$\dot{y}(t) = x(\tau) - kx^3(\tau) \tag{9}$$

The Hamiltonian associated with the unperturbed system, denoted as $\ddot{x}(\tau) - x(\tau) + kx^3(\tau) = 0$, is given by

$$H(x,\ \dot{x}) = \frac{1}{2}\dot{x}^2 - \frac{1}{2}x^2 + \frac{k}{4}x^4 \tag{10}$$

The following are the well-known formulations for the homoclinic orbits of the unperturbed Hamiltonian system $\ddot{x}(\tau) - x(\tau) + kx^3(\tau) = 0$:

$$x_0(\tau) = \sqrt{\frac{2}{k}}\operatorname{sech}(\tau), \quad y_0(\tau) = \sqrt{\frac{2}{k}}(-\operatorname{sech}(\tau)\tanh(\tau)) \tag{11}$$

A ship system that is undisturbed and does not have any water on the deck has a stable upright equilibrium, denoted as $(x,\dot{x}) = (0,0)$. When transitioning from the state of no water on the deck to the static effect of having a significant amount of water on the deck, the slope of the ship's righting arm curve at the origin, known as the initial stability height *GM*, decreases and eventually becomes negative. When this phenomenon takes place, the stable upright equilibrium, denoted as $(x,\dot{x}) = (0,0)$, undergoes a bifurcation, resulting in an unstable upright equilibrium and two stable equilibria located at positive and negative loll angles. The bifurcation being referred to is the classical pitchfork bifurcation. The phase picture of the bifurcated system in the vicinity of the loll angles, $\pm x_l$, exhibits two homoclinic orbits, as elucidated by Eq. (11). These orbits serve as connections between the unstable saddle located at the origin and itself. The pair of homoclinic orbits comprises the stable and unstable manifolds. In the case of an unperturbed ship system, which is characterized by the absence of damping and external forcing, it can be shown that the stable and unstable manifolds exhibit a coinciding behavior.

If we assume that $x = Q$ and $\dot{x} = P$, then we can rewrite Eq. (5) so that it takes the form of a perturbed Hamiltonian system:

$$\begin{cases} \dot{Q} = \dfrac{\partial H}{\partial P} = P \\ \dot{P} = -\dfrac{\partial H}{\partial Q} + \varepsilon\left[-\delta\dfrac{\partial H}{\partial P} - \delta_3\dfrac{\partial H}{\partial P}\left|\dfrac{\partial H}{\partial P}\right| + \gamma\cos(\Omega\tau)\right] \end{cases} \tag{12}$$

The Hamiltonian associated with the unperturbed system, denoted as $\ddot{x}(\tau) - x(\tau) + kx^3(\tau) = 0$, is given by

$$H(Q,P) = \frac{1}{2}\dot{x}^2 - \frac{1}{2}x^2 + \frac{k}{4}x^4 \tag{13}$$

The following are the well-known formulations for the homoclinic orbits of the unperturbed Hamiltonian system $\ddot{x}(\tau) - x(\tau) + kx^3(\tau) = 0$:

$$Q_0(\tau) = \sqrt{\frac{2}{k}}\operatorname{sech}(\tau), \quad P_0(\tau) = \sqrt{\frac{2}{k}}(-\operatorname{sech}(\tau)\tanh(\tau)) \tag{14}$$

The calculation of the Melnikov function for the perturbed system (Eq. (7)) can be performed according to the methodology proposed by Wang[2].

$$\begin{aligned} M(\tau_0) &= \int_{-\infty}^{\infty}\frac{\partial H}{\partial P}\left[-\delta\frac{\partial H}{\partial P} - \delta_3\frac{\partial H}{\partial P}\left|\frac{\partial H}{\partial P}\right| + \gamma\cos(\Omega(\tau+\tau_0))\right]\mathrm{d}\tau = \\ &-\delta\int_{-\infty}^{\infty}\left(\frac{\partial H}{\partial P}\right)^2\mathrm{d}\tau - \delta_3\int_{-\infty}^{\infty}\left(\frac{\partial H}{\partial P}\right)^2\left|\frac{\partial H}{\partial P}\right|\mathrm{d}\tau + \int_{-\infty}^{\infty}\frac{\partial H}{\partial P}\gamma\cos(\Omega(\tau+\tau_0))\mathrm{d}\tau = \overline{M}(\tau_0) - \overline{M} \end{aligned} \tag{15}$$

in which

$$\overline{M} = \delta\int_{-\infty}^{\infty}\left(\frac{\partial H}{\partial P}\right)^2 \mathrm{d}\tau + \delta_3\int_{-\infty}^{\infty}\left(\frac{\partial H}{\partial P}\right)^2\left|\frac{\partial H}{\partial P}\right|\mathrm{d}\tau \tag{16}$$

$$\overline{M}(\tau_0) = \int_{-\infty}^{\infty}\frac{\partial H}{\partial P}\gamma\cos(\Omega(\tau+\tau_0))\mathrm{d}\tau \tag{17}$$

are, in order, the component parts of the Melnikov function known respectively as the mean and the oscillatory part. The following results are obtained by inserting Eq. (14) into the above equations:

$$\overline{M} = \frac{4}{3k}\delta + \frac{2}{k}\sqrt{\frac{2}{k}}\delta_3\frac{4}{15} \tag{18}$$

$$\overline{M}(\tau_0) = \sqrt{\frac{2}{k}}(\gamma\pi\Omega)\sin(\Omega\tau_0)\mathrm{sech}\left(\frac{\pi\Omega}{2}\right) \tag{19}$$

Therefore, the full expression of the Melnikov function is as follows:

$$M(\tau_0) = \sqrt{\frac{2}{k}}(\gamma\pi\Omega)\sin(\Omega\tau_0)\mathrm{sech}\left(\frac{\pi\Omega}{2}\right) - \frac{4}{3k}\delta - \frac{2}{k}\sqrt{\frac{2}{k}}\delta_3\frac{4}{15} \tag{20}$$

The following is a requirement that must be met for the Melnikov function to have simple zeros:

$$\sqrt{\frac{2}{k}}(\gamma\pi\Omega)\mathrm{sech}\left(\frac{\pi\Omega}{2}\right) - \frac{4}{3k}\delta - \frac{2}{k}\sqrt{\frac{2}{k}}\delta_3\frac{4}{15} = 0 \tag{21}$$

When introducing damping and external excitations to the unperturbed system, the perturbed stable and unstable manifolds will no longer exhibit coincidence. The spatial separation between the entities is contingent upon their respective locations on the undisturbed manifolds, and is formally referred to as the Melnikov distance. In the context of this study, it can be shown that the Melnikov distance exhibits a direct proportionality to the absolute value of the Melnikov function, as determined by the utilization of Eq. (20). When the Melnikov function possesses simple zeros, the stable and unstable manifolds exhibit transverse intersections. According to a theory credited to Poincaré (Guckenheimer and Holmes[22]), in the event that the stable and unstable manifolds cross each other once, it follows that they will intersect an unlimited number of times, hence giving rise to the formation of homoclinic tangles.

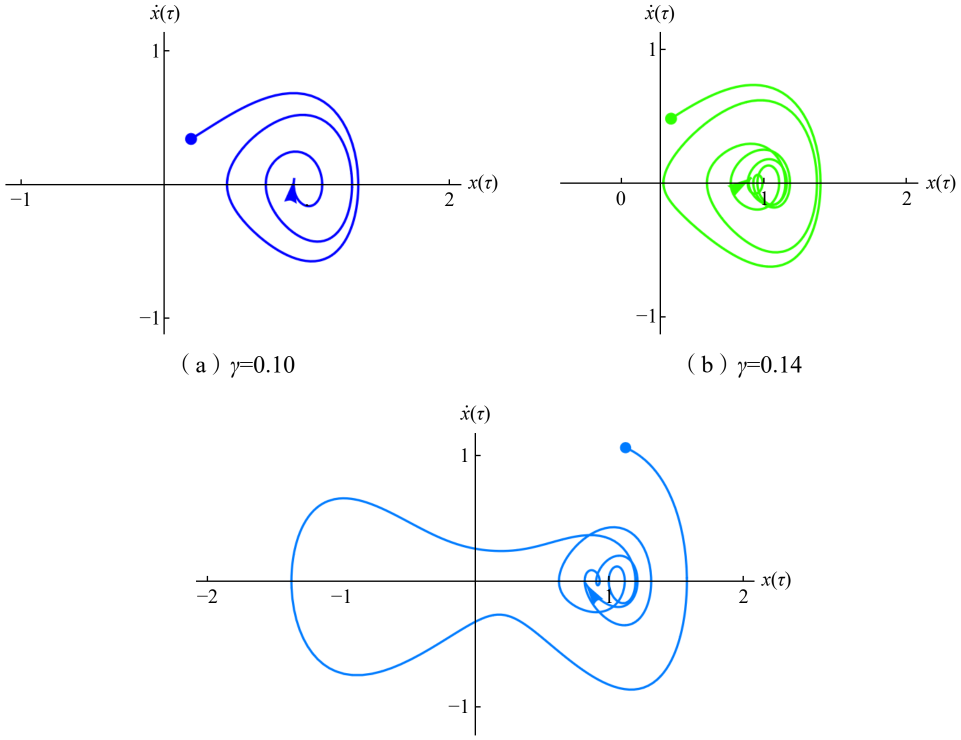
For the purpose of example, we have selected the system parameters as $k = 1$, $\delta = 0.1$, $\delta_3 = 0.3$, $\Omega = 0.8$. Then Eq. (21) results in a critical value of the amplitude of wave excitation, denoted as $\gamma_{\text{critical}} = 0.132$, at which homoclinic tangles are simultaneously formed on both sides of the origin. This occurrence is attributed to the presence of symmetry.

It is important to point out that, when deciding on the values for the damping coefficients $\delta = 0.1$ and $\delta_3 = 0.3$, references were looked for with regard to some credible data that had been used previously by a number of ship researchers (Spyrou et al[18]; Liu and Tang[23]; Wang and Tan[24]). In a similar manner, the value $\Omega = 0.8$ is a piece of data that was derived from the published research (Spyrou et al[18]).

It is probable that chaos will manifest when the forcing amplitudes surpass the critical threshold. It is important to acknowledge that the presence of the Melnikov boundary is a prerequisite for the manifestation of chaos, although it alone does not guarantee its occurrence. In other words, the Melnikov boundary serves as a minimum requirement for the emergence of chaos, but additional factors may be essential for its actual observation. When wave amplitudes exceed the critical threshold, the ship's dynamics caused by the homoclinic intersection would manifest as an unpredictable oscillation between the two loll angles. Additionally, the ability to anticipate which side the ship will lean towards would be compromised. While this conduct may not be classified as catastrophic, it is undeniably disconcerting.

3 Validation of the Melnikov criterion by phase plane diagrams and Poincare sections

The equation of motion for ship rolling (Eq. (5)) is also examined through the utilization of phase plane diagrams and Poincare maps. In order to investigate the motion responses of the vessel, we systematically manipulate the wave excitation amplitude (γ) in Eq. (5), while keeping the system parameters ($k = 1$, $\delta = 0.1$, $\delta_3 = 0.3$, $\Omega = 0.8$) constant. Initially, the value of $\gamma = 0.10$ is selected, ensuring that it is below the critical wave excitation amplitude as determined by the Melnikov criterion. The numerical integration of Eq. (5) is subsequently performed over a specified time interval (0, 100), utilizing randomly selected initial conditions of $(x(0), \dot{x}(0)) = (0.2, 0.35)$. Fig. 1(a) displays the phase plane diagram that has been acquired. It is evident from Fig. 1(a) that the ship's rolling reactions exhibit a high degree of regularity. Subsequently, the wave excitation amplitude is augmented to a value ($\gamma = 0.14$) slightly exceeding the critical wave excitation amplitude as predicted by the Melnikov criterion. Eq. (5) is thereafter subjected to numerical integration within a specified time interval (0, 100), with randomly selected initial conditions of $(x(0), \dot{x}(0)) = (0.07, 0.47)$. The phase plane diagram depicted in Fig.1(b) illustrates the results obtained. It is observed that the paths of the ship's rolling reaction begin to overlap. In order to conduct a comprehensive analysis of the impact of wave excitation amplitude on ship response, we continually select a specific value, denoted as $\gamma = 0.24$, and proceed to numerically integrate Eq. (5) during a specified time interval of (0, 100). The phase plane diagram obtained is depicted in Fig. 1(c). It is observed that the trajectories of the ship's rolling responses exhibit a lack of organization and present challenges in terms of interpretation.

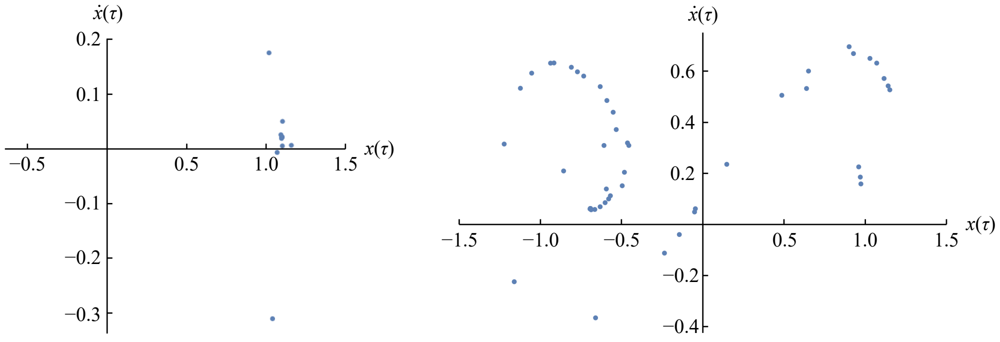


（a）γ=0.10 （b）γ=0.14

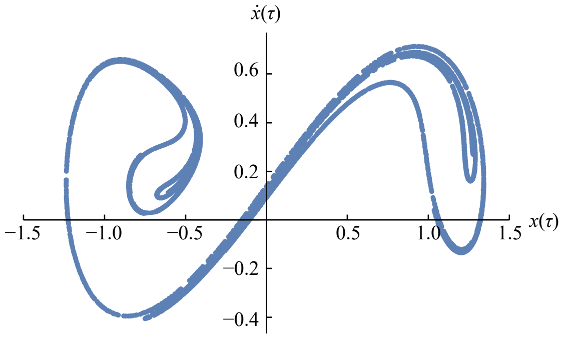
（c）γ=0.24

Fig.1 Phase plane diagrams of the ship rolling equation of motion (Eq. (5))

Finally, we make use of Poincare maps in order to investigate the rolling of the ship and verify the Melnikov criterion derived in Chapter 2. First, while keeping the system parameters ($k = 1$, $\delta = 0.1$, $\delta_3 = 0.3$, $\varOmega = 0.8$) unaltered, we select the value of $\gamma = 0.10$ which is less than the critical wave excitation amplitude predicted by the Melnikov criterion. This is done so that we can move on to the next step. After that, Eq. (5) is numerically integrated over a time interval of 100 000 and with the initial conditions being chosen at random to be ($x(0)$, $\dot{x}(0) = (0)$). The Poincare map that was obtained is displayed in Fig. 2(a). As can be seen in Fig. 2(a), the system has not yet descended into a chaotic state at this point in time. The fact that there are six points grouped together in a very limited region of the phase plane is evidence that the system is still operating in a quasi-periodic fashion. Because the Melnikov boundary is a required but not sufficient condition for observable chaos (that is, it is a lower bound), this circumstance does not contradict the outcome that was predicted with our Melnikov criterion in Chapter 2. Fig. 2(b) depicts the impending appearance of a chaotic attractor as the value of the wave excitation amplitude is increased to $\gamma = 0.14$. In the end, a clear picture of a chaotic attractor has been obtained and is displayed in Fig. 2(c) as a result of increasing the value of the wave excitation amplitude to $\gamma = 0.24$.



(a) γ=0.10　　(b) γ=0.14



(c) γ=0.24

Fig.2 Poincare maps of the ship rolling equation of motion (Eq. (5))

The conclusion arrived at by making use of our Melnikov criterion in Chapter 2 has been qualitatively confirmed by the results of the numerical integration presented above. For wave amplitudes that are greater than the critical value, the forecast of chaotic ship dynamics by employing the Melnikov criterion leads to an erratic oscillation between the two loll angles and a lack of prediction regarding which side the ship would lean to. Therefore, it is possible to say that the Melnikov criterion discussed in this article is capable of producing relevant findings when attempting to forecast the erratic rocking of a ship while water is trapped on deck.

4 Concluding remarks

For the purpose of investigating the rolling motion of a ship when water is present on its deck and regular beam waves are present, a nonlinear equation of motion with a cubic-type viscous damping term was developed and presented in this article. The Melnikov function was used to derive a ship stability criterion, which was then used to set an upper bound on the domain of the potentially chaotic rolling motion. Using the Melnikov criterion to anticipate the chaotic ship dynamics results in an unpredictable oscillation between the two loll angles, as well as a loss of the ability to predict which side of the ship will lean to.

The value of the wave excitation amplitude in the motion equation of the ship was systematically adjusted, and the equation of motion was numerically integrated in order to get phase plane diagrams and Poincare maps. This was done in order to validate the Melnikov criterion developed in this study. The conclusion reached with the help of our Melnikov criterion were qualitatively confirmed by the results of the numerical integration. When water is allowed to accumulate on the deck of a ship, the Melnikov criterion described in this article has the potential to produce accurate predictions of the ship's erratic rocking.

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对甲板带水的船舶稳性进行梅尔尼科夫分析的新方法

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摘要：为了研究甲板带水时船舶的横摇运动，在运动方程中采用线性加平方型阻尼项。船舶模型试验表明，线性加平方型阻尼项在船舶横摇方程中保留了物理系统的关键动力学特性，为此倾向于采用该阻尼项。作为潜在混沌横摇运动(无规律摇摆)域的上限，本文采用一种基于梅尔尼科夫函数的新方法来建立船舶稳定性衡准。此外，开发的梅尔尼科夫衡准通过使用相平面图和庞加莱映射进行验证，本研究进行了系统改变横摇方程中系统参数的首次尝试。

关键词：船舶稳性；梅尔尼科夫衡准；无规律摇摆；庞加莱映射

中图分类号：O352　　**文献标识码：**A

基金项目：国家自然科学基金资助项目(51979165)

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