

Parameterization, sensitivity, and uncertainty of 1-D thermodynamic thin-ice thickness retrieval

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Abstract

Retrieval of Thin-Ice Thickness (TIT) using thermodynamic modeling is sensitive to the parameterization of the independent variables (coded in the model) and the uncertainty of the measured input variables. This article examines the deviation of the classical model's TIT output when using different parameterization schemes and the sensitivity of the output to the ice thickness. Moreover, it estimates the uncertainty of the output in response to the uncertainties of the input variables. The parameterized independent variables include atmospheric longwave emissivity, air density, specific heat of air, latent heat of ice, conductivity of ice, snow depth, and snow conductivity. Measured input parameters include air temperature, ice surface temperature, and wind speed. Among the independent variables, the results show that the highest deviation is caused by adjusting the parameterization of snow conductivity and depth, followed ice conductivity. The sensitivity of the output TIT to ice thickness is highest when using parameterization of ice conductivity, atmospheric emissivity, and snow conductivity and depth. The retrieved TIT obtained using each parameterization scheme is validated using *in situ* measurements and satellite-retrieved data. From *in situ* measurements, the uncertainties of the measured air temperature and surface temperature are found to be high. The resulting uncertainties of TIT are evaluated using perturbations of the input data selected based on the probability distribution of the measurement error. The results show that the overall uncertainty of TIT to air temperature, surface temperature, and wind speed uncertainty is around 0.09 m, 0.049 m, and −0.005 m, respectively.

Key words: Arctic sea ice, 1-D thermodynamic ice model, thin-ice thickness, sea ice parameterization

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1 Introduction

Thin sea ice (less than 0.5 m) is crucial for the energy flux between ocean and atmosphere in the winter of polar regions, and usually exists in the form of leads, polynyas, or a marginal ice zone (Stroeve et al., 2012). Arctic polynyas and leads represent only 3%–4% of the sea ice area in winter, but they account for about 50% of the heat transfer to the atmosphere (Maykut, 1982). The heat flux through thin ice is strongly affected by the ice thickness. For example, 5-cm thick ice can conduct about 270 W/m² turbulent heat flux (Ebner et al., 2011). The sensible heat flux from polynyas is accompanied by the reformation of new ice, so that polynyas are an important sea ice production source in the polar winter. The accurate determination of ice production or the accompanying salt rejection magnitude requires precise estimation

of Thin-Ice Thickness (TIT) and area. Therefore, TIT information is important for understanding the sea ice-atmosphere-ocean interaction through heat flux. When studying thermodynamic TIT modeling, the sensitivity and uncertainty are of great significance to the study of polar climate. In the context of thin ice, very few field measurements of its physical properties are available, due to its inaccessibility and the fragility of its natural environment. Remote sensing data are a very useful tool in this regard.

Estimation of sea ice thickness of up to about 0.5 m thick can be accomplished using the L-band (1.4 GHz) Passive Microwave Radiometer (PMR) onboard the Soil Moisture and Ocean Salinity (SMOS) mission (Kaleschke, 2013; Zine et al., 2008) and the Soil Moisture Active Passive (SMAP) satellites (Huntemann et al.,

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2014; McHedlishvili et al., 2022; Pařilea et al., 2019; Tian-Kunze et al., 2014). TIT retrieval has been commonly obtained from other PMR satellites, namely SSM/I, AMSR-E, and AMSR2 (Iwamoto et al., 2014; Martin et al., 2004; Ohshima et al., 2020). The empirical relationships between TIT and the observed brightness temperature are usually used to estimate TIT (Kashiwase et al., 2021; Nakata et al., 2019). However, the coarse special resolution of the PMR sensors does not allow estimation of TIT in narrow leads, water channels, or small polynyas. The statistical relationships between spaceborne Synthetic Aperture Radar (SAR) backscatter and sea ice thickness have also been used to estimate TIT at a finer spatial resolution (Kern et al., 2006; Toyota et al., 2011; Karvonen et al., 2012, 2017; Similä et al., 2013). However, the application of SAR is hindered by the low coverage and long revisit interval. The spaceborne altimeters, e.g., ICESat (Kwok and Rothrock, 2009; Nishashi et al., 2018), ICESat-2, and/or CryoSat-2 (Kacimi and Kwok, 2020; Koo et al., 2021a; Kwok et al., 2020), have also provided valuable sets of sea ice thickness estimates over the polar regions, but with large uncertainty in the TIT retrieval (Koo et al., 2021b; Kwok et al., 2019).

Since thin ice growth is usually controlled by thermodynamic processes and the ice surface energy balance principle, the ice surface temperature (T_s) is linked with TIT up to about 0.5 m (Maykut, 1982; Yu and Rothrock, 1996), which makes it feasible to achieve thermodynamic model based TIT retrieval with the satellite-measured T_s from spaceborne Thermal Infrared (TIR) imagery, e.g., Advanced Very High-Resolution Radiometer (AVHRR) or Moderate Resolution Imaging Spectroradiometer (MODIS) imagery (Drucker et al., 2003; Willmes et al., 2010; Yu and Rothrock, 1996). One of the most popular one-dimensional (1-D) thermodynamic sea ice models used for TIT retrieval was introduced by Maykut (Maykut, 1978, 1982) (Section 3.1). This model employs the energy balance at the ice surface, and has been validated using TIR-TIT retrieval (Adams et al., 2012; Paul et al., 2015b; Preußer et al., 2019).

With the advantages of a short revisit period, extensive swath coverage, and a spatial resolution of as high as 1 km, spaceborne TIR images are frequently used for TIT retrieval and polar ice production calculation based on a 1-D model (Aulicino et al., 2018; Mäkyinen et al., 2013; Paul and Huntemann, 2021; Paul et al., 2015a, b; Preußer et al., 2016). Moreover, the TIR-TIT can

be used to construct the empirical relationship between Passive Microwave (PM) brightness temperature and TIT (Mäkyinen and Similä, 2019). However, some of the model's variables, e.g., the air emissivity, can be parameterized using different equations (see Appendix for details) and this changes the output TIT. In addition, the inevitable uncertainty in the measured input variables is reflected in the uncertainty of the TIT retrieval. Therefore, for accurate TIR-TIT retrieval, it is important to quantify the discrepancies of TIT caused by using different parameterizations on independent variables in the model and errors in the input variables.

Previous studies have investigated the uncertainty of the modeled TIT caused by the uncertainty of the measured input parameters of air temperature T_a and ice surface temperature T_s (Table 1). However, assessment of the uncertainty caused by parameterization of the model's independent variables is still lacking. Furthermore, the commonly adopted errors in the input variables (e.g., the values on the right side of “ $\pm$ ” in the second column of Table 1) are estimated under all ice conditions, and not specifically for thin-ice conditions. Therefore, there is a need to clarify the errors of the input variables under thin-ice conditions. To sum up, the motivation for this work is twofold. The first motivation is to evaluate the deviation of the retrieved TIT up to 0.5 m using different parameterization schemes for seven independent variables (given in Section 3.1) and to examine the sensitivity of the retrieval to the ice thickness. The second motivation is to estimate the uncertainty of the retrieved TIT in response to the uncertainty of the three input variables of T_a , T_s , and surface wind speed (u).

The datasets and the proposed method are described in Sections 2 and 3, respectively. Section 4 gives the main results of this study. Discussion and our conclusions are presented in Sections 5 and 6, respectively.

2 Datasets

2.1 Ice surface temperature data

The ice surface temperature from MODIS rather than that from AVHRR was adopted in this study because the former has a better cloud mask effect during the night than the latter. The ice surface temperature data from the MODIS 5-min L2 swath 1km

Table 1. Summary of the previous uncertainty in TIR-TIT retrieval

Reference	Data source (data volume)	Uncertainty analysis scheme	Finding
Yu and Rothrock (1996)	AVHRR (13 images)	$T_s \pm 1 \text{ K}$, $T_a \pm 1.6 \text{ K}$, $u \pm 2 \text{ m/s}$	(1) The uncertainty of the TIT caused by the error in the variables increases with the ice thickness. (2) The uncertainty of the cumulative distribution is no more than 3% for ice thinner than 0.2 m.
Willmes et al. (2010)	AVHRR + NCEP (1 image)	$T_a \pm 5 \text{ K}$, $u \pm 3 \text{ m/s}$	Results in max. TIT errors of $\pm 20\%$ (for TIT $\leq 0.5 \text{ m}$)
Wang et al. (2010)	MODIS (case study)	$T_s \pm 2 \text{ K}$, $u \pm 1 \text{ m/s}$	For TIT $< 0.3 \text{ m}$, TIT is -0.172 m or $+0.179 \text{ m}$ when $T_s + 2 \text{ K}$ or $T_s - 2 \text{ K}$; TIT is $+0.166 \text{ m}$ or -0.133 m when $u + 1 \text{ m/s}$ or -1 m/s .
Mäkyinen et al. (2013)	MODIS + HIRLAM (199 images)	–	The largest TIT uncertainty comes from air temperature.
Adams et al. (2013)	MODIS + NCEP/COSMO (two-winter data)	$T_s \pm 1.6 \text{ K}$, $T_a \pm 4.5 \text{ K}$; $u \pm 1.3 \text{ m/s}$, NCEP vs. COSMO	(1) For all the variables' uncertainties, the TIT varies by $\pm 0.37 \text{ m}$ for TIT of $\leq 0.5 \text{ m}$. (2) The NCEP T_a leads to overestimated ice thicknesses, in comparison to COSMO T_a .
Zeng et al. (2016)	MODIS + ERA-interim (four images)	Altering the value of $T_a - T_s$	(1) For $T_a - T_s > +3 \text{ K}$, the $T_a - T_s$ error of only $+1 \text{ K}$ causes the TIT to vary by $+0.1 \text{ m}$. (2) For $T_a - T_s \leq 0 \text{ K}$ or $T_a - T_s > +2 \text{ K}$, the TIT decreases by 1 cm to 2 cm or 9 cm when u increases from 0 m/s to 12 m/s .

Note: The symbols are specified in Table A1 in Appendix. The data sources of AVHRR or MODIS are used for T_s , and the National Centers for Environmental Prediction (NCEP), High Resolution Limited Area Model (HIRLAM), or European Centre for Medium-Range Weather Forecasts (ECMWF) Re-analysis (ERA)-Interim reanalysis data are the other meteorological input variables of the 1-D model. In the third column, the regular font denotes the analysis scheme for the errors in the input variables, the values on the right side of the positive and negative signs “ $\pm$ ” are the analyzed variable's errors, and the bold font is for comparing the sources of the different input variables. – means no uncertainty analysis scheme.

MOD29 and MYD29 datasets version 61 provided by the National Snow and Ice Data Center (NSIDC) were adopted (Hall and Riggs, 2021; Hall et al., 2004). The nominal spatial resolution at nadir is $1 \text{ km} \times 1 \text{ km}$ with a coverage of $1\,354 \text{ km}$ (across track) $\times$ $2\,030 \text{ km}$ (along track), and the nominal uncertainty under clear sky conditions is $1\text{--}3 \text{ K}$ (Hall and Riggs, 2021). Clouds in the ice surface temperature swath data are masked with no-data pixels. In total, 10% of all the night data north of 55°N in November to March from 2002/2003 to 2020/2021 were monthly stratified and randomly selected, and a total of 11 748 images were obtained for analysis. For the TIT retrieval, only 1% of the pixels with TIT less than 0.5 m were randomly sampled from each swath, and a total of 442 million pixels were used, averaging 377 pixels for each scene.

2.2 Meteorological data

The ERA5 reanalysis data were selected as the meteorological data source. The 2-m air temperature (T_a), 2-m dewpoint temperature (T_d), 10-m wind speed (denoted as a proxy for wind speed u), mean sea-level pressure (p), and the total cloud cover (C) from ERA5 were used. Other data, including the Skin Temperature (SKT), were also used for comparison. The ERA5 gridded data with a spatial resolution of 0.25° and a temporal interval of 1 h were adopted. The specifications of these variables can be found in the ECMWF (2021). For the swath-based TIT retrieval,

the temporally nearest ERA5 meteorological data were spatially interpolated to the same resolution as the MODIS ice surface temperature swath. The time interval between the ice temperature and the ERA5 atmospheric variables used for the TIT retrieval was less than half an hour.

2.3 In situ TIT measurements

For the TIT validation, measurements from Up Looking Sonar (ULS) were mainly used, with a few IceBridge TIT measurements as a supplement. The specifications of the ULS sonar are summarized in Table 2. The geographic distribution of the sonar sites is displayed in Fig. 1a. The sonar measures ice draft, which is the vertical distance between the bottom of sea ice and the local sea level directly above the ULS. The ice draft usually serves as a close proxy of ice thickness, which can be empirically transferred to ice thickness using a multiplication factor of about 1.2 (Fukamachi et al., 2017; Rothrock et al., 2008). As documented in Table 2, these ULS ice draft measurements have an estimated error of less than 10 cm . In addition, the footprint of the sonar is less than 2 m in diameter (depending on the depth of the instrument), and it can resolve some of the thinnest ice categories. We used the ice draft datasets released by the data sources described in Table 2 directly, without retrieving the ice draft ourselves.

Operation IceBridge (IB), operated by NASA, is the largest airborne survey of Earth's polar regions, which yields an unprece-

Table 2. Specifications of the sonar measurements used in this study

Data source	Data form	Number of sites	Data period	Number of samples	Nominal error/cm
NPEO	10-min ice draft data	2	2002, 2005	10	10
BGEP	daily average draft statistics	34	2003–2021	737	5–10
IOS-EBS	4-min ice draft data	1	2003	1 724	5
IOP	5-min ice draft data	5	2005–2008	3 305	10

Note: NPEO is the abbreviation for the North Pole Environmental Observatory (Morison, 2009). BGEP denotes the Beaufort Gyre Exploration Project. IOS and EBS are the short forms for the Institute of Ocean Sciences and the Eastern Beaufort Sea, respectively (Melling and Riedel, 2008). IOP denotes the Integrative Observational Platforms from University of Washington.

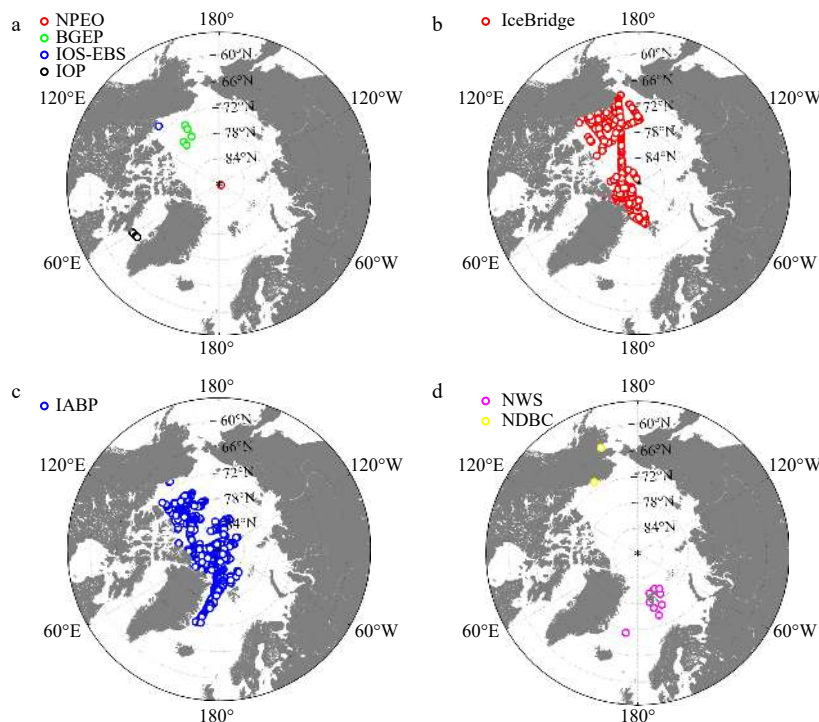


Fig. 1. The geographic locations of the sonar sites (a), the IceBridge footprints (b), the IABP buoys (c), and the wind speed stations (d) used in this study.

ented three-dimensional (3-D) view of the Arctic and Antarctic ice sheets, ice shelves, and sea ice (IceBridge Mission Overview, https://www.nasa.gov/mission_pages/icebridge/mission/index.html). Measurements from 10 562 sample points for ice less than 0.5 m thick, obtained during 14 campaigns in March during 2010–2017, were selected for the comparison (Fig. 1b). The ice thickness from IceBridge has a spatial resolution of 40 m along track, together with thickness uncertainty (denoted as 1σ) estimation for each pixel (Kurtz et al., 2015).

2.4 SMOS/SMAP TIT

In addition to the *in situ* TIT measurements, the SMOS/SMAP TIT products were also compared with the TIT retrievals. The operational daily 12.5 km SMOS/SMAP TIT (up to 0.5 m) product for the Arctic from 2015 to present is generated from measurements obtained by the L-band SMOS and SMAP spaceborne PM sensors and is offered through Bremen University. The horizontal and vertical polarized brightness temperatures in the incidence angle range of 40° to 50° for SMOS are transferred to the fixed incidence angle of SMAP using empirical fit functions, and the SMAP brightness temperatures are calibrated to SMOS to provide a merged and improved SMOS-SMAP sea ice thickness product (Huntemann et al., 2014; Pařileu et al., 2019; Tian-Kunze et al., 2014). All the available SMOS/SMAP TIT data from 2015 to 2021 in November were used for the comparison.

2.5 *In situ* T_a , T_s , and u

To evaluate the accuracy of the ERA5 T_a and MODIS T_s over thin ice, all the available T_a and T_s measurements under thin-ice conditions from the International Arctic Buoy Program (IABP) (Fig. 1c) in Level 2 were used after eliminating obvious spikes. The comparison was conducted for the nearest data pairs (total of 1 724) within a half an hour time interval. In addition, *in situ* wind speed measurements from nine Norwegian Weather Stations (NWS) and two National Data Buoy Center (NDBC) sites were obtained to estimate the uncertainties in the ERA5 10 m wind speed u . The specifications and locations of these stations are given in Table 3 and Fig. 1d. All the wind speed data were transferred to 10 m based on the logarithmic wind profile law with an aerodynamic roughness length of 0.003.

3 Methods

3.1 1-D Model

The 1-D model [Eq. (1)] used for TIT retrieval was introduced by Maykut (Maykut, 1978, 1982) following the general form of the energy balance equation at the ice surface. This model is based

Table 3. Station ID and geographic locations of the wind speed observation stations

Data source	Station ID	Latitude and longitude	Temporal resolution
NWS	99950	71.00°N, 8.46°E	1 h
	99710	74.50°N, 19.00°E	1 h
	99720	76.51°N, 25.01°E	1 h
	99740	78.91°N, 28.89°E	1 h
	99752	76.47°N, 16.54°E	1 h
	99790	78.07°N, 13.63°E	1 h
	99927	80.06°N, 16.24°E	1 h
	99935	80.65°N, 25.00°E	1 h
	99938	80.10°N, 31.46°E	1 h
NDBC	ULRA2	63.87°N, 160.78°W	6 min
	PRDA2	70.40°N, 148.53°W	6 min

on the assumptions that the total energy flux from the surface to the atmosphere is balanced by the conductive heat flux F_c through the ice, and that the temperature profile through the ice cover is linear. All the heat or radiative flux components in Eq. (1) are in unit of W/m^2 .

$$F_c = (1 - \alpha_s) F_r + I_0 + F_{dn} + F_{up} + F_s + F_e, \quad (1)$$

where $(1 - \alpha_s) F_r$ is the net incoming shortwave radiation, α_s is the surface albedo, and I_0 is the part that penetrates the ice surface. The equation is applied only during polar night, and hence F_r , α_s , and I_0 are set to zero. F_{dn} and F_{up} are the atmospheric downward and surface-emitted longwave radiation, which can be estimated by the Stefan-Boltzmann's law given in Eqs (2) and (3), respectively.

$$F_{dn} = \varepsilon_a \sigma T_a^4, \quad (2)$$

$$F_{up} = \varepsilon_s \sigma T_s^4, \quad (3)$$

where σ is the Stefan-Boltzmann constant [$5.670 \times 10^{-8} W/(m^2 \cdot K^4)$], and T_a and T_s are the temperature of the atmosphere and the surface in unit of K, respectively. ε_a and ε_s are the longwave emissivity of the atmosphere and surface, respectively. The emissivity ε_s of sea water, ice, and snow are similar to that of a black body in the infrared band, with the emissivity varying from 0.98 to 1 (Murray et al., 2020), which represents a maximum 2% fluctuation of F_{up} , based on Eq. (3). We ignored the variation of the ice emissivity ε_s and set it to 0.988 because the surface type was not distinguished in our analysis. Moreover, the surface F_{up} emission is low in magnitude during the winter season.

F_s and F_e are the turbulent sensible and latent heat fluxes, which are estimated from a bulk flux algorithm (Launiainen and Vihma, 1990) based on Monin-Obukhov similarity theory, taking the forms shown in Eqs (4) and (5), respectively.

$$F_s = \rho_a c_p C_s u (T_s - T_a), \quad (4)$$

$$F_e = \rho_a L_s C_e u (q_s - q_a), \quad (5)$$

where ρ_a (kg/m^3) is the air density, and c_p [$J/(kg \cdot K)$] is the specific heat of air at constant pressure. L_s (J/kg) is the latent heat of sublimation. C_s and C_e are the transfer coefficients for sensible heat and latent heat, respectively, which were assigned with the same constant value of 0.003 used by Yu and Rothrock (1996). u (m/s) is the effective wind speed at the same height as T_a . We utilize the 10-meter wind speed to represent the effective wind speed. q_s and q_a are the specific surface and air humidities, respectively. $q_s - q_a \approx 0.622(e_s - e_a)/p_0$, where e_a and e_s are the water vapor pressure of air and surface, respectively, and p_0 (hPa) is the surface atmospheric pressure.

The term F_c is the conductive heat flux through the ice and the overlaid snow. It is equivalent to the last four terms of Eq. (1), i.e., the radiative and heat fluxes during polar night. Assuming that the temperature profile through the ice is linear, then F_c can be written as follows (Maykut, 1978):

$$F_c = \lambda \frac{dT}{dz} = \left(\frac{k_i \cdot k_s}{k_s \cdot h_i + k_i \cdot h_s} \right) \cdot (T_s - T_f), \quad (6)$$

where λ is the thermal conductivity, and $\frac{dT}{dz}$ is the temperature gradient through the ice-snow slab. T_f (K) is the water freezing point (271.35 K), and k_i [W/(m·K)] and k_s [W/(m·K)] are the thermal conductivity of ice and snow, respectively. h_i (m) and h_s (m) are the thickness of ice and snow, respectively. The unknown in the model Eq. (1) is h_i , which appears in Eq. (6). We adopt the convention that a flux toward the surface is positive and a flux away from the surface is negative. F_r and F_{dn} are always positive, F_{up} is the opposite, and F_s and F_e can be positive or negative.

Parameterizations are needed for the variables involved in Eqs (2)–(6). A literature review was performed to identify the variables that need parameterization and the commonly used parameterization equations. Seven independent variables were identified: atmospheric emissivity (ε_a), air density (ρ_a), specific heat of air (c_p), latent heat of sublimation (L_s), ice conductivity (k_i), snow depth (h_s), and snow conductivity (k_s). A total of 14 parameterization schemes (either equations or values) were selected, as shown in Table 4, and the equations are presented in Appendix. Note that some of the parameters in Table 4 are parameterized in terms of other parameters. For example, air emissivity is a function of water vapor pressure. In this case, the parameterized equations in Table 4 are presented in two parts, with the second part between brackets. For example, an equation such as ε_{a_JX06} [$+E'_{AE96}$, Eq. (A5)] means that the atmospheric emissivity (ε_a) is parameterized with equation ε_{a_JX06} [Eq. (A3)], and the water vapor parameter embedded in it is parameterized by equation E'_{AE96} [Eq. (A6)]. For each tested parameter, the relevant component in the model is shown in the fifth column of Table 4, and the parameterized equations are given in Appendix. The equation or the constant value of each parameter that was most commonly used in previous studies is considered to be the default scheme/value for that parameter.

3.2 Study scheme

The study scheme, which is composed of two tasks, is presented in the flowchart in Fig. 2. The first is about the effect of each parameterization scheme (coded in the model) on the model's output of TIT. A parameterization scheme is associated with each one of the seven variables listed above. The sensitivity of the TIT output from using each scheme to the TIT value (presented as

bins) is included in this task (the left side of the flowchart). Validation of the output using the different parameterization schemes is included in this task. In the second task, the uncertainties of the three measured input variables T_a , T_s , and u are determined by comparison against *in situ* or satellite-derived data, generating the distribution of the error. The effect of the error in each parameter on the uncertainty of the output TIT is then estimated using the distribution and Monte Carlo random number generator (the right side of the flowchart).

3.2.1 Deviation and sensitivity of the output TIT using different parameterization schemes

For each one of the parameterization variables in Fig. 2 and each parameterization scheme shown in Table 4 (i.e., T1–T4 for ε_a), the TIT was calculated and the deviation [Eq. (7)] from the value obtained using the default scheme was determined. This revealed how close (or how far) the results obtained from using the test scheme were to the default scheme.

The deviation from using each one of the 14 tested schemes listed in Table 4 (T1–T14), was determined using the control variates method. In this method, the influence of multiple independent variables on a given dependent variable is determined by detecting the variability of the dependent variable (i.e., TIT) after changing the scheme of one of the independent variables while keeping the default schemes of the other independent variables. To explain, each one of the 14 test schemes was used together with the default schemes of the other six variables. The percentage deviation of the model output f_{Ti} using the i -th test scheme from the output using the default scheme f_{def} was calculated using the following equation:

$$\text{Dev}_{T_i} = \frac{f_{T_i} - f_{def}}{f_{def}} \times 100\%, \quad (7)$$

where Dev_{T_i} is the deviation of the i -th test scheme ($i = 1, 2, \dots, 14$) from the default scheme. The sensitivity of the TIT output using each parameterization scheme to successive intervals of TIT was then obtained by examining the slope of the deviation.

3.2.2 Validation of the TIT output

The validation of the TIT was performed by comparison with

Table 4. The parameterized variable test schemes

Test scheme	Parameter	Default scheme	Parameterization scheme	
			Parameter name	Equation number
T1	atmospheric emissivity (ε_a)	ε_{a_JX06} ($+E'_{AE96}$)	ε_{a_EF61} ($+E'_{AE96}$)	Eq. (A1) [$+E_{AE96}$]
T2	atmospheric emissivity (ε_a)	ε_{a_JX06} ($+E'_{AE96}$)	ε_{a_KL94}	Eq. (A2)
T3	atmospheric emissivity (ε_a)	ε_{a_JX06} ($+E'_{AE96}$)	ε_{a_JX06} ($+E_{M78}$) (denoted as $\varepsilon_{a_J/M}$)	Eq. (A3) [$+E_{AE96}$]
T4	atmospheric emissivity (ε_a)	ε_{a_JX06} ($+E'_{AE96}$)	ε_{a_JX06} ($+E_{AE96}$) ($\varepsilon_{a_J/A}$)	Eq. (A3) [$+E_{AE96}$]
T5	air density (ρ_a , kg/m ³)	$\rho_a = 1.3$ kg/m ³	ρ_{a_GL}	Eq. (A7)
T6	specific heat of air [c_p , J/(kg·K)]	$c_p = 1\,004$ J/(kg·K)	c_{pw}	Eq. (A8)
T7	latent heat of sublimation (L_s , J/kg)	$L_s = 2.5 \times 10^6$ J/kg	L_{sw}	Eq. (A9)
T8	ice conductivity [k_i , W/(m·K)]	$k_i = 2.03$ W/(m·K)	k_{i_U64} ($+k_{0_N64} + S_{i_J94}$) (k_{i_UNJ})	Eq. (A10) [$+E_{AE96}$]
T9	ice conductivity [k_i , W/(m·K)]	$k_i = 2.03$ W/(m·K)	k_{i_U64} ($+k_{0_S78} + S_{i_J94}$) (k_{i_USJ})	Eq. (A10) [$+E_{AE96}$]
T10	ice conductivity [k_i , W/(m·K)]	$k_i = 2.03$ W/(m·K)	k_{i_U64} ($+k_{0_C10} + S_{i_J94}$) (k_{i_UCJ})	Eq. (A10) [$+E_{AE96}$]
T11	ice conductivity [k_i , W/(m·K)]	$k_i = 2.03$ W/(m·K)	k_{i_U64} ($+k_{0_C10} + S_{i_C74}$) (k_{i_UCC})	Eq. (A10) [$+E_{AE96}$]
T12	snow depth (h_s , m)	$h_s = 0$ m	h_{s_D71} ($+k_{s1}$)	Eq. (A16)
T13	snow depth (h_s , m)	$h_s = 0$ m	h_{s_M19} ($+k_{s1}$)	Eq. (A17)
T14	snow conductivity [k_s , W/(m·K)]	$k_{s1} = 0.33$ ($+h_{s_M19}$)	$k_{s2} = 0.21$ ($+h_{s_M19}$)	[$+E_{AE96}$]

Note: All the symbols and equations in the third and fourth columns are introduced in Appendix.

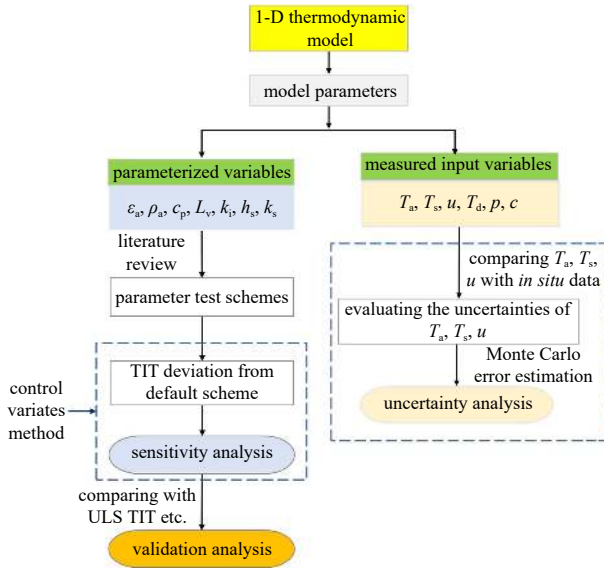


Fig. 2. Flowchart of the study scheme.

in situ measurements of TIT from the ULS (Table 2). In addition a few field-measured TIT values from IceBridge and the satellite-retrieved TIT from SMOS/SMAP were also used. The original ULS TIT (<0.5 m) which is sampled in minutes, was first averaged into hourly data. It was then compared with the spatially and temporally nearest model-retrieved TIT.

The uncertainty of the IceBridge TIT (presented as 1σ) is larger in some cases. To refine the comparison between the model TIT and IceBridge TIT, the IB-TIT for which the spatially and temporally nearest model TIT within $IB-TIT \pm 1\sigma$ was selected for the comparison. In total, 44.03% of the IB-TIT satisfied this criterion. To compare the retrieved TIT with the SMOS/SMAP TIT, the retrieved TIT at a 1-km resolution was first resampled to the SMOS/SMAP ice thickness grid resolution of 12.5 km. The spatially nearest daily median retrieved TIT was then used to compare with the daily SMOS/SMAP TIT when it was less than 0.5 m.

3.2.3 Uncertainty of TIT due to uncertainty of the measured input variables

The measured input variables (as specified in Fig. 2) are subject to uncertainty. This can be due to unknown surface conditions, point sampling from a spatially variant domain, and possible noise/drift in the measurements. This uncertainty is reflected in the uncertainty of the model's output TIT. The uncertainty of only three input parameters was considered: T_a from ERA5, T_s from MODIS, and u from ERA5. The uncertainties caused by errors in the meteorological factors, such as dewpoint temperature (T_d), air pressure (p), and cloud fraction (C), were not investigated.

The uncertainties of ERA5 T_a and MODIS T_s were determined as the difference between the measured value of each parameter and the estimate from the IABP buoy measurements over thin ice using the spatially and temporally (less than 0.5 h) nearest matching pairs. For the MODIS T_s , its discrepancy with respect to IABP T_s was found to be very large. Therefore, it was additionally compared with the ERA5 skin temperature. Similarly, the uncertainty of the ERA5 u was determined as being the difference with respect to wind speed measurements from the NWS and NDBC stations, also using the spatially and temporally nearest matching pairs.

The probability density function of the uncertainty of each parameter was generated, and the Monte Carlo error estimation method (Adams et al., 2013; Mäkinen et al., 2013) was performed to assess the impact of the uncertainties of the measured input variables on the uncertainty of the model's TIT. This method involves generating random values (errors) for each variable by using the inverse of the cumulative distribution of the probability density function of its error (Shokr et al., 2008). This ensures that the generated random values will follow the same probability distribution as the uncertainty of the input variable. Each random value was assigned to a sampled pixel in the MODIS scene, which has a retrieved TIT of <0.5 m using the combined parameterization scheme. A new TIT map was then generated using the original values of ERA5 T_a , MODIS T_s , and ERA5 u , plus a random number of the relevant variable (considered to be a perturbation). This process was performed using the random numbers of ERA5 T_a while keeping the original input of T_s and u , and was then repeated using the random numbers of T_s , and then u , while keeping the original input of the other two variables unchanged in each case. Finally, the difference between the new TIT and the original TIT (using the original values of T_a , T_s , and u) was considered to represent the uncertainty of the output TIT due to the uncertainty (error) of the relevant input variable.

4 Results

4.1 Deviation of the TIT output using parameterization schemes from the output using the default scheme

The deviation of the retrieved TIT using each one of the 14 test parameter schemes from the value retrieved using the default scheme is given in Table 5. Data were obtained for successive thickness bins and are presented as the percentage of deviation, as given by Eq. (7).

For the four schemes involving atmospheric emissivity (T1 to T4), Table 5 shows that the water-vapor-dependent atmospheric emissivity T1 [ϵ_{a_EF61} , i.e., Eq. (A1)] produces positive deviation (i.e., overestimated TIT) within the range of 11.18% to 14.34% for different intervals of TIT. The use of the cloud-dependent atmospheric emissivity scheme T2 [ϵ_{a_KL94} , i.e., Eq. (A2)] produces a larger deviation. In both cases, the deviation increases as the ice thickness increases. When the default atmospheric emissivity

Table 5. The mean deviation of TIT for the 14 parameterization schemes with respect to the default scheme

Test scheme	Mean deviation of TIT/%			
	TIT=0–0.1 m	TIT=0.2–0.3 m	TIT=0.4–0.5 m	TIT=0–0.5 m
T1	11.18	12.81	14.34	12.49
T2	16.11	22.20	17.20	19.25
T3	0.10	−0.15	−0.46	−0.16
T4	−0.10	−0.14	−0.20	−0.14
T5	−2.99	−3.40	−1.92	−3.00
T6	−0.03	−0.01	0.00	−0.01
T7	−0.09	−0.08	−0.02	−0.07
T8	−9.77	9.31	10.50	4.79
T9	−23.40	−1.40	0.95	−5.95
T10	−13.84	10.09	11.65	4.29
T11	−10.72	9.93	11.70	5.05
T12	−8.09	−23.52	−38.09	−26.02
T13	−8.09	−33.34	−35.63	−28.23
T14	−9.25	−41.79	−46.52	−36.44

Note: Calculations are for the three shown ice thickness bins as well as the average TIT over the entire thickness range of 0–0.5 m.

[Eq. (A3)] is used with the two forms of water vapor saturation equations [Eqs (A4) and (A5)] within the schemes denoted $\varepsilon_{a,J/M}$ and $\varepsilon_{a,J/A}$, (i.e., schemes T3 and T4), the results become nearly the same as those from the default scheme. This means that the water vapor saturation has no influence on the TIT output.

The air density parameterization (T5) underestimates TIT, while parameterization of the specific heat of air (T6) or latent heat of sublimation of ice (T7) produces almost zero deviation from the default scheme. The use of the parameterization equations for the ice conductivity (T8–T11) leads to significant deviation from the default scheme, ranging between underestimation and overestimation (i.e., negative and positive deviation, respectively). The deviation fluctuates greatly between -8% and 24% . No correlation between the deviation and ice thickness is apparent. The two parameterization schemes for the snow depth (T12 and T13) as well as the scheme for the snow conductivity (T14) produce high deviation (mostly underestimation of TIT). Clearly, the parameterizations that produce minimum deviation from the default scheme are candidates for optimal selection. However, this should be confirmed based on the evaluation of each scheme against a set of ground-truth data (Section 4.3). These parameterization candidates include the following parameters: atmospheric emissivity (T1 and T2), ice conductivity (T8–T11), snow depth (T12 and T13), and snow conductivity (T14).

4.2 Sensitivity of TIT to the different parameterization schemes

Figure 3 shows the deviation of the model output TIT using each one of the 14 parameterization schemes from the value using the default scheme (as in Table 5), but for successive thickness bins incremented by 0.1 m. Data for the entire TIT range of 0–0.5 m are also shown. Schemes T1 and T2 show deviations between 10% and 20% but with high variability. Schemes T3 to T7 show no deviation, while schemes T8 to T11 show deviations between 2% and -22% , depending on the ice thickness, but with moderate variability. Schemes T12 to T14 show deviations between -10% and -35% , also with moderate variability.

For each parameterization scheme, the change of the average TIT output (solid circles in Fig. 3) in response to the incremental change of TIT defines the sensitivity of the output to the change of ice thickness. This can be determined from Fig. 3 as the slope of the mean values (the solid circles) within each scheme. The slopes are presented numerically in Table 6. Schemes T1 and T2 show mild sensitivity to ice thickness, with slight increase of the

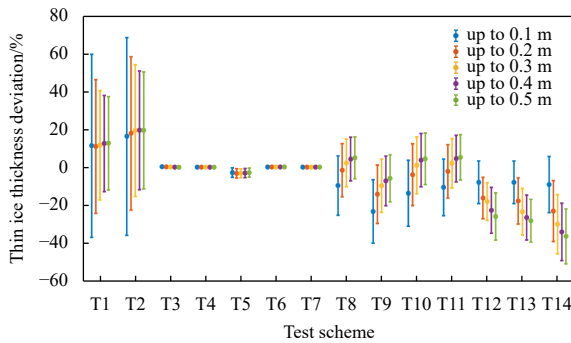


Fig. 3. Relative deviation of the model output TIT using each test scheme (T1, T2, ..., T14) from the TIT obtained using the default scheme. The solid points are mean values of the deviation, and the vertical lines represent one standard deviation. The colors denote the upper limit of the different TIT bins, from 0.1 m to 0.5 m.

deviation with thickness, and their sensitivity measures 0.33% and 0.79%, respectively. Schemes T3 to T7 show no sensitivity (and no deviation). The rest of the schemes, i.e., T8–T14, are sensitive to TIT. For the ice conductivity schemes T8 to T11, the sensitivity is high (3.64% to 4.53%), with the deviation of TIT from the default value continuing to decrease with ice thickness up to 0.4 m before it stabilizes. This means that these schemes produce better results for thicker ice. This is opposite to the snow depth and snow conductivity schemes (T12 to T14), where the sensitivity is still high (-4.48% to -6.8%), but the deviation increases with TIT, i.e., each scheme produces results closer to the default values when TIT is small. The most sensitive scheme in this group is T14. As the results show the different sensitivities of the different schemes to ice thickness, it is difficult to recommend a set of schemes to produce the best model results. However, the statistical criteria for selection of the best scheme for the overall TIT range are presented in the next section.

4.3 Comparison of the calculated TIT with ULS measurements

In this section, we describe how the output TIT from the model was assessed by comparison with the TIT obtained from the ULS data, which is considered to be the “ground-truth” data, obtained at the finest spatial resolution. As mentioned in Section 3, the TIT was calculated from the 1-D model using the input of ice surface temperature T_s from MODIS. Therefore, the output was generated at the spatial resolution of MODIS. The difference in the resolution between the ULS data and the model data should be taken into consideration.

Table 6. The sensitivity of the model’s output to increments in TIT of 0.1 m for the 14 parameterization equation schemes

Test scheme	Sensitivity/%	Test scheme	Sensitivity/%
T1	0.33	T8	3.64
T2	0.79	T9	4.36
T3	-0.07	T10	4.53
T4	-0.01	T11	3.94
T5	0	T12	-4.48
T6	0	T13	-5.03
T7	0	T14	-6.80

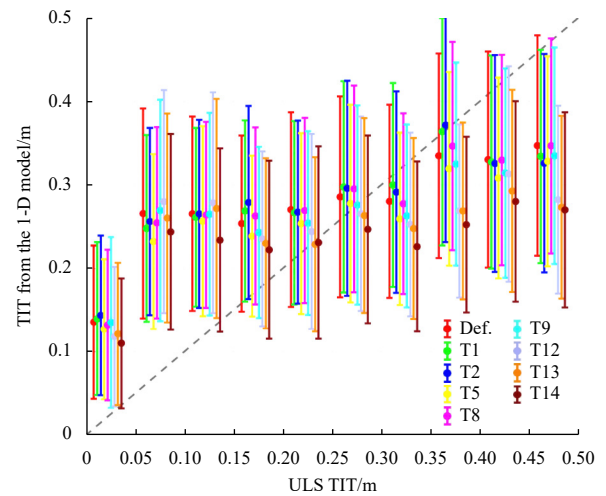


Fig. 4. TIT from the 1-D model versus the ULS data, obtained for different TIT intervals. The solid dots represent the mean value from the model, and the error bars represent 0.5 standard deviation. Dotted line indicates diagonal line.

Figure 4 is a plot of the model TIT obtained using the different parameterization schemes as well as the default scheme (Def.) versus TIT from the ULS. The ULS data are divided into 10 bins with an interval of 0.05 m, i.e., 0–0.05 m, 0.05–0.10 m, ... 0.45–0.50 m. The TIT data were grouped within each bin, and the standard deviation within each group was calculated and included in the figure. Figure 4 reveals the dependence of the model output on ice thickness. The most accurate TIT retrieval, regardless of the parameterization scheme, is found to occur around 0.25 m thickness (Fig. 4). The TIT from the model is overestimated and underestimated below and above this value, respectively. This indicates that 0.25 m may happen to be the balance point for the external meteorological factors. A striking observation in Fig. 4 is the similar trend of the deviation of the model's TIT from the ULS data using any scheme, including the default scheme. This reflects the limitation of the 1-D model rather than the use of the parameterization scheme or the use of the coarse-resolution input data from MODIS and ERA5. A more detailed discussion of these results is presented in Section 5.

The deviation of the model TIT from the ULS TIT is also presented in Table 7 for the different parameterization schemes. The data can be used to develop correction factors to bring the model's output closer to the ULS TIT data by dividing the model TIT by 1 + percentage deviation. For ice thinner than approximately 25 cm, the default scheme has the best agreement with the ULS data. For ice ranging from 25 cm to 40 cm, schemes T1, T2, T5, and T9 yield better results than the other schemes. Once again, this reveals the challenge in deciding on a set of parameterization schemes suitable for the entire range of TIT. The bias of each scheme is larger than 24 cm in the 40 cm to 45 cm bin, which is half the thickness of the ice bin.

Table 8 lists the statistics of the difference between the model

and the ULS TIT obtained using the default scheme and the eight shown test schemes. The purpose of this is to identify the best parameterization schemes to use in the model. Results with smaller bias, Root-Mean-Square Error (RMSE), and Mean Absolute Error (MAE) but higher coefficient of determination (R^2) and Pearson's linear correlation coefficient (ρ) are better. Compared to the default scheme, scheme T1 performs worse, with a 0.4 cm higher bias, even though it has a 0.2 cm (i.e., the difference between 0.103 m and 0.101 m) lower RMSE and MAE. Scheme T2 is worse, judging by all the statistical parameters. T5 is better in all the statistical parameters. T8 is similar to the default scheme, and T9 produces a 0.9 cm lower bias but with a 0.3 cm higher RMSE and MAE. Schemes T12, T13, and T14 produce a smaller bias but perform worse than the other schemes in terms of RMSE, MAE, R^2 , and ρ . Their smaller bias may be a result of the offset of the positive bias for ice less than 0.2 m and the negative bias for ice thicker than about 0.2 m. In short, combining schemes T5 and T9 represents a worthwhile attempt to obtain more accurate TIT retrievals. The following section addresses the results obtained from this combination, as it represents a potential for model improvement.

4.4 Comparison of the output obtained using a combined parameterization scheme with the validation datasets

Parameterization schemes T5 and T9, along with the default parameterization for all the other parameters, were used in the TIT model, and the results were compared to the TIT values obtained from the three validation datasets, i.e., the ULS, IceBridge, and SMOS/SMAP products (Fig. 5). The spatial resolution of these datasets is less than 2 m, 40 m, and 12.5 km, respectively. Data are presented for the different ice thickness bins. Once again, the model TIT (with T5 and T9) is the most consistent with

Table 7. Deviation of the model TIT with respect to the ULS TIT based on the different parameterization schemes

Scheme	Deviation of the model TIT/%								
	Ice bin = 0–5 cm	Ice bin = 5–10 cm	Ice bin = 10–15 cm	Ice bin = 15–20 cm	Ice bin = 20–25 cm	Ice bin = 25–30 cm	Ice bin = 30–35 cm	Ice bin = 35–40 cm	Ice bin = 40–45 cm
Def.	9.35	1.92	0.94	0.28	0.06	–0.04	–0.25	–0.18	–0.32
T1	10.65	2.43	1.05	0.39	0.16	0.00	–0.18	–0.13	–0.25
T2	10.21	2.13	1.02	0.48	0.14	0.05	–0.11	–0.05	–0.25
T5	10.73	2.27	1.02	0.54	0.15	0.04	–0.14	–0.03	–0.26
T8	9.62	1.99	0.98	0.31	0.09	–0.03	–0.23	–0.17	–0.30
T9	10.59	2.28	1.04	0.45	0.16	0.04	–0.17	–0.09	–0.24
T12	10.38	2.48	1.04	0.33	0.09	–0.03	–0.23	–0.16	–0.28
T13	10.17	2.64	1.18	0.28	0.02	–0.08	–0.25	–0.31	–0.29
T14	10.40	2.36	1.11	0.24	–0.04	–0.09	–0.27	–0.31	–0.34

Note: The deviation was calculated according to Eq. (7) in Section 3.

Table 8. Statistics of the difference between the model and ULS TIT based on the different parameterization schemes

Scheme	Bias/m	RMSE/m	MAE/m	R^2	ρ
Def.	0.027	0.103	0.088	0.741	0.861
T1	0.031	0.101	0.086	0.766	0.875
T2	0.032	0.106	0.089	0.693	0.832
T5	0.010	0.100	0.087	0.766	0.875
T8	0.028	0.101	0.088	0.755	0.869
T9	0.018	0.106	0.091	0.663	0.814
T12	0.004	0.120	0.102	0.367	0.606
T13	–0.004	0.120	0.102	0.407	0.638
T14	–0.018	0.118	0.102	0.511	0.715

Note: RMSE is root-mean-square error, MAE is mean absolute error, R^2 is the coefficient of determination, ρ is the Pearson's linear correlation coefficient. The p -value for each scheme is no more than 0.05.

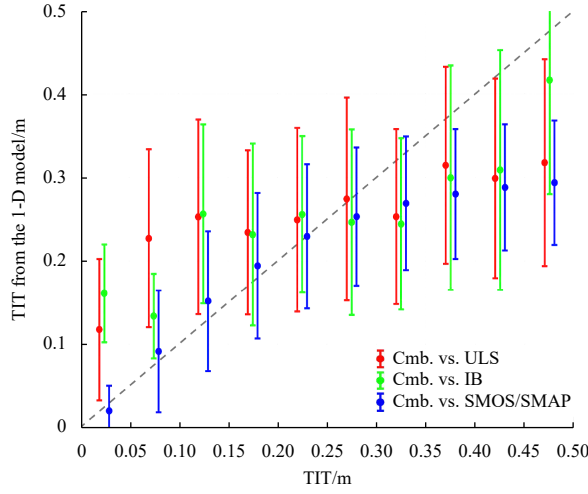


Fig. 5. Comparison between the model-retrieved TIT using the combination of schemes T5 and T9 and the TIT from the ULS, IceBridge (IB), and SMOS/SMAP products, respectively. Cmb. denotes the combination scheme. The interval between the data on the horizontal scale is 0.05 m. The error bars represent 0.5 standard deviation. Dotted line indicates diagonal line.

the validation datasets in the 0.2–0.3 m ice thickness range, but tends to overestimate (underestimate) the thickness for thinner (thicker) ice, which is similar to Fig. 4, suggesting that the accuracy of the 1-D model depends on the ice thickness range. Figure 4 also shows the nearly full agreement of the model TIT with the SMOS/SMAP products in the range of 0–0.3 m thickness, although both underestimate the thickness compared to the true values from the ULS. The data in Fig. 5 show that the selection of validation dataset is crucial in assessing the model's output.

4.5 Quantification of the uncertainty of the input variables

The uncertainty of T_a obtained from ERA5, as well as T_s obtained from MODIS, was estimated by comparison with *in situ* data obtained from the IABP measurements. The uncertainty of the wind speed u from ERA5 was estimated by comparison with the observations from the NWS and NDBC sites. The data were obtained for TIT of ≤ 0.5 m resulting from the model. The results are given in Tables 9–10 and Fig. 6.

For $T_a > 245$ K, there is fairly good correlation between T_a from ERA5 and the IABP measurements (Fig. 6a). The correlation coefficient (ρ) is around 0.86, with a minimum bias of -2.24 for the data of TIT < 0.1 m. The high correlation is more-or-less maintained for the thicker TIT, but with larger bias (underestimation), as shown in Table 9. There is no pattern of bias with temperature, but the best agreement is found for the temperature range of 245–250 K, with an overall bias of -2.11 K. For $T_a < 245$ K, ERA5 shows higher T_a than IABP, but this is reversed for $T_a > 245$ K. Figure 6b shows the probability distribution of the difference between T_a from ERA5 and IABP. This distribution has an approximately Gaussian shape, and 95% of the differences are in the range of -12.53 K to 2.23 K, with an overall average difference of -3.62 K.

The comparison of T_s from MODIS with that from IABP reveals overestimation of the former by a few degrees for $T_s < 245$ K and agreement in the range of 245–255 K for all ice thickness bins (Fig. 6c and Table 9). As IABP T_s is considered to be the ground-truth data, the discrepancy can be attributed either to the much coarser resolution of MODIS or the presence of metamorphosed snow, which is not accounted for in the MODIS TIT retrieval. The probability distribution of the difference between T_s from MODIS and that from IABP (Fig. 6d) is very close to a Gaussian distribution when the entire TIT range is considered (0–0.5 m). In total, 95% of the differences are in the range of -21.03 K to 3.13 K, with an overall bias of -9.04 K, but it is just -1.38 K for T_s in the range of 240–260 K. The wind speeds from ERA5 show agreement with the observations from the NWS data at low speeds ($u <$

Table 9. Bias of T_a (ERA5 minus IABP measurements) and T_s (MODIS minus IABP) for the different ice bins and temperature bins

Input variable	Ice bin/m	Bias for different temperature bins/K						Bias for all temperature bins/K	ρ
		240–245 K	245–250 K	250–255 K	255–260 K	260–265 K	265–270 K		
T_a	0–0.1	–	–2.08	–1.17	–1.96	–2.84	–3.59	–2.24	0.86
T_a	0.1–0.2	–	–1.28	–2.68	–2.64	–4.91	–4.23	–3.19	0.81
T_a	0.2–0.3	6.62	–0.95	–4.01	–3.39	–5.53	–4.59	–3.81	0.82
T_a	0.3–0.4	3.42	–2.25	–3.39	–4.92	–5.93	–4.98	–3.80	0.76
T_a	0.4–0.5	3.77	–3.13	–3.36	–5.41	–5.57	–4.45	–4.05	0.71
T_a	0–0.5	4.06	–2.11	–3.36	–3.87	–5.31	–4.44	–3.62	0.75
T_s	0.2–0.3	5.71	3.94	–0.50	–3.69	–	–	–0.57	0.47
T_s	0.3–0.4	5.43	–0.02	–0.62	–5.13	–	–	–1.67	0.61
T_s	0.4–0.5	3.16	–0.90	–0.59	–6.54	–	–	–3.78	0.62
T_s	0–0.5	4.72	0.89	0.08	–3.88	–	–	–1.38	0.53

Note: Correlation coefficients ρ of the data from the two sources are also shown in the last column. – indicate no data.

Table 10. Statistics of the difference between ERA5 u and the observed u for different ice bins and wind speed bins

Input variable	Ice bin/m	Bias for different wind speed bins/(m·s ^{–1})				Bias for all wind speed bins/(m·s ^{–1})	ρ
		0–5 m/s	5–10 m/s	10–15 m/s	15–20 m/s		
u	0–0.1	0.93	–1.10	–3.61	–5.12	–1.17	0.65
u	0.1–0.2	1.16	–0.30	–3.76	–4.74	0.07	0.68
u	0.2–0.3	0.99	–0.54	–2.67	–4.95	–0.15	0.74
u	0.3–0.4	0.81	–0.94	–3.13	–5.71	–0.41	0.72
u	0.4–0.5	0.65	–0.99	–4.48	–3.80	–0.35	0.69
u	0–0.5	0.94	–0.93	–3.54	–5.07	–0.75	0.68

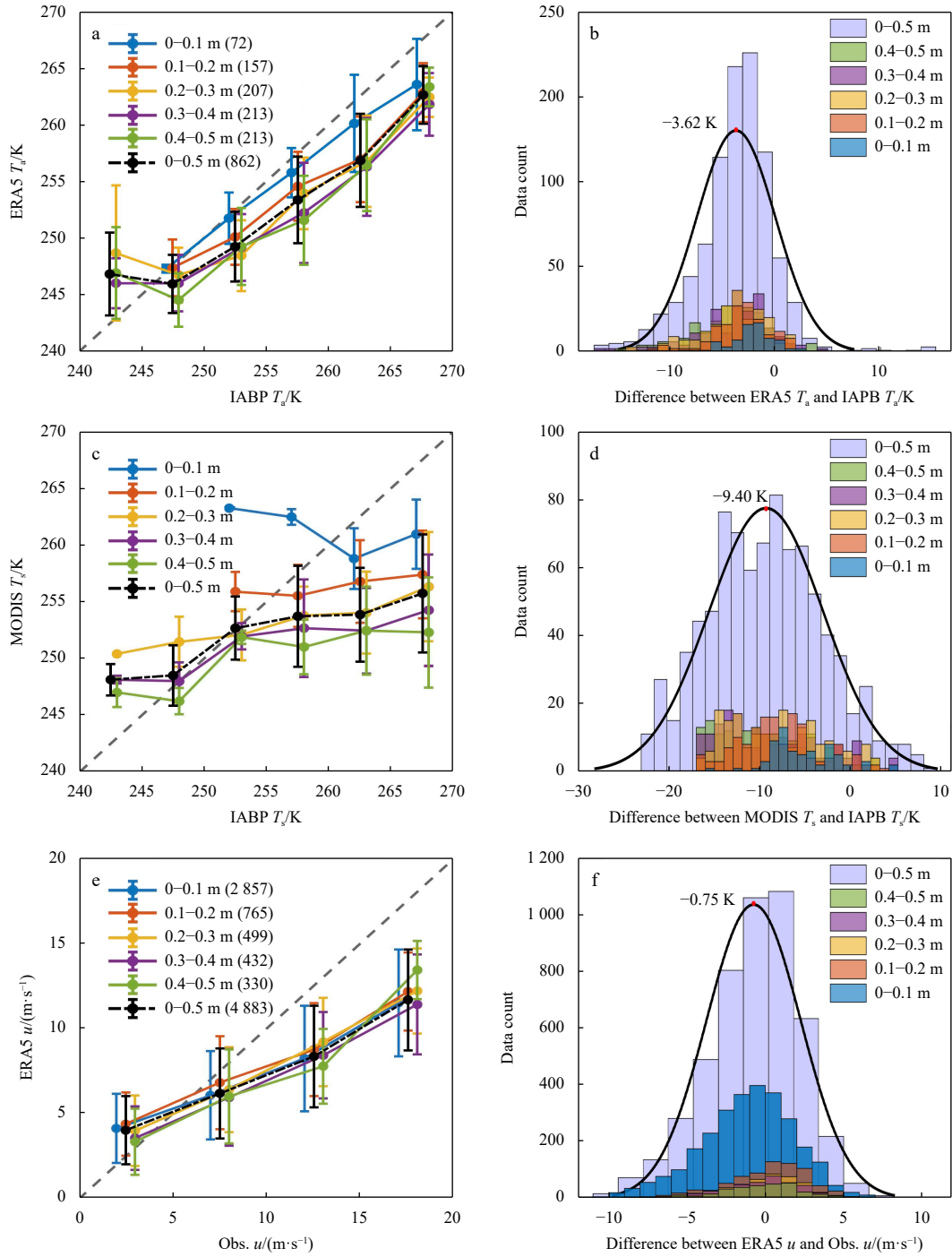


Fig. 6. Comparisons of ERA5 T_a vs. IAPB T_a (a), MODIS T_s vs. IAPB T_s (c), and ERA5 u vs. NWS-observed (Obs.) u (e), and their corresponding probability distributions of the differences, with fitted curves (black lines) (b, d, and f). The error bars represent one standard deviation. The numbers between brackets in a and e denote the data counts. a and c share the same data counts. The numbers in b, d, and f at the peak of each curve are the mean value of the differences. Dotted line in a indicates diagonal line.

7 m/s), but an increasing degree of underestimation beyond this limit (Fig. 6e and Table 10). All the ice bins share a similar pattern of relationship between the two datasets. The fitting curves of the distributions shown in Fig. 6 represent the uncertainty of the ERA5 T_a , and were used to generate the perturbations of T_a input to the model, and hence quantify the uncertainty of the TIT output due to the uncertainty of the three input parameters T_a , T_s , and u , as described in Section 3.2.3.

Since MODIS T_s deviates greatly from the IAPB data, a com-

parison between MODIS T_s with the ERA5 skin temperature was conducted. This will serve as another estimate of the uncertainty of T_s , which contributes to the uncertainty model's TIT output (Section 4.6). The results of comparing MODIS T_s with ERA5 T_s are given in Fig. 7. For the TIT range of 0–0.1 m, MODIS overestimates T_s , but agrees fairly well with the ERA5 skin temperature T_s beyond this thickness range. For all the ice bins and temperature bins, the overall bias between the MODIS and ERA5 T_s is 1.94 K, with a correlation coefficient of 0.89.

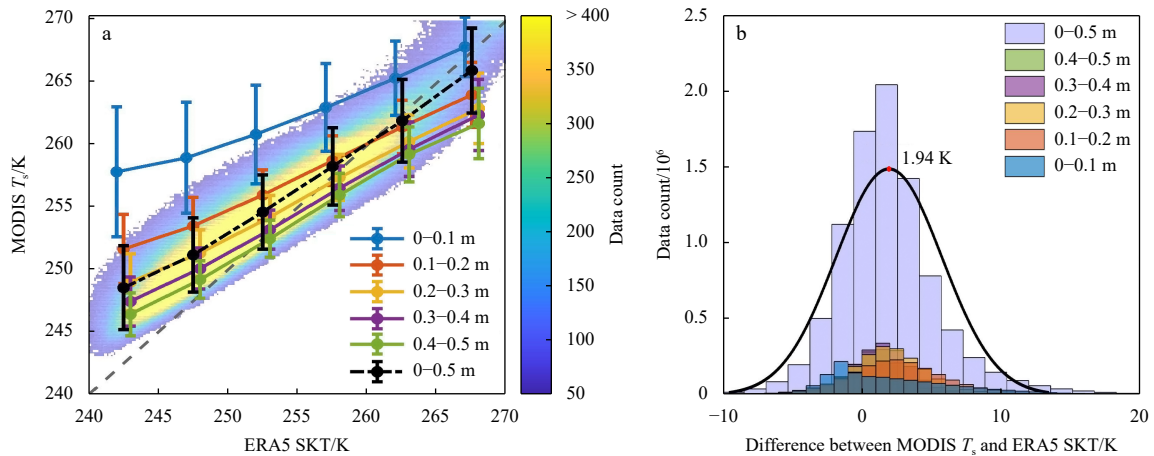


Fig. 7. Plot between T_s from MODIS and ERA5 (a), and histogram of the difference between the measurements (b). The shadow colors in a show the counts of data. The error bars represent one standard deviation. The black line in b is the fitted Gaussian curves, and the number at the peak is the mean difference. Dotted line in a indicates diagonal line.

In total, 95% of the differences are in the range of -4.64 K to 11.18 K (Fig. 7b). The small difference between MODIS T_s and ERA5 skin temperature can be attributed to their similar spatial resolution.

4.6 Uncertainty of the model's output in response to the uncertainty of the input variables

The uncertainty of the model TIT caused by the uncertainty in each individual input variable T_a , T_s , and u was estimated as described in Section 3.2.3. It is worth repeating that the perturbations that represent the uncertainty of each variable were generated according to the fitting distributions of the errors shown in Fig. 6. The results of the uncertainty of the model are given in Table 11 and Fig. 8, where the “bias” is calculated as the new TIT minus the original TIT.

The uncertainty in T_a causes thicker TIT for all the ice thickness bins, with an overall overestimation of 0.09 m, but this varies with the ice thickness bin. The smallest uncertainty is found for the $0-0.1$ m bin. The uncertainties are shown graphically in Fig. 8a. The overall uncertainty over the entire thickness range is 0.09 .

The uncertainty of the T_s measurements (with reference to the true values from the IABP T_s) causes larger TIT uncertainty, depicted as underestimation of TIT, as shown in Table 11 (third column) and Fig. 8b. On the other hand, if we take the ERA5 skin temperature as the true value, minor uncertainties for TIT are obtained (fourth column of Table 11 and Fig. 8d), but with large variance. The TIT uncertainty in this case (estimated for the entire TIT range) is 0.049 compared to -0.20 if the uncertainty of T_s

is based on the difference between the MODIS T_s and ERA5 skin temperature.

For both T_a and T_s , the uncertainty is minimum in the thickness range of $0-0.1$ m. For the uncertainty of T_s according to the IABP measurements, the uncertainty of the model's output increases with the ice thickness, and the opposite is noted for the uncertainty of T_s according to the ERA5 estimates. Figure 8c shows that the TIT uncertainty caused by the uncertainty in u is remarkably low, with an overall bias of -0.005 m for the $0-0.5$ m ice bin (fifth column of Table 11), along with small bias and standard deviation. The impact of u is the smallest in each ice bin.

5 Discussion

Wide spatial and temporal variability of the TIT of Arctic sea ice during winter has been reported (Bareiss and G6rgen, 2005; Preußer et al., 2019; Tamura and Ohshima, 2011; Yu and Rothrock, 1996). This has been the prime motivation to study the deviation, sensitivity, and uncertainty of modeled TIT using the traditional 1-D thermodynamic model, which is based on the surface energy balance (Maykut, 1982). In this study, 14 parameterization schemes for seven selected independent variables in the model were investigated. Most of these schemes were investigated for the first time.

Each parameterization scheme causes deviation of the output TIT, compared to using the default scheme, as can be seen in Table 7. The deviation defines how crucial the parameterization of the given parameter is within the scheme of the model. For the air emissivity schemes (T1 to T4), it is not surprising that T3 and T4 cause smaller deviations than T1 and T2, as the former two schemes share the same air emissivity equation as the default scheme. For T1 (air water vapor pressure dependent) and T2 (cloud dependent), the mean air emissivity difference with respect to the default emissivity is about 8.7% and 13.37% , respectively. This leads to a deviation of 12.49% and 19.25% for TIT (Table 7). Using the T1 scheme, TIT is overestimated by 5 cm with respect to using the default scheme. This is comparable to the value of 7 cm reported in Adams et al. (2013). For air density, the T5 scheme causes deviation of this parameter by 7.3% , compared to using the default value of 1.3 kg/m³, and this leads to underestimation of TIT by 3% . No appreciable deviation from using the default value is noticed for the specific heat of air scheme (T6) and the latent heat of sublimation scheme (T7). For the ice

Table 11. Uncertainty (bias) of TIT due to the uncertainty of the input variables of T_a , T_s , and u

Ice bin/m	Bias of TIT/m			
	T_a	T_s (vs. IABP)	T_s (vs. ERA5)	u
0–0.1	0.047	–0.027	0.068	0
0.1–0.2	0.117	–0.100	0.088	–0.002
0.2–0.3	0.127	–0.178	0.065	–0.006
0.3–0.4	0.104	–0.261	0.017	–0.009
0.4–0.5	0.064	–0.346	–0.045	–0.010
0–0.5	0.090	–0.200	0.049	–0.005

Note: Columns 2–5 correspond to the uncertainty of the input variables in Figs 6b, d, f, and 7b.

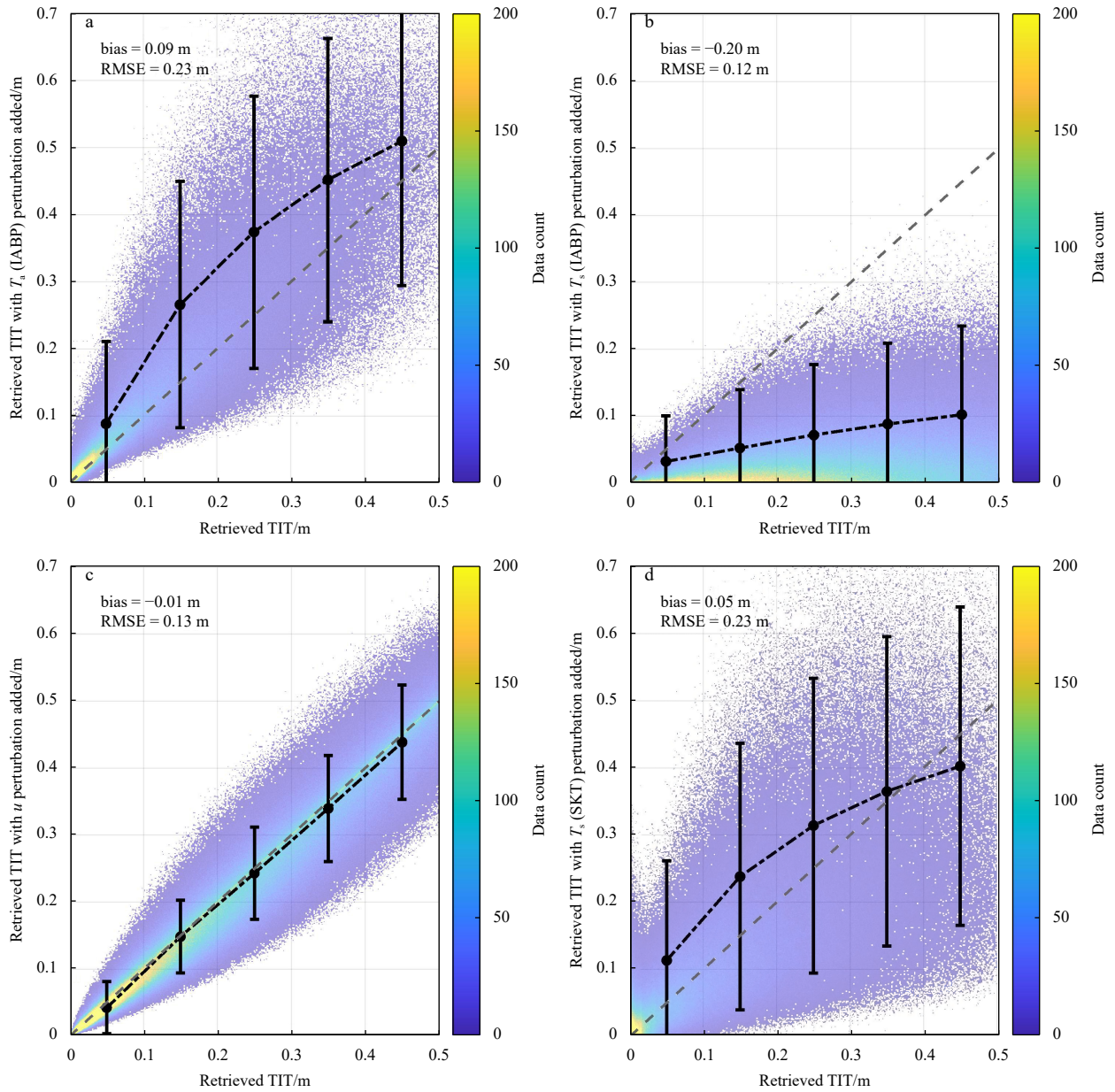


Fig. 8. Plots showing the retrieved TIT obtained using the original T_s , T_a , and u as inputs (values in the horizontal axis) and the retrieved TIT with perturbations added representing the error in measurements from T_a (with respect to IABP T_a) (a), T_s (with respect to IABP T_s) (b), u (with respect to the NWS/NDBC observations) (c), and T_s (with respect to ERA5 skin temperature) (d). The error bars represent one standard deviation. The shadow colors show the counts of the data points. Dotted lines indicate diagonal lines.

conductivity schemes (T8–T11), the deviation of the ice conductivity [with respect to the constant value of $2.03 \text{ W}/(\text{m}\cdot\text{K})$] is 4.65%, -6.67% , 4.27% , and 5.05% . These schemes produce deviation of TIT that is comparable to the results obtained using the default parameterization (Table 7).

Owing to its insulating properties, snow reduces the conductive heat flux through ice (F_c) and hinders its thermodynamic growth (Ledley, 1991; Maykut, 1978). Depending on the snow wetness and density (i.e., particle size), its conductivity can vary from $0.05 \text{ W}/(\text{m}\cdot\text{K})$ to $0.50 \text{ W}/(\text{m}\cdot\text{K})$ (Sturm et al., 2002). This wide range is likely responsible for the important role of the snow depth parameterization (T12 and T13), as well as the snow conductivity parameterization (T14). Numerically, the snow depth is parameterized as a piecewise linear function of ice thickness [Eq. (A16) for T12, and Eq. (A17) for T13], whose slopes increase

as the ice thickens, based on *in situ* observations (Doronin et al., 1970; Mäkinen et al., 2013). This leads to larger TIT deviation using the snow schemes (T12–T14), which is more noticeable for thicker ice. As a result, an increasing TIT deviation when using the snow scheme is observed as the ice thickens (Table 7). Moreover, the magnitude of the TIT deviation of the snow scheme depends on the slope of the snow function. As the ice thickens, the effect of thicker snow on the thermodynamic TIT retrieval increases (Sturm and Massom, 2009). The highest TIT deviation resulting from the snow conductivity scheme (T14) confirms its potentially serious implication for TIT retrieval, especially for ice thicker than 0.2 m (Table 7). *In situ* measurements have shown that the conductivity for new snow can be as low as 0.078 (Sturm et al., 2002). If this value is adopted, TIT will be underestimated from the model. Therefore, accurate estimation of

snow depth and composition, and hence conductivity, is extremely important for the accurate retrieval of TIT.

In this study, the thickness-dependent data of TIT deviation, compared to using the default scheme (Section 4.1), along with its sensitivity to thickness (Section 4.2), gives a new perspective that has not been presented in previous studies. It also serves to identify possible thickness-based patterns of the model's output. In general, no identifiable pattern of deviation is observed using any parameterization scheme, except for T9 (ice conductivity), T12 and T13 (snow depth), and T14 (snow conductivity). Using T9, the deviation compared to using the default scheme decreases as the thickness increases. The opposite is observed for T12–T14.

The model-retrieved TIT obtained using the default scheme and the different parameterization schemes (T1, T2, T5, T8, T9, T12 to T14) were validated against ULS measurements of ice draft (and hence thickness) (Section 4.3). This is an unprecedented validation method for model-retrieved thermodynamic TIT. By comparing the thermodynamic-driven TIT and ULS TIT during a polynya event in the St. Lawrence Island polynya in the Bering Sea, Drucker et al. (2003) showed that both datasets agreed for TIT in the 0–0.5 m range. In another study by Tamura et al. (2006), the authors showed that the average difference and standard deviation between the AVHRR-derived thermodynamic TIT (≤ 0.5 m) and *in situ* ice thickness was 3.3 cm and 2.0 cm, respectively. In the Bohai Sea, Zeng et al. (2016) showed that the bias and RMSE of the model-retrieved thermodynamic TIT (≤ 0.4 m) were -1.4 cm and 3.9 cm with respect to *in situ* TIT measurements.

In this article, we have presented more validation details for different ice bins (Fig. 4). However, it should be noted that the mechanical growth of ice thickness is neglected in the model but not in the ULS measurements. Mechanical growth can dominate as ice thickness increases under severe weather conditions with high winds and waves or at specific locations (von Albedyll et al., 2021). In order to account for this possibility, a correction scheme (linear model) is provided to adjust the model's output to the ULS measurements of TIT (Table 7). This supports the use of the model's TIT results based on the ERA5 meteorological data and MODIS ice surface temperature.

The validation result (Fig. 4) reveals the limitation of the 1-D model regardless of the parameterization scheme, which overestimates (underestimates) the model TIT when ice is thinner (thicker) than 0.25 m. An acceptable finding from using the 1-D thermodynamic model to retrieve TIT is that the retrieved TIT is underestimated when T_a decreases, and vice versa (Adams et al., 2013). On the other hand, the TIT is overestimated when T_s decreases. If we apply these findings and the uncertainties of the input variables in Section 4.5 to the data in Fig. 4, it can be said that the overestimation of TIT for thinner ice can be attributed to the relatively underestimated T_s (about 260 K to 270 K, Figs 6c and 7a) during the early growth phase of sea ice. Conversely, the underestimation of TIT for thicker ice can be attributed to the underestimated T_a associated with this thickness range (Fig. 6a).

As each scheme produces similar patterns in different thickness bins with respect to the ULS measurements (Fig. 4), it is difficult to generate an optimum set of parameterization schemes. Despite this, the combined scheme of T5 and T9 yields slightly better results with respect to various validation data, especially when compared with SMOS/SMAP TIT. The thickness-dependent data from these two datasets (model TIT and SMOS/SMAP TIT) are similar to the results in Huntemann et al. (2014), where thermodynamic TIT was retrieved based on the ice surface heat

balance equation, which uses parameterization schemes from MODIS T_s and HIRLAM atmospheric forcing (Mäkynen et al., 2013; Yu and Rothrock, 1996). In this study, the correlation coefficient r and RMSE were 0.68 and 11 cm for ice between 0 cm and 50 cm thick, respectively. These values correspond to 0.67 cm and 9.47 cm in the current study.

The uncertainties of the input variables of T_a (from ERA5), T_s (from MODIS), and u (from ERA5) have been summarized in this article. These uncertainties were used to estimate the resulting uncertainty in the model's output of TIT. For T_a , the overall average bias is -3.62 K, which is larger than the previous results of (1.76 ± 3.09) K (in November) and (2.89 ± 3.09) K (in March) when also compared with T_a from the IABP measurements, but over the entire ice thickness range (Yu et al., 2021). However, the worst bias (with respect to *in situ* measurements) is obtained for the thin-ice condition. For MODIS T_s , the overall bias with respect to IABP T_s is -9.04 K, which is larger than its nominal uncertainty of 1–3 K (Hall and Riggs, 2021), but is smaller (1.94 K) when compared to ERA5 T_s . The overall bias of ERA5 u over thin ice with respect to *in situ* measurements is -0.75 m/s. This is comparable with the ship-based measurements of about -0.88 m/s in the marginal ice zone (Renfrew et al., 2021).

The uncertainties of the ERA5 T_a , MODIS T_s , and ERA5 u lead to uncertainty of the model's output of TIT. As presented in Section 4.6, based on the original error (uncertainty) distributions of these three input variables, we found that the smallest TIT uncertainty is associated with the uncertainty of u , and the highest is associated with the uncertainty of T_a (based on IABP *in situ* data). Figure 6b shows that the uncertainty from T_a is 60% higher than that from T_s (Fig. 7b). Zeng et al. (2016) found that when $T_a - T_s \leq 0$ K, the 1-D model-retrieved TIT is underestimated by 1 cm to 2 cm with the increase of wind speed from 0 m/s to 12 m/s. Mäkynen et al. (2013) found that the TIT uncertainty is smallest under very cold low-breeze conditions. For the current study, 87.42% of the temperature pairs (i.e., the ice surface temperature and air temperature) are featured with $T_a - T_s \leq 0$ K, and 59.63% of the wind speed is less than 8 m/s, which may contribute to the smallest TIT uncertainty arising from u .

The sea ice surface heat balance method used in this study works under limited conditions, such as no cloud, nighttime, and cold weather conditions. The cloud contamination problem in the MODIS ice surface products has been well documented, and many previous studies have attempted to solve the problem (Paul and Huntemann, 2021; Paul et al., 2015a), but this was not considered in this study as the elimination of cloud is beyond the purpose of this article. The potential errors caused by the spatial or temporal interpolation of meteorological data were also not analyzed.

6 Conclusions

The accuracy of TIT retrieval based on the 1-D thermodynamic sea ice model (Maykut, 1982) is highly sensitive to the model parameterization and the accuracy of the input data. In this study, the deviation and sensitivity of the retrieved TIT (of up to 0.5 m) were studied for the seven independent variables of atmospheric emissivity (ϵ_a), air density (ρ_a), specific heat of air (C_p), latent heat of sublimation (L_s), ice conductivity (k_i), snow depth (h_s), and snow conductivity (k_s). We also estimated the uncertainty of the measured input variables, namely T_a (from ERA5), T_s (from MODIS), and u (from ERA5), and then proceeded to estimate the associated uncertainty of the output TIT. The findings are summarized in the following.

The deviation of the 14 parameterization schemes with re-

spect to the commonly used default scheme indicates the TIT variations caused by parameterization. The use of the atmospheric emissivity schemes of T1 and T2 can yield 12.49% to 19.25% thicker ice than the default scheme. The variation of the water vapor saturation equations [Eqs (A4)/(A5)/(A6) in the T2/T3/Def. scheme] has almost no effect on the TIT retrieval. Meanwhile, the default constant values of the specific heat of air and the latent heat of sublimation of ice do not produce results that are different from those obtained using their tested parameterization schemes, namely Eqs (A8) and (A9) in schemes T6 and T7, respectively. The use of air density generated from the ideal gas law [Eq. (A7) in scheme T5] underestimates TIT by 3% when compared to the commonly used constant air density value of 1.3 kg/m^3 . The parameterization of ice conductivity (k_{i_U64} , T8–T11) strikingly impacts the TIT retrieval, with a relatively moderate effect on thicker ice. The k_{i_U64} is a function of pure ice conductivity k_0 and sea ice salinity S_i . The equations of k_{0_N64} [Eq. (A11) in T8] and k_{0_C10} [Eq. (A13) in T10] result in almost no difference in the retrieved TIT, but the equation of k_{0_S78} [Eq. (A12) in T9] produces about 10% thinner ice. For the two parameterizations of S_i , S_{i_C74} [Eq. (A14) in T10] yields an average of 1.2% thinner ice than S_{i_J94} [Eq. (A15) in T11]. Snow is the most uncertain element embedded in thermodynamic TIT retrieval based on the 1-D model. The use of empirical equations for snow depth [Eqs (A16) and (A17)] underestimates TIT, especially as the ice thickens. As the ice thickens, the thinning effect due to the reduced snow conductivity is also accentuated.

The sensitivity of the retrieved TIT (based on each test scheme) to ice thickness ranges from 0% to –6.8%. It is relatively high for the parameterization schemes of ice conductivity (T8 to T11), snow conductivity (T12, T13), and snow density (T14), ranging from 3.64% to 6.80%. The remaining schemes for air density, the specific heat of air, and the latent heat of sublimation of ice show near-zero sensitivity.

When compared with ULS measurements, the model-retrieved TIT obtained using atmospheric emissivity ε_{a_JX06} [Eq. (A3), embedded in the default scheme] performs better than the other two equations [Eqs (A1) and (A2)], especially for ice of less than 0.25 m. On the other hand, the air density parameterization scheme ρ_{a_GL} (T5) and the ice conductivity scheme k_{i_U64} perform better than the default scheme for ice thicker than 0.25 m. The worst performance belongs to the three snow-related schemes, namely T12 to T14. The schemes of T5 and T9 show a better performance. When combined, they show slightly better TIT retrieval performance than all the test and default schemes. A modulation scheme (linear model) is also provided to adjust the model's output to ULS measurements of TIT.

By a comparison with IABP and NWS measurements, the error (uncertainty) of T_a and u from ERA5 was investigated. The overall average difference is –3.62 K and –0.75 m/s, respectively. A higher difference in T_a is found over thin ice compared to the difference over the entire ice cover. The errors of MODIS T_s with respect to the IABP T_s are large, but are smaller when compared with the ERA5 skin temperature. Based on the error distributions from T_a , T_s , and u , the retrieved TIT shows an overall TIT uncertainty of 0.09 m, 0.049 m, and –0.005 m, respectively.

The results of this study give a new detailed insight into the retrieved TIT obtained using the thermodynamic 1-D model in the Arctic during winter and provide simultaneous validation of the model-retrieved TIT using *in situ* measurements. These findings will facilitate the quantitative comparison of previous TIT retrieval results and the choice of parameterization equations in the 1-D model. In addition, these findings should also benefit the

construction and validation of the empirical relationships between thermodynamic TIT and brightness temperature from PMRs. Moreover, the uncertainty of the output TIT, triggered by the uncertainty of the input variables, should explain the wide range of results obtained using the model.

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Appendix:

In this appendix, we present the parameterization equations used for the following independent variables implied in Eqs (2)–(6) in the main text. Three sets of variables are parametrized. The first set includes the air emissivity ε_a used in the expression for atmospheric longwave radiation (F_{dn}) and the parametrized water vapor pressure (e_a) used in the calculation of ε_a , which is calculated using the saturation of vapor pressure E . The second set are the parameters used in calculating the sensible heat (F_s) and latent heat (F_e) components, namely air density, specific heat of air, and latent heat of sublimation of ice. The third set is made up of the three parameters used in obtaining the conductive heat flux, ice conductivity (k_i), snow conductivity (k_s), and snow depth (h_s). All these parameters are used in the deviation of the model's TIT output from the output using the default parameterization scheme, as well as the sensitivity of the model's output to ice thickness, as described in Section 3.2.1.

A1 Longwave radiation

A1.1 Air emissivity

The air emissivity ε_a is responsible for the accurate evaluation of F_{dn} when T_a is determined, which varies with clouds, water vapors, and aerosols (Garrett and Zhao, 2006). Previous studies used empirical estimates of ε_a in TIR, as given in Eq. (A1) (Efimova, 1961) used by Yu and Lindsay (2003), Eq. (A2) (König-Langlo and Augstein, 1994) used by Nihashi and Ohshima (2001), and Eq. (A3) (Jin et al., 2006) used by Adams et al. (2013).

$$\varepsilon_a = 0.746 + 0.006 \, 6e_a, \quad (\text{A1})$$

$$\varepsilon_a = 0.765 + 0.22C^3, \quad (\text{A2})$$

$$\varepsilon_a = [0.000 \, 3 \times (T_a - 273.16)^2 - 0.007 \, 9 (T_a - 273.16) + 1.298 \, 3](e_a/T_a)^{1/7}, \quad (\text{A3})$$

where e_a (hPa) is the air water vapor pressure, and C is the cloud cover, which ranges from 0 to 1. The air emissivity ε_{a_EF61} is more accurate in low humidity conditions which just take water vapor into account. ε_a in Eqs (A1), (A2), and (A3) are denoted as ε_{a_EF61} , ε_{a_KL94} , and ε_{a_JX06} , respectively. The function of ε_{a_KL94} is obtained from massive *in situ* observations and a variety of atmospheric emissivity models with the consideration of the cloud condition (König-Langlo and Augstein, 1994). The ε_{a_JX06} is determined from a physically rigorous equation of the atmospheric emissivity model and the Arctic rawinsonde data limited in the defined temperature range of 229–290 K.

A1.2 Water vapor pressure

The air water vapor pressure e_a in Eqs (A3) and (A5) over ice can be estimated based on the formula for saturation of vapor pressure E (hPa), i.e., $e_a = E(T_d)$ or $e_a = RH \cdot E(T_a)$, where T_d (K) is the dewpoint temperature, which can be obtained from the ERA5 reanalysis dataset, and RH (in ratio) is the relative humidity. To avoid introducing the relative humidity parameter, we adopt the former method (i.e., $e_a = E(T_d)$) to calculate the water vapor pressure e_a . The saturation of vapor pressure E , which is a function of temperature, was expressed in an early formula developed by Goff and Gratch (1945). Some simpler forms have since been developed, such as Eq. (A4) (Maykut, 1978) and Eq. (A5) (Alduchov and Eskridge, 1996).

$$E(T) = aT^4 + bT^3 + cT^2 + dT + f, \quad (\text{A4})$$

$$E(T) = 6.112 \, 1 \exp[22.587(T - 273.15)/(T + 0.71)], \quad (\text{A5})$$

where T is the temperature in unit of K. For Eq. (A4), the constant coefficients a , b , c , d , and f are $2.779 \, 820 \, 2 \times 10^{-6}$, $-2.691 \, 339 \, 3 \times 10^{-3}$, $0.979 \, 208 \, 49$, $-158.637 \, 79$, and $9 \, 653.192 \, 5$, respectively. Equation (A5) is suited for the $223.15 \, \text{K} \leq T \leq 273.15 \, \text{K}$ condition, which is documented with a higher accuracy than E_{M78} (Alduchov and Eskridge, 1996). $E(t)$ in Eqs (A4) and (A5) are denoted as E_{M78} and E_{AE96} , respectively. The departure of the ideal gas law for a mixture of air and water vapor leads to errors that can be eliminated by an enhancement or correction factor (Alduchov and Eskridge, 1996), and the surface pressure implied enhancement for Eq. (A5) is presented in Eq. (A6), where p_0 (hPa) is the sea-level pressure.

$$E = 0.998 \, 82 \exp(8 \times 10^{-6} p_0) \times 6.112 \, 1 \exp[22.587(T - 273.15)/(T + 0.71)], \quad (\text{A6})$$

E in Eq. (A6) is denoted as E'_{AE96} . The air emissivity ε_{a_EF61} , ε_{a_KL94} , and ε_{a_JX06} , together with the saturation vapor pressure E_{M78} , E_{AE96} , and E'_{AE96} , were used in the parameterization schemes (Table 2) to perform the sensitivity analysis.

A2 Turbulent and latent heat fluxes

A2.1 Air density

Most studies adopt a constant air density of $\rho_a = 1.3 \, \text{kg/m}^3$ under standard atmospheric pressure. However, in reality, air density

varies with air pressure and temperature, and the air over thin ice should be treated as wet air, which can be solved by the ideal gas law, as given in Eq. (A7).

$$\rho_{a_GL} = \frac{100 \times p_0}{R_d T_v}, \quad (A7)$$

where p_0 (hPa) is the surface atmospheric pressure, $R_d = 287.1$ J/(kg·K) is the dry gas constant, and $T_v = (1 + 0.608q_a)T_a$ is the surface air virtual temperature in unit of K.

A2.2 Specific heat of air and latent heat of sublimation

Previous studies have adopted the specific heat of air $c_p = 1\,004$ J/(kg·K) for F_e calculation. For wet air conditions, c_p is estimated as a function of the air-specific humidity q_a , as shown in Eq. (A8). The latent heat of sublimation L_s is often assigned with a constant value of 2.5×10^6 J/kg, which varies with the surface temperature and can be estimated by Eq. (A9).

$$c_p = 1\,004(1 + 0.86q_a), \quad (A8)$$

$$L_s = 2.5 \times 10^6 - 2\,323(T_s - 273.15). \quad (A9)$$

c_p in Eq. (A8) is denoted as c_{pw} and L_s in Eq. (A9) is denoted as L_{sw} .

A3 Conductive heat flux

A3.1 Ice conductivity

Some studies have used a constant ice conductivity value of 2.03 W/(m·K) to conduct TRI-TIT retrieval (Adams et al., 2013; Drucker et al., 2003). The ice conductivity property is actually affected by the entrapped liquid brine, which changes rapidly during the thin ice growth, so the conductivity of ice k_i can be approximately estimated by the ice salinity and temperature, as given in Eq. (A10) (Untersteiner, 1964):

$$k_i = k_0 + \beta S_i / (T_i - 273.15), \quad (A10)$$

where β [W/(m²·kg)] is a constant value of 0.117 W/(m²·kg) (Maykut, 1978) or 0.13 W/(m²·kg) (Yu and Lindsay, 2003; Yu and Rothrock, 1996). k_i in Eq. (A10) is denoted as k_{i_U64} . We adopted the latter value in this study. T (K) is the ice internal temperature, which we replaced with the ice surface temperature T_s with a 5% potential error, as documented by Yu and Rothrock (1996). The pure ice conductivity k_0 [W/(m·K)] is often treated as a constant of 2.03 W/(m·K) (Maykut, 1978) or 2.034 W/(m·K) (Yu and Rothrock, 1996), which is actually a function of sea ice temperature. In this study, we also treated it as a function of the ice surface temperature T_i , which is given in Eq. (A11) (Nazintsev, 1964) used by Wang et al. (2010), or Eq. (A12), which was developed from laboratory experiment data (Sakatume and Seki, 1978), or Eq. (A13) (Cuffey and Paterson, 2010). The latter two have never been used in TIR-TIT retrieval, so we used them to test the relative accuracy of the former k_{0_N64} .

$$k_0 = 2.22[1 - 0.001\,59(T_i - 273.15)], \quad (A11)$$

$$k_0 = 1.905 - 0.008\,6(T_i - 273.15) + 2.9 \times 10^{-5}(T_i - 273.15)^2, \quad (A12)$$

$$k_0 = 9.828 \exp(-0.005\,7T_i). \quad (A13)$$

k_0 in Eqs (A11), (A12), and (A13) are denoted as k_{0_N64} , k_{0_S78} , and k_{0_C10} , respectively.

The mean sea ice salinity S_i can be empirically estimated as a function of ice thickness, as given in Eq. (A14) (Cox and Weeks, 1974), based on observations of the Arctic sea ice used by Yu and Rothrock (1996) or Eq. (A15) (Jin et al., 1994). We used these two formulas to test the influence of S_i on TIR-TIT retrieval in this study.

$$S_i = \begin{cases} 14.24 - 19.39h_i, & h_i \leq 0.4 \text{ m}, \\ 7.88 - 1.59h_i, & h_i > 0.4 \text{ m}, \end{cases} \quad (A14)$$

$$S_i = \begin{cases} 17 - 31.63h_i, & h_i \leq 0.3 \text{ m}, \\ 8 - 1.63h_i, & h_i > 0.3 \text{ m}. \end{cases} \quad (A15)$$

S_i in Eqs (A14) and (A15) are denoted as S_{i_C74} and S_{i_J94} , respectively.

A3.2 Snow conductivity

The frequently adopted value of the snow thermal conductivity k_s for TIR-TIT retrieval is 0.33 W/(m·K) (denoted as k_{s1}), which is solved for the inferred average value of snow thermal conductivity with a snow depth deeper than 0.2 m (Sturm et al., 2002). According to Eq. (A17), the hypothesized snow depth over thin ice (<0.5 m) is up to 0.045 m for newly deposited fresh snow, and it is reasonable to assume a lower conductivity value of about 0.21 W/(m·K) (Sturm et al., 2002). The influence of a lower constant value of $k_s = 0.21$ W/(m·K) (denoted as k_{s2}) was tested in this study.

A3.3 Snow depth

Snow is an integral element of the polar sea ice systems, and is one of the variables that controls the heat and energy balance between the ice, ocean, and atmosphere. The snow layer on top of the Arctic sea ice is an important component of the climate system. In winter, the insulating effect of snow slows down the rate of ice growth. Snow depth has been related empirically to ice thickness, as given in Eq. (A16) (Doronin et al., 1970). This equation is widely used for TIR-TIT retrieval (Yu and Lindsay, 2003; Yu and Rothrock, 1996), and was modulated and improved by Mäkynen and Similä (2019) as shown in Eq. (A17) by using *in situ* observations from the Soviet Union's airborne Sever expeditions. The departure caused by using different h_s parameter schemes was analyzed in this study.

$$h_s = \begin{cases} 0, & h_i < 0.05 \text{ m}, \\ 0.05h_i, & 0.05 \text{ m} \leq h_i \leq 0.2 \text{ m}, \\ 0.1h_i, & 0.2 \text{ m} < h_i, \end{cases} \quad (\text{A16})$$

$$h_s = \begin{cases} 0, & h_i < 0.05 \text{ m}, \\ 0.05h_i, & 0.05 \text{ m} \leq h_i \leq 0.1 \text{ m}, \\ 0.13h_i - 0.008, & 0.1 \text{ m} < h_i \leq 0.2 \text{ m}, \\ 0.09h_i, & 0.2 \text{ m} < h_i. \end{cases} \quad (\text{A17})$$

h_s in Eqs (A16) and (A17) are denoted as h_{s_D71} and h_{s_M19} , respectively.

Table A1. List of symbols

Symbol	Definition	Unit	Symbol	Definition	Unit
F_{dn}	atmospheric downward longwave radiation	W/m ²	ρ_a	air density	kg/m ³
F_{up}	surface-emitted longwave radiation	W/m ²	c_p	specific heat of air	J/(kg·K)
F_s	sensible heat flux	W/m ²	L_s	latent heat of sublimation	J/kg
F_e	latent heat flux	W/m ²	u	wind speed	m/s
F_c	conductive heat flux	W/m ²	C_s	sensible transfer coefficient	–
ε_a	longwave emissivity of the atmosphere	–	C_e	latent heat transfer coefficient	–
ε_s	longwave emissivity of the surface	–	q_a	specific air humidity	–
T_a	atmosphere surface temperature	K	q_0	specific surface humidity	–
T_s	ice surface temperature	K	λ	thermal conductance of the ice-snow slab	–
e	water vapor pressure	hPa	k_i	conductivity of ice	W/(m·K)
C	total cloud cover	–	k_0	conductivity of pure ice	W/(m·K)
E	saturation vapor pressure	hPa	k_s	conductivity of snow	W/(m·K)
T	temperature in Kelvin	K	S_i	sea ice salinity	–
p_0	sea-level air pressure	hPa	h_i	thickness of ice	m
T_d	dewpoint temperature	K	h_s	thickness of snow	m
RH	relative humidity	%	T_f	freezing temperature of sea water	K

Note: – denotes no unit.