

Numerical investigation of solitary wave run-up attenuation by patchy vegetation

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Abstract

Coastal vegetation is capable of decreasing wave run-up. However, because of regrowth, decay or man-made damage, coastal vegetation is always distributed in patches, and its internal distribution is often non-uniform. This study investigates the effects of patchy vegetation on solitary wave run-up by using a numerical simulation. A numerical model based on fully nonlinear Boussinesq equations is established to simulate the wave propagation on a slope with patchy vegetation. By using the model, the process of solitary wave run-up attenuation due to patchy vegetation is numerically analysed. The numerical results reveal that patchy vegetation can considerably attenuate the wave run-up in an effective manner. In addition, high-density patched vegetation can attenuate the solitary wave run-up more effectively than low-density patched vegetation can. For the same density, patchy vegetation with a uniform distribution has a better attenuation effect on wave run-up compared to that of patchy vegetation with a non-uniform distribution.

Key words: solitary wave, run-up, patchy vegetation, Boussinesq equation

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1 Introduction

Tsunami run-ups are associated with massive destructive power and can cause extensive damage to life and property. In 2004, the 9.0 magnitude earthquake that occurred in Sumatra initiated a tsunami which killed over 283 000 people and incurred enormous financial losses (Tanaka, 2009). In 2011, a 9.0 magnitude earthquake occurred in the north of Japan and generated the third-largest tsunami in the past ten years; the tsunami submerged 400 square kilometres of land and killed over 20 000 people (Mori et al., 2011). Hence, it is necessary to determine effective methods to decrease tsunami run-up. A traditional and direct method to protect coastal regions from wave run-up is the construction of hard structures, such as seawalls and breakwaters; however, this approach is expensive and harmful to the environment. Therefore, research regarding the mitigation of damage due to tsunami run-ups by using structures with low environmental impact has become quite prevalent. Coastal vegetation can realize reasonable wave run-up dissipation with less environmental disruption than that associated with artificial engineering (Zhang et al., 2010, 2013; Duarte et al., 2013; Temmerman et al., 2013). However, in nature, coastal vegetation is rarely distributed uniformly and continuously; instead, it tends to be distributed in patches owing to regrowth, decay and man-made damage. Tsunamis are always simulated by a solitary wave; thus, it is necessary to investigate the attenuation of a solitary wave run-up by patchy vegetation to analyse the influence of patchy vegetation on the tsunami run-up.

In recent years, several researchers have investigated the ef-

fect of vegetation on wave run-up attenuation. Irtem et al. (2009) found that wave run-up was reduced by approximately 45% on a vegetated slope compared to that on a landscape without trees. Tang et al. (2013) investigated the sensitivity of solitary wave run-up to plant height, diameter and stem density by comparing the numerical results pertaining to different patterns of vegetation. Tang et al. (2017) studied the effects of vegetation on long-period wave run-up via a numerical simulation by using the nonlinear shallow water equations, and the results indicated that the attenuation of long-period wave run-up due to vegetation is sensitive to the variation of the incident wave period; in addition, the attenuation of wave run-up did not increase or decrease monotonically with the incident wave period. Tsai et al. (2017) studied the damping effect of solitary wave propagation on moving emergent vegetation by using a numerical model based on the general Reynolds-averaged Navier-Stokes equations; the results indicated that the effect of moving vegetation on the damping of the wave height was notably less than the corresponding effect of stationary vegetation. Yao et al. (2018), on the basis of a modified Boussinesq equation, investigated the effects of beach slope, forest density and tree distribution on the solitary wave drag coefficient. Thuy et al. (2018) proposed a numerical model based on 1D nonlinear longwave equations, and the model was applied to investigate the effects of forest width, tree density, incident tsunami period and height on the height of wave run-up. The existing have focused mainly on the effect of continuous and uniform vegetation on solitary run-up. In nature, coastal vegetation is always distributed in patches due to regrowth, decay or man-

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made damage, and its internal distribution is often non-uniform (Maza et al., 2016); however, only a few studies have focused on patchy vegetation. Fonseca and Koehl (2006) and Vandenbruwaene et al. (2011) studied the effects of patch sizes and inter-patch distance on currents. Irish et al. (2014) evaluated the reduction in the intensity of a tsunami via patchy vegetation by conducting a series of experiments. Maza et al. (2016) proposed a new parameter termed the equivalent length, which can take the effect of patchiness of vegetation into analysis. Using this parameter, the authors investigated the effect of the submergence ratio and the field characteristic of circular vegetation patches on wave attenuation. Yang et al. (2017) investigated the behaviour of a tsunami when coming ashore through patchy vegetation and the variation of this behaviour with the roughness level, patch spacing and patch size. Zainali et al. (2018) performed numerical simulations of long waves interacting with arrays of emergent cylinders inside regularly spaced patches. The abovementioned studies revealed the effects of uniform or patched vegetation on the attenuation of solitary wave run-up. However, the effects of density and internal distribution of vegetation patches on solitary wave run-up remain yet unclarified.

In this study, the effect of patchy vegetation on the attenuation of solitary wave run-up was investigated by using a wave-vegetation numerical model based on COULWAVE (which is based on nonlinear Boussinesq equations). The paper is organized as follows. Section 2 presents the governing equations for wave propagation in vegetation zones, which are based on the Boussinesq model. Section 3 describes the analysis of the accuracy of the numerical model by comparing the numerical results with the laboratory data pertaining to wave propagation with continuous vegetation on a flat bottom surface and patched vegetation on a slope; this section also describes the investigation pertaining to the sensitivity of solitary wave run-up to the distribution of patchy vegetation. Section 4 presents the conclusions derived from this study and the limitations.

2 Governing equation

Boussinesq equations have been widely used for modelling coastal waves (Liu and Fang, 2016, 2019; Liu et al., 2018; Yang et al., 2018). The governing equations of COULWAVE are high-order nonlinear Boussinesq equations. In this study, the governing equations based on COULWAVE, including the vegetation term can be described as follows (Lynett et al., 2002; Kim et al., 2009):

$$\frac{\partial H}{\partial t} + \frac{\partial HU_\alpha}{\partial x} + \frac{\partial HV_\alpha}{\partial y} + D_c = 0, \quad (1)$$

$$\frac{\partial HU_\alpha}{\partial t} + \frac{\partial HU_\alpha^2}{\partial x} + \frac{\partial HU_\alpha V_\alpha}{\partial y} + gH \frac{\partial \zeta}{\partial x} + gHD_x + U_\alpha D_c + f_{xveg}H = 0, \quad (2)$$

$$\frac{\partial HV_\alpha}{\partial t} + \frac{\partial HU_\alpha V_\alpha}{\partial x} + \frac{\partial HV_\alpha^2}{\partial y} + gH \frac{\partial \zeta}{\partial y} + gHD_y + V_\alpha D_c + f_{yveg}H = 0, \quad (3)$$

where ζ is the wave elevation; $H=\zeta+h$ is the total water depth, where h is the still water depth; g is the gravity acceleration; U_α and V_α respectively represent the horizontal velocity in the x and y directions at any horizontal plane; D_x and D_y respectively refer to the second-order term of the horizontal momentum equation in the x and y directions of average depth; and f_{veg} is the force acting on the vegetation.

In this study, the plants are represented by cylinders, and the Morison equation is adopted to indicate the force acting on the vegetation. The inertial term is ignored, as the diameter of vegetation is considerably small compared to the wavelength. The force acting on vegetation can be expressed as

$$f_{veg} = \frac{1}{2} C_d N_v b_v \vec{u}_p \left| \vec{u}_p \right|, \quad (4)$$

where C_d is the drag coefficient, N_v is the vegetation density, b_v is the vegetation diameter, and $\vec{u}_p$ is the horizontal velocity in the vegetation region. The relationship between $\vec{u}_p$ and $\vec{u}$ can be expressed as $\vec{u}_p = \vec{u} / (1 - \phi)$, where ϕ is the vegetation volume fraction, $\phi = V_s / V$, V_s is the vegetation volume and V is the control volume.

In the governing equation, the vegetation drag coefficient is the most important parameter for simulating the vegetation drag force, and its magnitude depends on the characteristics of the fluid and vegetation. In this study, the equation derived by Kobayashi et al. (1997) is employed:

$$C_d = 1.8 \zeta R_d^{-3/50} [1 + 0.45 \ln(1 + 100\phi)] \times (0.8 + 0.2F - 0.15F^2), \quad (5)$$

where $R_d = |u_c| b_v / \gamma$, $F = |u_c| / (gd)^{1/2}$, $\gamma = 1.01 \times 10^{-6} \text{ m}^2/\text{s}$ is the hydrodynamic viscosity, $\zeta = 0.8$, u_c is the maximal velocity in the vegetation region (Stone and Shen, 2002), R_d is the Reynolds number and F is the Froude number.

When simulating wave propagation, the existence of offshore structures and the change in submarine topographies can influence the process of wave propagation, and some of these aspects can even cause wave breaking. Hence, it is vital to simulate the bottom fraction, wave breaking and boundary conditions accurately. The wave breaking term and bottom fraction term were introduced in the momentum equation:

$$\frac{\partial u}{\partial t} + \dots + R_f - R_b = 0, \quad (6)$$

where R_f is the bottom fraction term, and R_b is the wave breaking term.

For the bottom fraction term, R_f can be expressed as:

$$R_f = \frac{f}{H} \vec{u}_b \left| \vec{u}_b \right|, \quad (7)$$

where f is the bottom fraction coefficient, H is the total water depth and $\vec{u}_b$ is the bottom horizontal water velocity. The bottom fraction coefficient f is related to Chezy's coefficient, $f = g/C^2$; when the seabed is rough, $C < 10$, and when the seabed is typical, $20 < C < 60$.

For the wave breaking term, R_b can be expressed as:

$$R_b = R_{bx}i + R_{by}j, \quad (8)$$

where

$$R_{bx} = \frac{1}{H} \left\{ [v(Hu_1)_x]_x + \frac{1}{2} [v(Hu_1)_y + v(Hv_1)_x]_y \right\}, \quad (9)$$

$$R_{by} = \frac{1}{H} \left\{ \left[v(Hu_1)_y \right]_y + \frac{1}{2} \left[v(Hu_1)_x + v(Hu_1)_y \right]_x \right\}, \quad (10)$$

where ν is the motion viscosity coefficient, and $\nu = BH\zeta_r$. The value of variable B can be determined as follows:

$$\begin{aligned} B &= \delta, & 2\zeta_t^b &\leq \zeta_t, \\ B &= \delta (\zeta_t / \zeta_t^b - 1), & \zeta_t^b &\leq \zeta_t \leq 2\zeta_t^b, \\ B &= 0, & \zeta_t &\leq \zeta_t^b, \end{aligned}$$

where δ is an amplification factor, and ζ_t is defined as:

$$\begin{aligned} \zeta_t^b &= \zeta_t^{(F)}, & T^b &\leq t - t_0, \\ \zeta_t^b &= \zeta_t^{(I)} + \frac{t - t_0}{T^b} (\zeta_t^{(F)} - \zeta_t^{(I)}), & 0 &\leq t - t_0 \leq T^b, \end{aligned}$$

where $\zeta_t^{(I)}$ is the initial displacement when wave breaking occurs, $\zeta_t^{(F)}$ is the minimum displacement to maintain wave breaking, t_0 is the time when wave breaking is initiated, and T^b is the instantaneous time. In terms of contrast with the laboratory data (Hansen and Svendsen, 1979), the parameters above can be verified as:

$$\delta = 6.5, \quad \zeta_t^{(I)} = 0.65\sqrt{gH}, \quad \zeta_t^{(F)} = 0.08\sqrt{gH} \quad \text{and} \quad T^b = 8.0\sqrt{H/g}.$$

3 Numerical results and analysis

First, the accuracy of the model is validated by simulating a solitary wave run-up on patchy vegetation on flat bottom and continuous vegetation on slope. Second, the model is used to study the effect of patchy vegetation on wave run-up on a slope by performing a simulation of wave propagation in the presence of different internal distributions.

3.1 Validation of the numerical model

The numerical model is validated in considering the laboratory data for solitary wave propagation with patchy vegetation, which were obtained by Maza et al. (2016), and the data for solitary wave propagation with emergent continuous vegetation on a slope bottom, which were obtained by Yao et al. (2015).

Maza et al. (2016) performed experiments in a wave basin with a width, length, and depth of 8.6 m, 24.1 m and 1.0 m, respectively. The three different cylinder configurations con-

sidered in these experiments were built using wood cylinders, and their locations and distributions are shown in Fig. 1 and Fig. 2. The platform was located at the centre of the wave basin, 4.3 m from the vertical walls. The cylinders had a diameter of 3 cm and height of 50 cm, and they were uniformly distributed with a separation of 9 cm between centres. The experimental results for C2 were used to test the accuracy of the numerical model for the simulation of solitary wave propagation in the presence of patchy vegetation on a flat bottom. The numerical model was performed for a region with a length and width of 36 m and 8.6 m, respectively; a uniform spatial grid system was employed in which the spatial steps $\Delta x = \Delta y = 0.04$ m, and the total simulation time was 20 s, with time steps $\Delta t = 0.008$ s. The simulation water depth was set as 0.3 m. The solitary wave was generated at a distance of 6 m from the left side, and the incident wave height was 0.1 m. Sponge layers were set on both the left and right sides. The results of the comparison between the model and the experimental data are shown in Fig. 3. This figure shows that the predictions of the model are in agreement with the experimental data, and the model can satisfactorily simulate the wave propagation corresponding to patchy vegetation on a flat bottom.

Yao et al. (2015) conducted experiments in a wave basin with a width, length, and depth of 0.5 m, 32 m and 0.6 m, respectively. The rigid vegetation was simulated by using PVC cylinders with a diameter of 0.01 m. The length of the emergent vegetation was 0.5 m. Three different distributions were considered. In this study, a comparison was performed with case A, for which the volume portion was 0.175. The distribution for case A and experimental setup are shown in Fig. 4. The numerical model was run in a region with a length of 20 m and width of 0.5 m with a uniform spatial grid system, for which the spatial steps $\Delta x = \Delta y = 0.0199$ m, and the total simulation time was 25 s, with time steps $\Delta t = 0.016$ s. The simulation water depth was set as 0.15 m. The solitary wave was generated at 6 m from the left side, and the incident wave height was 0.038 m. The results of the comparison between the measured results and simulation results are shown in Fig. 5. The figures indicate that the predictions of the model are in agreement with the experimental data, and that the proposed model is valid for simulating wave propagation on a slope with continuous vegetation.

3.2 Numerical investigation of the effect of patchy vegetation on wave run-up on slope

After the validation of the present numerical model, the model was used to perform several numerical tests to investigate the effects of patchy vegetation on solitary wave run-up on a slope. In

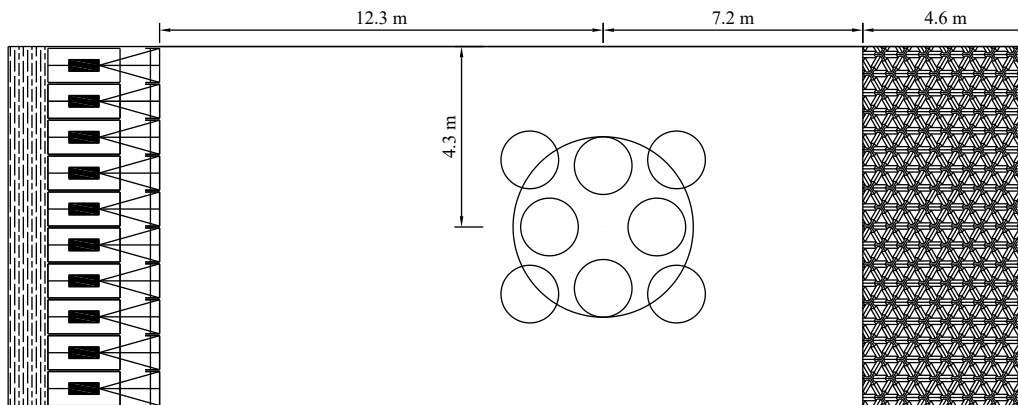


Fig. 1. Wave basin and locations of the three-cylinder configurations.

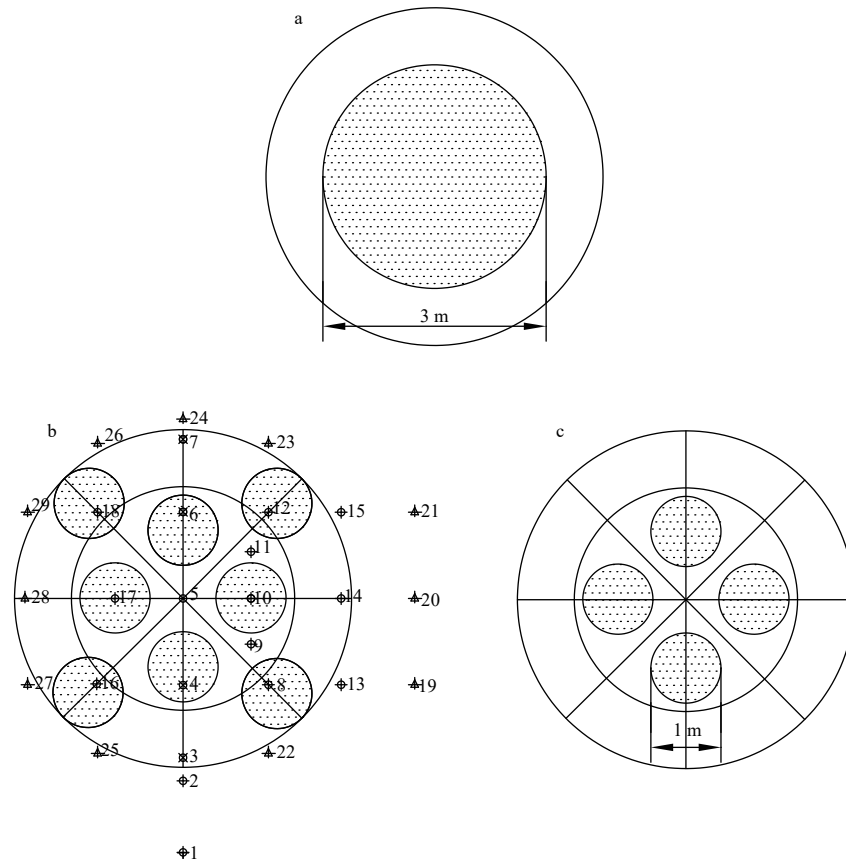


Fig. 2. Distribution of the three different cylinders in experiments conducted by Maza et al. (2016). a. C1 consisted of 880 cylinders disposed in a single circular shape with a diameter of 3 m, b. C2 was composed of 8 circular cylindrical patches with a diameter of 1 m each. Each patch was made of 112 cylinders, leading to the total number of cylinders being 896, c. C3 was formed by the four inner patches of the second configuration.

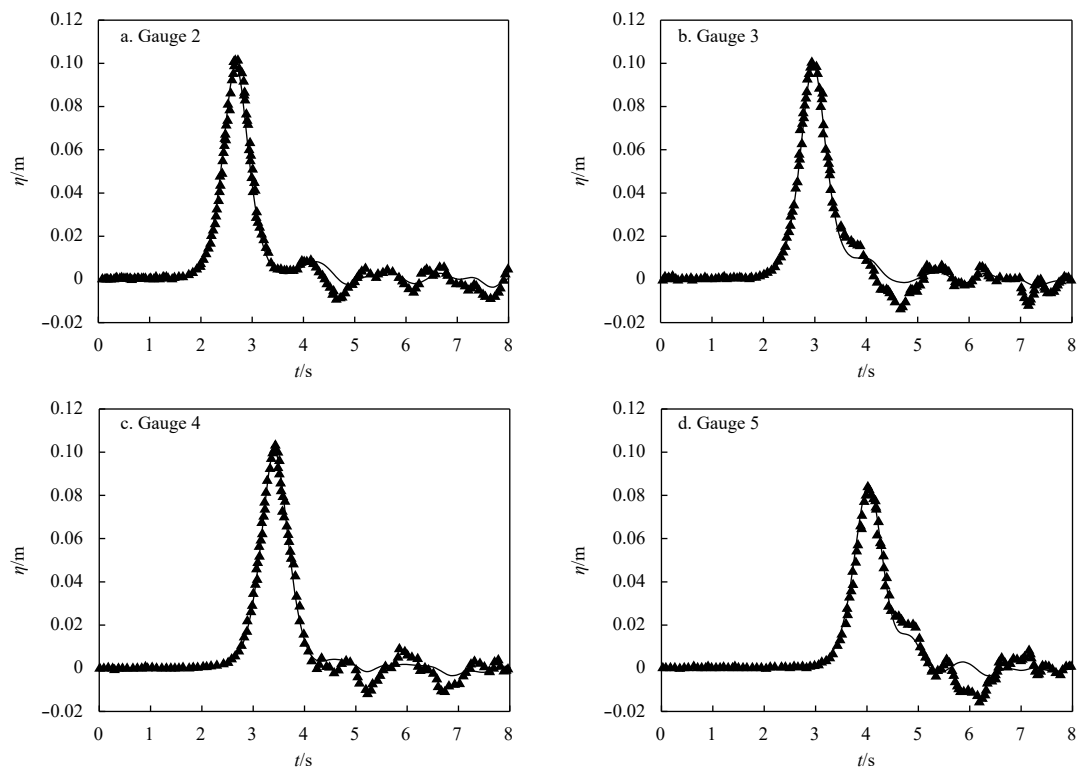


Fig. 3.

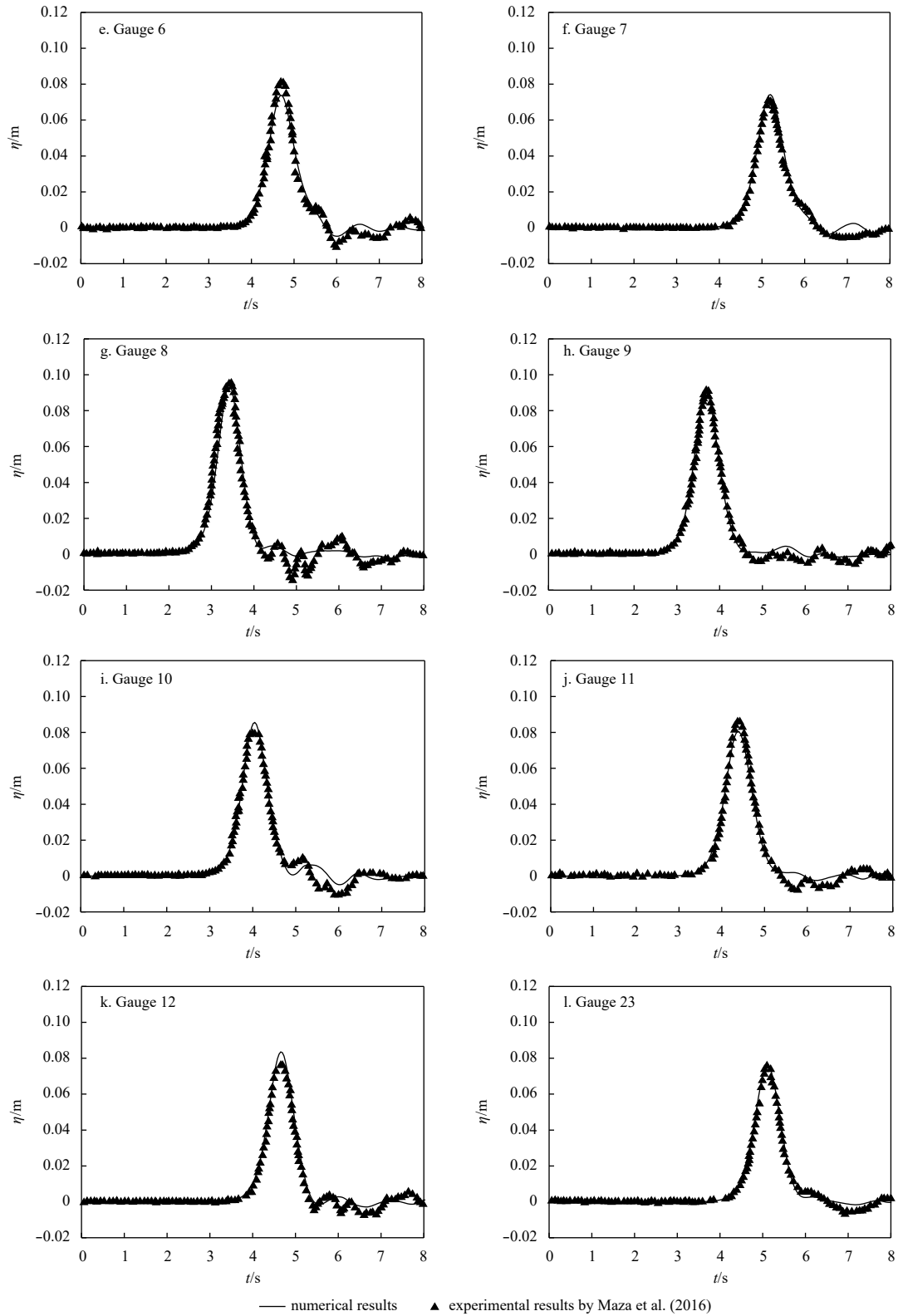


Fig. 3. Comparison between the relative surface of numerical results and laboratory data obtained by Maza et al. (2016) for solitary wave propagation with patchy vegetation for C2.

the tests, the numerical region is 30 m long and 3 m wide, and the detailed arrangement is shown in Fig. 6. The still water depth is 0.15 m, and the incident solitary wave height is 0.038 m. The wave maker is set at 6 m from the left side in the x direction, and a sponge layer is set on the left side.

The numerical model was run on a uniform spatial grid system with spatial steps $\Delta x = \Delta y = 0.049$ m and time step $\Delta t = 0.01$ s. Six internal distributions for the vegetation patch were considered, specifically, non-vegetation, uniform low density, uniform medium density, uniform high density and non-uniform

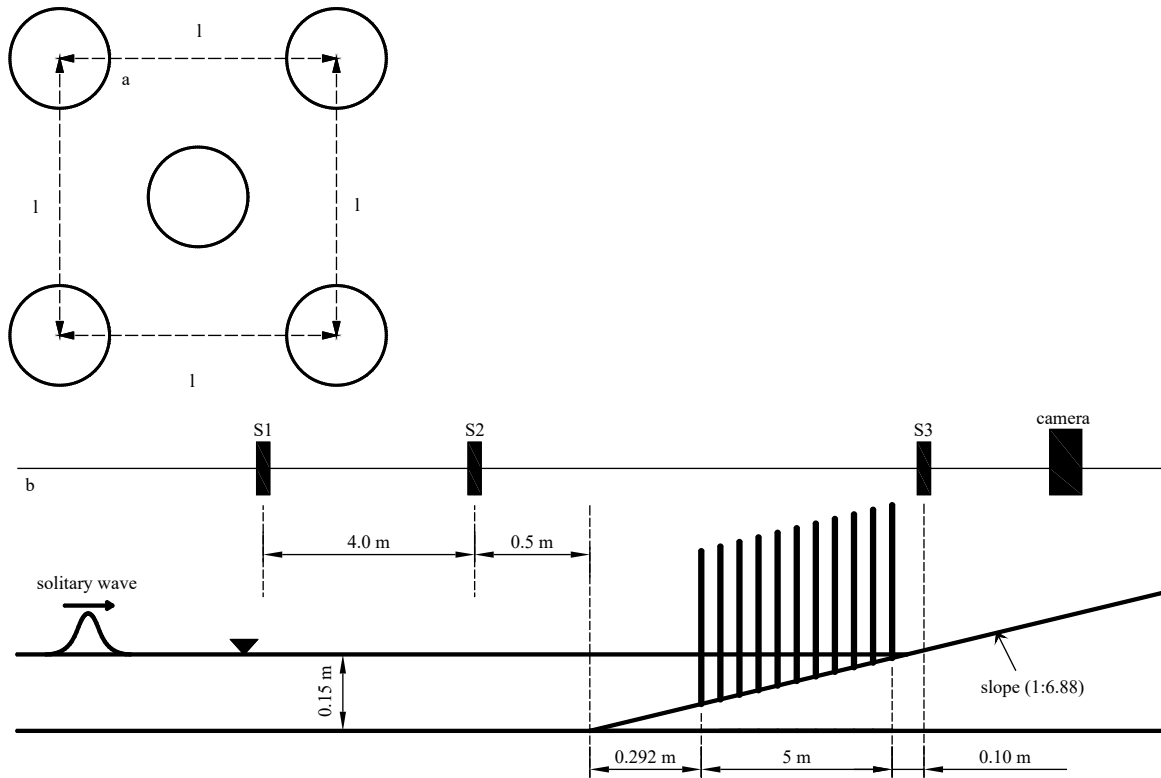


Fig. 4. Experimental setup of Yao et al. (2015). a. Model configuration of vegetation stems and b. sectional view of experimental basin.

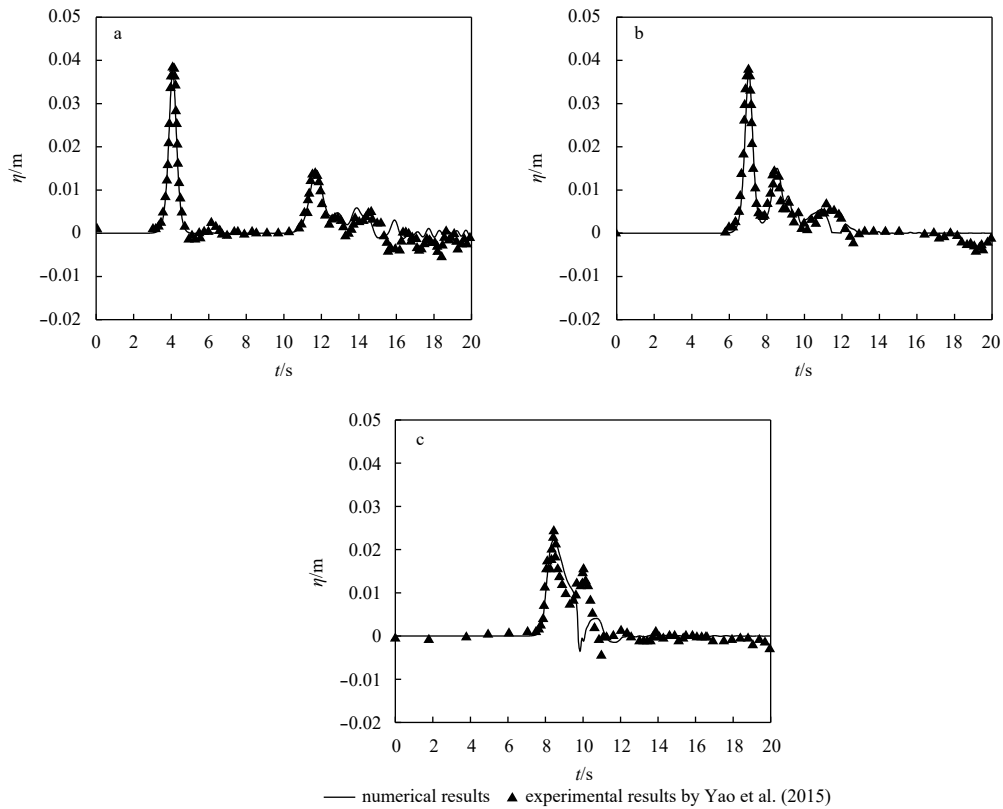


Fig. 5. Free surface simulation for case A. Comparison of free surface at gauge S1 (a), gauge S2 (b) and gauge S3 (c).

medium density (“low-high density” and “high-low density”). The arrangement region for these six distributions was 2 m long and 1 m

wide, and the details are presented in Table 1 and Fig. 7. The arrangement of the six wave gauges is described in Table 2.

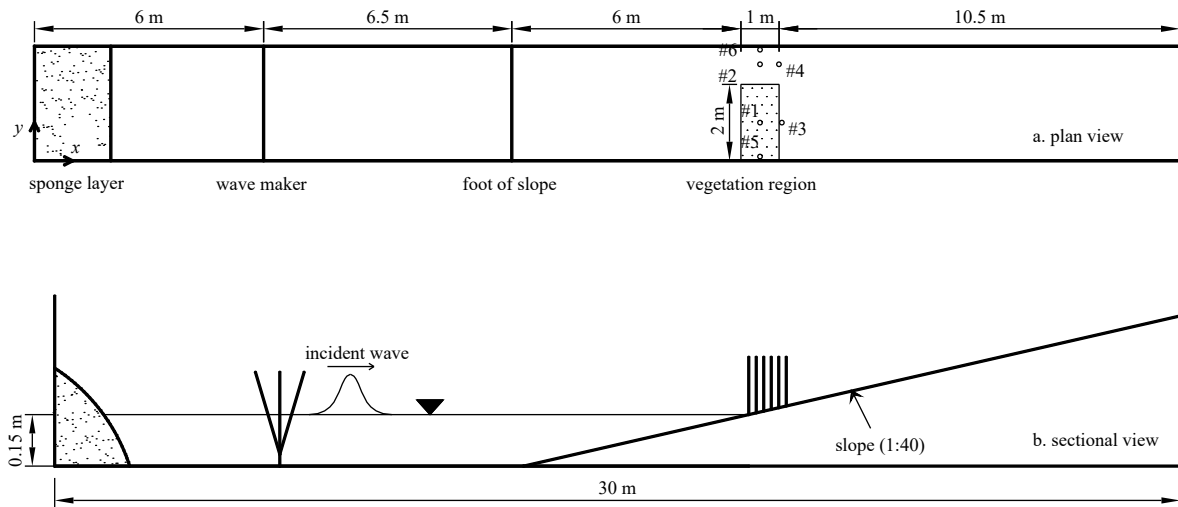


Fig. 6. Setup for the numerical experiment.

Table 1. The arrangement of patchy vegetation cases

Case	The number of cylinders	Density /m ²	Volume portion (ϕ)
A. Non-vegetation	0	0	0
B. Low density	600	300	0.023 6
C. Medium density	1 600	800	0.062 8
D. High density	3 200	1 600	0.126 0
E. High-low density	1 600	800	0.094 0–0.031 4
F. Low-high density	1 600	800	0.031 4–0.094 0

Figures 8 and 9 show the solitary wave run-up results for the six cases listed in Table 1 at $y=0.3$ m (vegetation region) and $y=2.85$ m (channel); the specific figures are presented in Table 3 and Table 4. The wave run-up heights for cases A and B are 0.05 m and 0.036 m, respectively; these values indicate that patchy vegetation has a considerable attenuation effect on solitary wave run-up.

For uniform distributions, the solitary wave run-up locations

for Cases B, C and D are, respectively, 19.88, 19.60 and 19.40 m; these values indicate that a larger density corresponds to a smaller solitary wave run-up. For non-uniform distributions, the solitary wave run-up location for Case E is 19.69 m, while that for Case F is 19.79 m. These values are considerably larger than those for Case C, for which the density was the same as that for Cases E and F; however, the internal distribution is uniform. This finding indicates that the wave run-up is different for vegetation

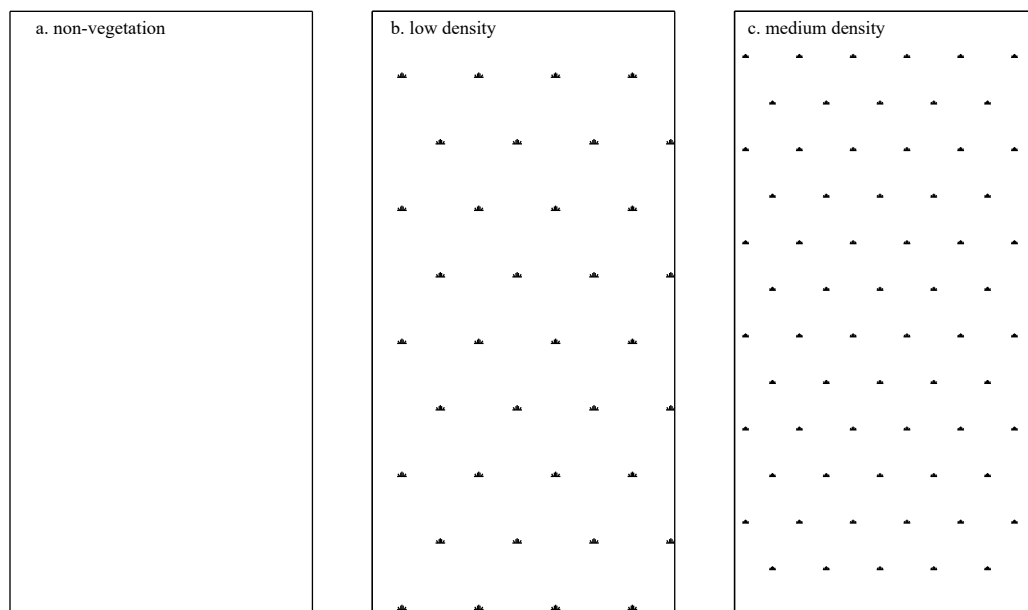


Fig. 7.

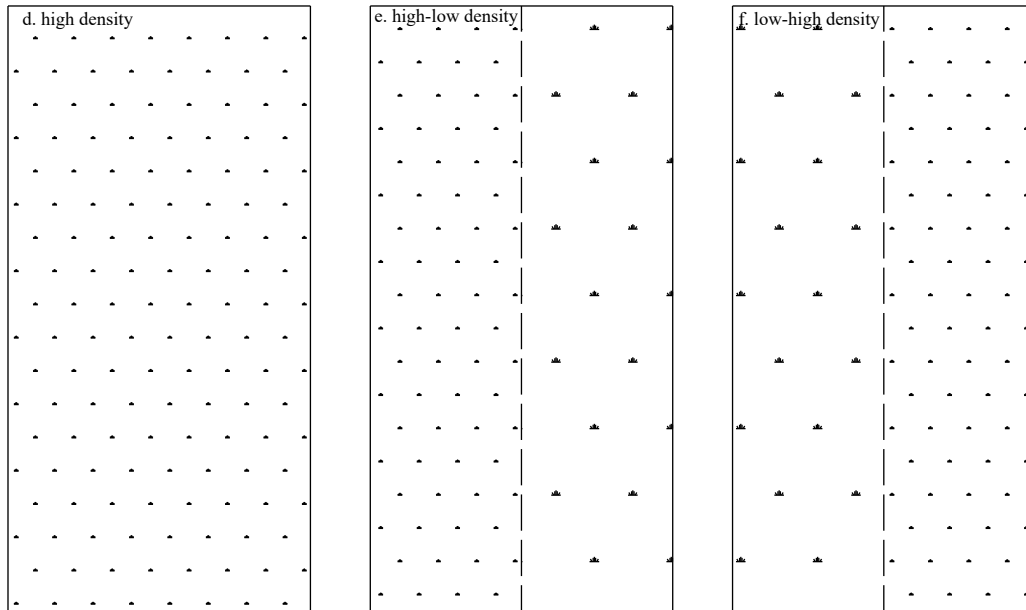


Fig. 7. Six distributions for the vegetation patch.

Table 2. Wave gauges location

	WG1	WG2	WG3	WG4	WG5	WG6
x/m	19.00	19.00	19.60	19.60	19.00	19.00
y/m	1.00	2.50	1.00	2.50	0.30	2.85

with the same density but different internal distributions. A uniform distribution exhibits a better wave run-up attenuation effect than a non-uniform distribution does.

However, vegetation patches did not exert a notable influence on solitary wave run-up in the channel. For all Cases except Case B, the run-up locations are 20.46 m. The wave run-up loca-

tion of Case B is 20.51 m, which is higher than that of Case A, possibly because the diffraction of a high-density vegetation patch is more significant.

4 Conclusions

In this study, the effects of patchy vegetation on solitary wave run-up were investigated by using a numerical simulation. The numerical model is based on an implementation of Morison's formulation for rigid-structure-induced drag stresses in the COULWAVE model. The accuracy of the model was examined by conducting laboratory experiments pertaining to solitary wave propagation through patchy vegetation on a flat bottom and con-

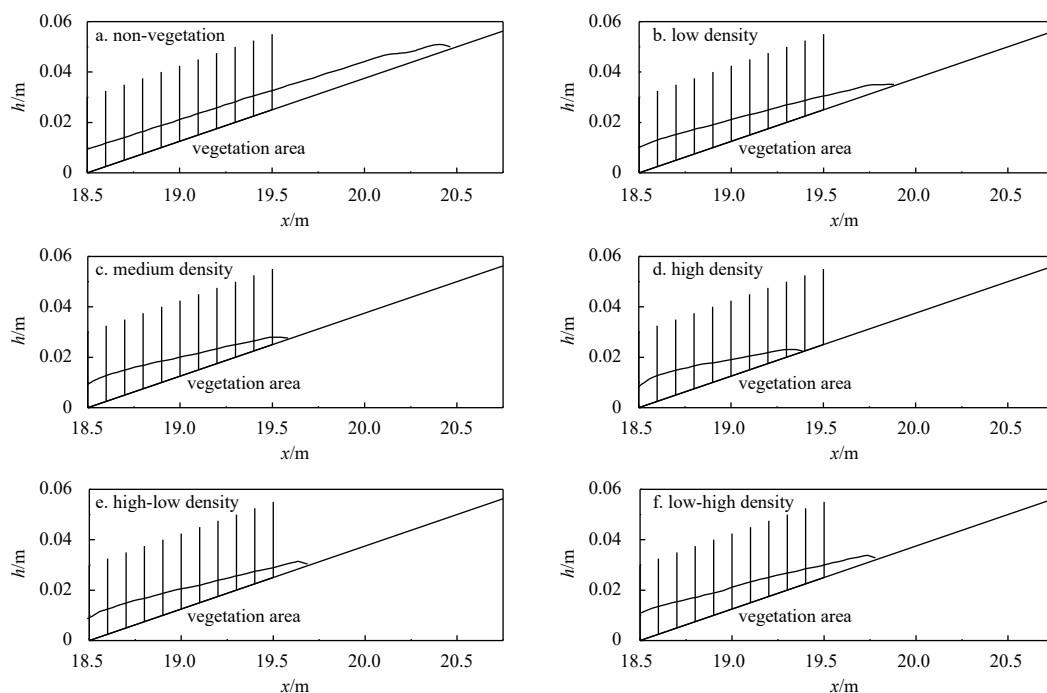


Fig. 8. Comparison of solitary wave run-up at vegetation area ($y=0.3$ m) for different cases.

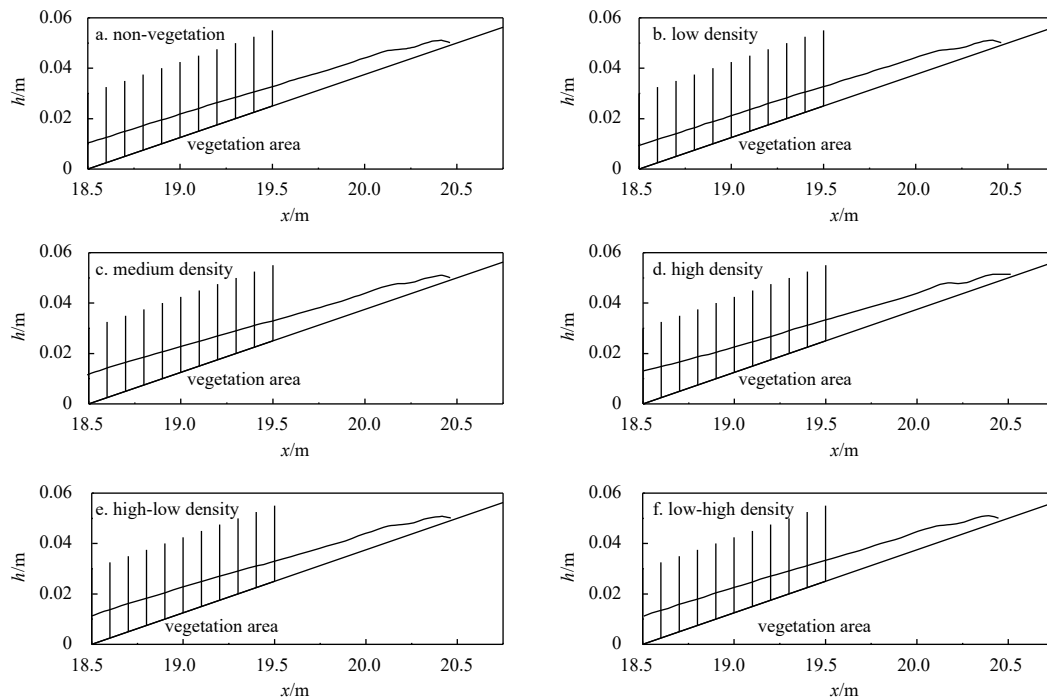


Fig. 9. Comparison of solitary wave run-up at channel ($y=2.85$ m) for different cases.

Table 3. The wave run-up locations and heights at $y=0.3$ m

Case	Run-up location (x/m)	Run-up height (y/m)
A	20.46	0.050
B	19.88	0.036
C	19.60	0.029
D	19.40	0.024
E	19.69	0.031
F	19.79	0.034

Table 4. The wave run-up locations and heights at $y=2.85$ m

Case	Run-up location (x/m)	Run-up height (y/m)
A	20.46	0.050
B	20.46	0.050
C	20.46	0.050
D	20.51	0.052
E	20.46	0.050
F	20.46	0.050

tinuous vegetation on a slope. Subsequently, the model was applied to an investigation of the solitary wave run-up through a patchy vegetation slope. The simulation results indicated that patchy vegetation could lead to a considerable reduction in the wave run-up. Patchy vegetation can reduce the wave run-up to approximately 60% when compared to condition without vegetation. Furthermore, it was noted that the internal distribution can influence the attenuation effect of the solitary wave run-up, and high-density patched vegetation can attenuate the solitary wave run-up more effectively than low-density patched vegetation can. For vegetation patches with the same density, patched vegetation with uniform distribution has a better effect on the attenuation wave run-up when compared to the effect of a non-uniform distribution.

It should be noted that in the present study, the vegetation is

represented by rigid sticks, which is a simplification compared to real vegetation. Natural vegetation is more complex due to the presence of plant branches and leaves, the obvious non-rigidity of the plants, and the spatial or seasonal variations in plant properties. In future work, laboratory data using real vegetation or field data may be used to further validate the proposed model and to confirm the validity of our findings under naturally vegetated coastal environments.

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