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A novel method for internal wave monitoring based on expansion of the sound speed profile

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Abstract

For acoustic detection of internal waves, the core issue is to obtain the temporal and spatial distribution of the sound speed profile (SSP). In the inversion process, the SSP is usually expressed by a few parameters through expansion. However, information about internal waves may sometimes be hard to read directly from the inversion results. The aim of this paper is to characterize the internal waves directly though expansion coefficients. By deducing the dynamic equations of the internal waves, an orthogonal basis called the hydrodynamic normal modes (HNMs) can be extracted from a certain number of SSP samples. Unlike the existing widely used empirical orthogonal functions (EOFs), the HNMs have a more explicit physical meaning that is directly related to internal wave activity. The HNMs are then used to expand the SSP time series, and the expansion coefficients are derived. Eventually, information about internal waves can be read directly from the time derivative of the expansion coefficients of the first two modes. In this study, this method is applied to thermistor string profiles from the northern shelf of the South China Sea, where the SSP shows evident spatial and temporal variations due to internal waves. The results show that the SSP can be described approximately by the first two modes with adequate precision. The special oscillation structure of the time derivative of the expansion coefficients can be used to detect internal solitary waves. The expansion coefficients can also give information on internal solitary wave amplitude and width. According to theoretical and experimental analysis, it can be concluded that the internal waves monitoring method introduced in this paper is effective. The HNMs method is simple to apply and depends less on sample data than EOFs. It could be used as an efficient alternative to EOFs to expand the use of the SSP in highly variable areas, where internal waves are intensive.

Key words: internal waves, hydrodynamic normal modes, internal solitary waves, South China Sea

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1 Introduction

Internal wave activity in shallow seas has a significant effect on sound propagation, resulting in an abnormally large attenuation in propagation (Zhou and Zhang, 1991), variation in the range-frequency interference striation slope (Yang, 2014; Yang et al., 2016), decorrelation of normal mode amplitude (Rouseff and Turgut, 2002), fluctuations in sound arrival time (Li et al., 2013), and a decrease in the radius of correlation time in matched field processing (Ji et al., 2008; Ren et al., 2010). By measuring these effects, disturbances in the sound speed profile (SSP) structure can be inverted and then used to monitor internal waves.

Theoretically, SSP variation in time and space can be described by a matrix. However, such a matrix requires a large number of parameters, which makes it difficult to invert the SSP. Hence, empirical orthogonal functions (EOFs) have been widely used to solve inversion problems related to SSP (Tolstoy et al., 1995; Yu et al., 2010; Li et al., 2015). Using EOFs and projection coefficients, the number of inversion parameters can be greatly reduced, making stable and convenient inversion possible. However, when the inversion results are used to monitor internal waves, it is difficult to explain the internal wave intuitively using several projection coefficients obtained from the inversion. Moreover, the EOFs method is restricted by the completeness and measurement time of the data sample, which presents a difficult challenge for monitoring internal waves.

To establish a direct relationship between the inversion results and internal waves, an orthogonal basis called the hydrodynamic normal modes (HNMs) was used in this study. Unlike the method involving feature extraction from EOFs, the HNMs are derived from hydrodynamic equations. The basis of EOFs reflects the principal components of SSP samples and is the result of mathematical statistics. Differently, the basis of HNMs reflects the motion law of water particles and is the result of oceanographic dynamics analysis. Because the HNMs are derived from the internal-wave mode and discloses the hydrodynamic mechanism which is the cause of SSP's change, the HNMs are directly

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related to internal wave activity and have a more explicit physical meaning than EOFs. Based on a simple analysis of the expansion coefficients obtained from the inversion, the basic features of the internal wave can be obtained. For each occurrence of internal solitary wave, the time derivative of the first two orders of expansion coefficient has a typical double-oscillation structure. By analyzing the change rates of the first two orders of expansion coefficient with time, the internal solitary waves can be neatly identified. The double-oscillation structure also gives information about the wave duration and strength, by measuring the duration and amplitude of the oscillation structure. This novel method was applied to the SSP collected on a thermistor string on the continental shelf of the northern South China Sea. It was shown that the first two orders of expansion coefficient captured enough variability in the SSP to monitor internal waves and their time derivative took higher absolute values during internal wave events. The first two orders of expansion coefficient also provided the double-oscillation structure as a good detector of internal solitary waves, and each internal solitary wave can be straightforwardly identified. As the duration and amplitude of the oscillation structure can be easily measurable, the wave duration and strength were estimated. The internal wave was well identified, thus verifying the validity of the method.

2 HNM method

In regions with frequent internal wave activity, the SSP of seawater possesses a prominent time-varying property. In accordance with the cycle length, the SSP variation can be divided into seasonal time variation at large time scale and time variation of ocean dynamic activity at small time scale. The seasonal variation mainly affects the steady-state characteristics of the SSP, such as layer depth and thickness, over a long time period, whereas the mesoscale dynamic activity of the internal wave can induce a change in the SSP within a short time, but has no effect on the background steady-state properties of the SSP (Jensen et al., 2011).

In this study, the set of HNMs was solved based on the background steady-state SSP. It is worth noting that the background steady-state SSP is usually unknowable, the most common method is to use the average SSP to approximate the background SSP. There are many techniques to reduce dependence on samples to obtain the average SSP. Using the average cycle of fluctuations in the SSP or the SSP at slack tide, the orthogonal basis could be solved. Compared with the traditional EOFs method, this approach was less dependent on sample size.

After this, simple derivation was conducted for the HNM method; for a detailed introduction, the reader is referred to Song et al. (2014). Taking into account the disturbances caused by ocean dynamic activity, the following formula can be derived:

$$\left(\frac{\partial^2}{\partial t^2}+\omega_j^2\right)\frac{\partial^2 w}{\partial z^2}+\left(N^2+\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)w = \frac{N^2}{g}\left(\frac{\partial^2}{\partial t^2}+\omega_j^2\right)\frac{\partial w}{\partial z}, \tag{1}$$

where w is the vertical velocity of a water particle, ω is the frequency, and N is the buoyancy frequency, which can be approximately expressed as a gradient function of temperature T along the vertical direction z:

$$N \approx \alpha g\frac{\partial T}{\partial z}, \tag{2}$$

where α is the thermodynamic compressibility and g is the gravitational acceleration. Assuming that the linear internal wave at the measurement point is the principal factor in SSP disturbances, w can be written as

$$w = W\mathrm{e}^{\mathrm{i}(k_x x+k_y y-\omega t)}, \tag{3}$$

where k_x and k_y are the horizontal wavenumbers in the x- and y-directions, and W is the amplitude. The motion equation of a water particle under the linear internal wave is obtained by substituting Eq. (3) into Eq. (1) to yield:

$$\frac{\partial^2 w}{\partial z^2}+\frac{N^2-\omega^2}{\omega^2-\omega_j^2}k^2 w = 0, \tag{4}$$

where $k=\sqrt{k_x^2+k_y^2}$ is the horizontal wavenumber of the internal wave. At the sea surface and on the seabed, because w must satisfy the boundary condition, that is, be equal to 0, the eigenfunction φ_j of the linear internal wave can be solved. Assuming that the change in the eigenfunction with frequency can be ignored (Song et al., 2014), Eq. (3) can be expressed as the modal superposition of the linear internal wave as follows:

$$w = \sum_j W_j\varphi_j\mathrm{e}^{\mathrm{i}(k_x^j x+k_y^j y-\omega t)}, \tag{5}$$

where W_j is the amplitude of the eigenfunction φ_j for mode j. If the movement of the internal wave is regarded as adiabatic motion, then the temperature gradient and the vertical movement of water particles are the main causes of variation in temperature with time t, and the following formula is obtained:

$$\frac{\partial T}{\partial t}+w\frac{\partial T}{\partial z} = 0. \tag{6}$$

When Eq. (6) is substituted into Eq. (5), the result is

$$\frac{\partial T}{\partial t} = -\frac{\partial T}{\partial z}\sum_j W_j\varphi_j\mathrm{e}^{\mathrm{i}(k_x^j x+k_y^j y-\omega t)}. \tag{7}$$

Temperature is expressed as the disturbance variable ΔT caused by the internal wave superposed over the seasonal steady-state temperature profile T_0. It can be approximately assumed that:

$$\frac{\partial T}{\partial z}\approx\frac{\mathrm{d}T_0}{\mathrm{d}z}. \tag{8}$$

By integrating Eq. (7), the following can be obtained:

$$T(z,t) = T_0(z)+\frac{\mathrm{d}T_0}{\mathrm{d}t}\sum_j A_j\varphi_j\mathrm{e}^{[\mathrm{i}(k_x^j x+k_y^j y-\omega t)]}, \tag{9}$$

where A_j=W_j/iω. The temperature can be expressed as the superposition of n-order normal modes:

$$T = T_0+\begin{bmatrix} B_1 & B_2 & \cdots & B_n \end{bmatrix}\begin{bmatrix}\psi_1\\ \psi_2\\ \vdots\\ \psi_n\end{bmatrix}, \tag{10}$$

where the basis function is ψ_n:

$$\psi_n = \frac{\mathrm{d}T_0}{\mathrm{d}z}\varphi_n. \tag{11}$$

The n-th coefficient B_n is:

$$B_n = A_n \exp[\mathrm{i}(k_x^n x + k_y^n y - \omega t)]. \tag{12}$$

The sound speed increases about 4 m/s for every 1°C rise in seawater temperature, while every 1 rise in seawater salinity just increase the sound speed about 1.14 m/s. In general, water temperature is considered to dynamically vary with depth. On the contrary, salinity remains relatively constant over ocean zones (Lü et al., 2004). Therefore, the sound speed change caused by salinity is much smaller than that caused by temperature in the general case of shallow water. The SSP can also be expanded by the set of orthogonal bases as follows:

$$C(t) = C_0 + \begin{bmatrix} \eta_1 & \eta_2 & \cdots & \eta_n \end{bmatrix} \begin{bmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_n \end{bmatrix}, \tag{13}$$

where C_0 is the background SSP and can usually take on the value of the average SSP. The expansion coefficient η_n can be obtained by SSP inversion in practical application.

Based on the HNM method in Eq. (13), the orthogonal hydrodynamic basis can be obtained by simply measuring the temperature and calculating the temperature gradient and the internal-wave modes. Because the orthogonal basis is established on the basis of the hydrodynamic equations of particles in the linear internal wave, it is the mode that represents the physical properties of the internal wave and naturally has a direct relationship with the internal wave characteristics. When the orthogonal basis is used, the results obtained in the SSP inversion will establish a good relationship with the internal wave properties. In the experiments described below, this process can be clearly seen.

3 Data processing

3.1 *Experimental data*

This research used a set of thermistor string data at location (21°33′N, 117°21′E) on the continental shelf of the South China Sea in spring of 2001. The measurement time extended from 12:00 on May 16 to 12:00 on May 19. The sampling interval was 1 min. The thermistor string was composed of 13 measurement units at depths from 21.7 to 135.1 m, with an average vertical resolution of about 10 m. Because the deepest unit of the temperature series was close to the seabed, it can be assumed that its depth was the depth of the sea at that point.

Figure 1 shows obvious linear internal wave and high-frequency internal solitary wave activity. The deepest internal wave can descend to about 120 m and almost reaches the seabed. The thermocline covers the water body below 20 m.

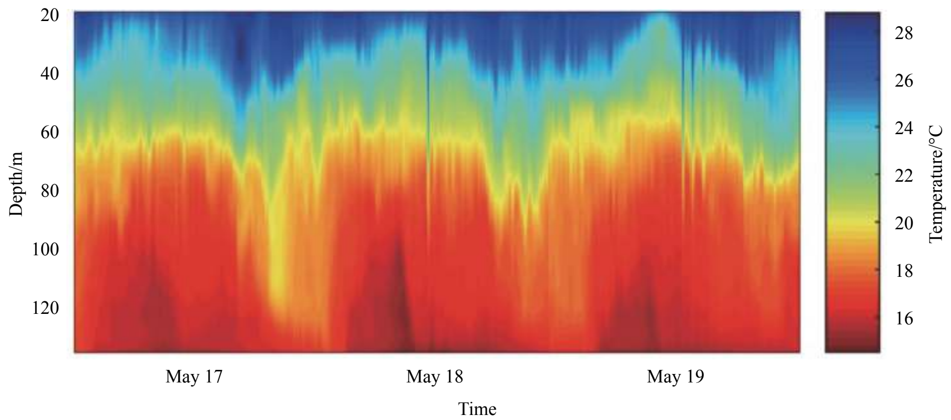

Fig. 1. Temperature data series obtained from 12:00 on May 16 to 12:00 on May 19.

Figure 2 shows the background temperature profile and the average salinity profile. As proved in an earlier experiment (Song et al., 2014), the average temperature profile can be used to approximate the background temperature and to calculate the background SSP. The average temperature profile was mainly calculated from the thermistor string data. Eleven CTD casts, which were within 100 km of the thermistor string, performed during the research period. As the thermistor string data were just available between 21.7 and 135.1 m in depth, the average temperature profiles were then extrapolated for the layer up to the surface based on the CTD profiles. Because the average depth of the mixing layer was about 13 m in the CTD measurement, the average temperature profile from surface to 13 m was approximated to the average temperature of the mixing layer as measured by the CTD. Form 13 m to 21.7 m, the average temperature profile was obtained by cubic interpolation. The average salinity pro-

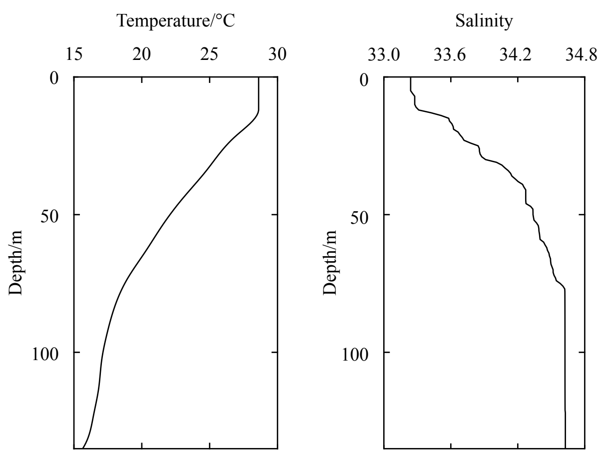

Fig. 2. Background temperature and average salinity profiles.

file was calculated from the CTD profiles. Salinity showed no obvious change during measurement. Temperature still played a decisive role in the SSP.

It is worth noting that the background temperature profile outside the range of the thermistor string measurement may have some deviation. The small numbers of the CTD casts are difficult to represent the background temperature profile. Similarly, the sea depth used (135.1 m) may result a certain error due to temporal variations in depth. In the experiment, the intended depth of deployment was 125 m, but the actual depth of deployment was 139 m. Strong tides were also observed at the time. The small errors of water depth and temperature extrapolation may lead to slight deviation of internal-wave mode in the depth near the sea surface and bottom. For these reasons, the expansion and reconstruction of the SSP in the following sections were both based on the data within the range of the thermistor string measurement. The SSP reconstruction results were able to prove that the HNMs calculated from the background temperature are effective under the temperature extrapolation and depth assumption used.

3.2 *Expansion of the sound speed profile*

Using the background temperature profile in Fig. 2, the linear internal-wave normal modes (LIWNMs) and the HNMs were calculated, as shown in Figs 3a and b.

To illustrate the advantage of the HNM method for monitoring internal waves, the expansion coefficients were calculated directly. The sound speed $c(z)$ as a function of the depth z is given by Medwin's formula (Medwin, 1975):

$$c(T,S,z) = 1\,449.2+4.6T-0.055T^2+0.000\,29T^3+ (1.34-0.01T)(S-35)+0.016z, \tag{14}$$

where T is the temperature and S is the salinity. The background SSP is calculated from the background temperature profile and the average salinity profile. Based on the HNMs, the expansion coefficients can be calculated. In practical application, the expansion coefficients are directly acquired by SSP inversion

Table 1 shows the root-mean-square errors (RMSE) between restructured and real SSP for different orders. The results show that the RMSE of the two methods were close. The RMSE of the EOFs method was slightly smaller than that of the HNMs method. Comparing with the previous work of SSP reconstruction in the continental shelf of the South China Sea (Shen et al., 1999), it has been found that the first two orders of HNMs can provide a reasonable precision. Because the method of internal wave monitoring in this study is based on the expansion coefficients of the first two modes, the first two orders of HNMs were selected for reconstruction. Figure 4 shows examples of representative SSPs reconstructed by the first two orders. These suggests that the HNMs can accurately describe the thermocline disturbance character-

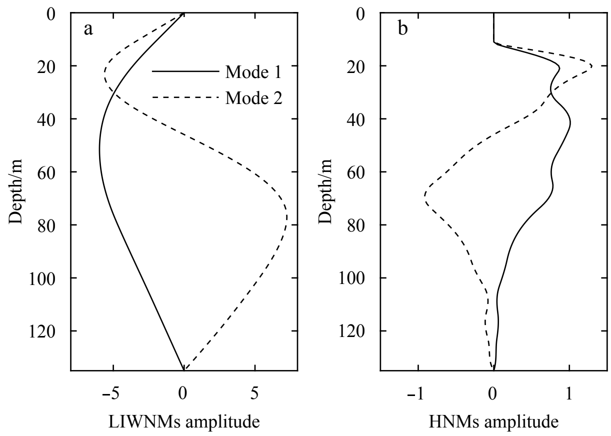

Fig. 3. Eigenfunctions and HNMs of the linear internal wave.

Table 1. The root-mean-square errors (RMSE) between restructured and real SSP for different orders

	1 order	2 orders	3 orders	4 orders	5 orders
The RMSE of the HNMs method/m·s^{-1}	1.41	1.09	0.80	0.59	0.43
The RMSE of the EOFs method/m·s^{-1}	1.25	0.97	0.59	0.46	0.35

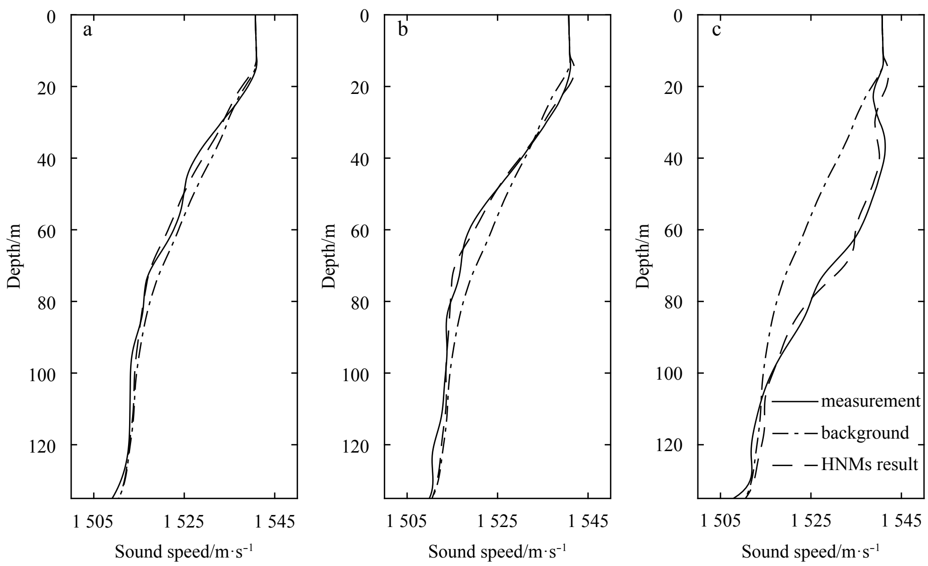

Fig. 4. Reconstruction of the SSP. From left to right, the thermocline locates in the average position, is shifted up, and is shifted down.

istics of the SSP at steady-state depth (Fig. 4a), with an upward shift (Fig. 4b), and even with a drastic downward shift of the internal solitary wave (Fig. 4c). Hence, internal wave monitoring was performed mainly using the expansion coefficients of the first two orders of HNMs, as described below.

3.3 *Internal wave monitoring*

Unlike the traditional EOFs method, the HNMs are directly obtained by differentiating the linear internal wave, and therefore it is easy to establish a relationship between the projection coefficients and the vertical oscillation of the equal sound-speed line in the internal wave activity. To illustrate the connection between the projection coefficients and internal wave activity, a six-hour period from 21:00 on May 18 to 03:00 on May 19 was selected for analysis. Figure 5 shows that during the measurement period, there was obvious sound speed disturbance resulting from internal waves, particularly two internal solitary wave series. The two internal solitary waves had larger amplitudes and were followed by a period of downward small displacement of isotherms. By comparing the variations over time in the first two orders of expansion coefficient, a relationship between the amplitude variations of the expansion coefficient and the internal solitary waves was found. In particular, the variation curve of the first-order coefficient was almost consistent with the variation curve of depth over time at a sound speed of 1 525 m/s. Physically, the first-order HNM represents the vertical oscillation of the equal sound-speed line. The solid line in the SSP map in Fig. 5 is very similar to A_1. By calculating Pearson correlation coefficient, it could be found that the difference between these two solid lines came from the errors between the restructured and real SSP. The correlation coefficient between the real depth at the sound speed 1 525 m/s and the value of the first-order coefficient was 0.89, which means that these two sold lines were strongly correlated. For the restructured SSP, the correlation coefficient between the depth at the sound speed 1 525 m/s and the value of the first-order coefficient was 0.98, the vertical oscillation of the equal sound-speed line would be almost identical with the variation curve of the first-order coefficient. Figure 3 shows that the first-order mode is mainly distributed in the range of 20–80 m and is mainly caused by internal wave activity. When internal solitary wave activity occurs, higher amplitudes for both the first-order and second-order modes are observed.

As for the first two orders of expansion coefficient when internal solitary wave activity occurs, they both exhibit larger amplitudes. An evaluation method for the internal solitary wave is further proposed here. First, the change rates of the first two orders of coefficient with time are obtained by differentiating with respect to time. When the internal solitary wave appears, there is considerable oscillation in the amplitude of the thermocline. Hence, a large-amplitude pulse waveform appears for the first-order expansion coefficient. The amplitude of the derivative increases rapidly and then decreases to a small amplitude after reindexing. When the internal solitary wave occurs, the second-order coefficient also shows a similar increase in amplitude, and therefore its derivative will also exhibit a similar change pattern. Through observation of the first two HNM mode orders, it can be established that only when the projection coefficients have opposite sign can orthogonality be ensured. Therefore, the derivatives should have opposite sign. Taking both these effects into account, when an internal solitary wave appears, the derivatives of

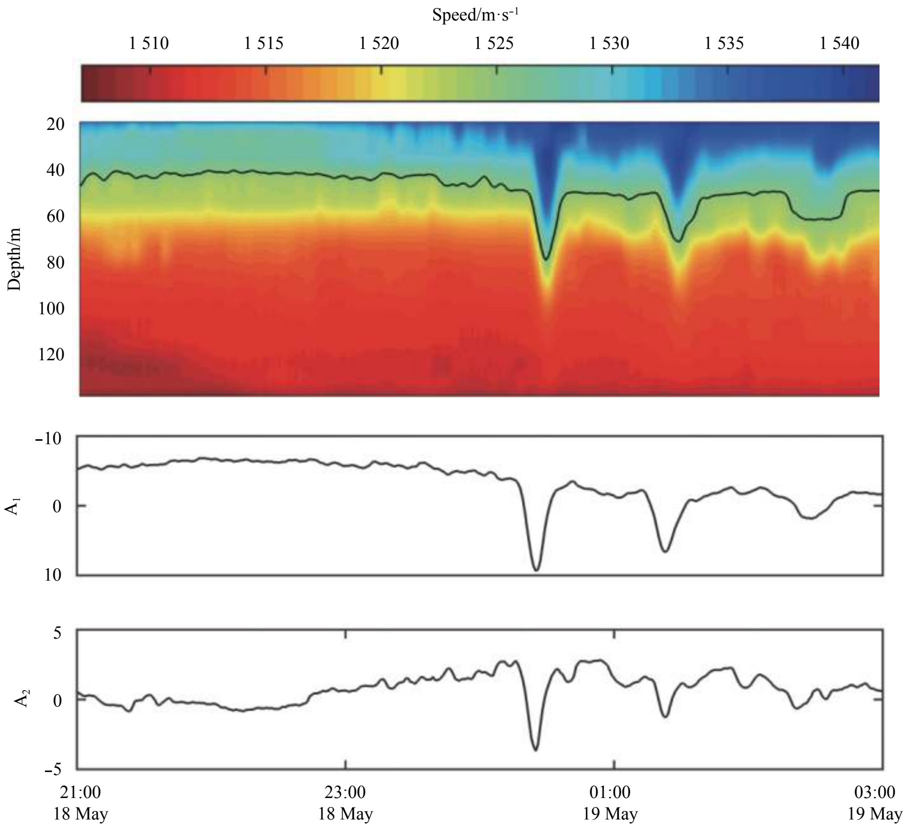

Fig. 5. SSP and variations in the first-order and second-order expansion coefficients with time. In the SSP map, the black line shows the depth variation with time at a sound speed of 1 525 m/s.

the first-order and second-order coefficients with time will exhibit a similar oscillation structure, that is, increased amplitude, opposite sign, and decreased amplitude. Furthermore, the first-order and second-order oscillation structures have opposite sign. Here, this structure is defined as a double-oscillation structure.

Figure 6 shows the monitoring of the internal solitary wave using the double-oscillation structure. When two internal solitary waves with larger amplitudes appear, a double-oscillation structure arises. With the downward shift in the thermocline at smaller amplitude, a shock structure with smaller amplitude can also be observed.

In summary, by observing the time derivatives of the first-order and second-order expansion coefficients, the internal solitary wave can be monitored by the double-oscillation structure. The duration and strength of this wave can also be monitored by the duration and amplitude of this structure.

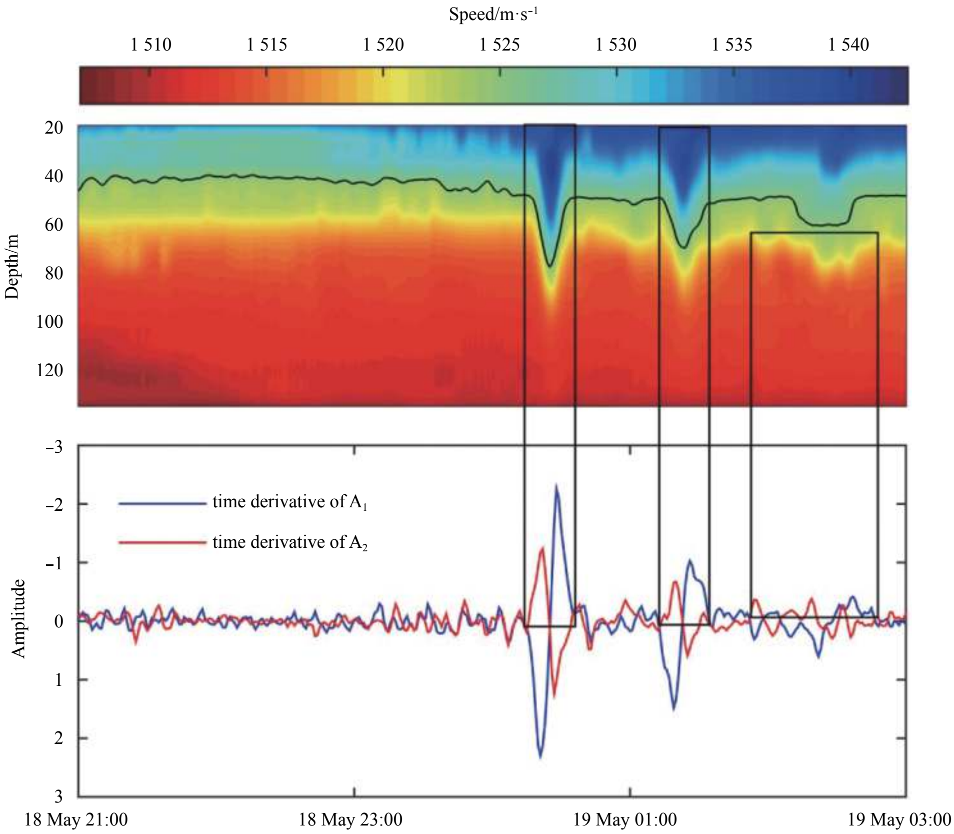

Fig. 6. Evaluation of an internal solitary wave by the double-oscillation model.

4 Conclusions

In this study, the HNM method has been used as an orthogonal basis to expand the SSP. Moreover, a measurement method based on the double-oscillation structure was proposed to observe internal solitary waves.

Similarly to the EOFs method, the orthogonal basis of the HNMs can also describe SSP disturbances with a small number of parameters. In the internal wave environment of the northern South China Sea, the SSP can be accurately reconstructed using two orders of coefficient. Although the HNM method was derived based on the linear internal wave, due to the internal relationships of water particles, a definite connection between the expansion coefficient and the internal wave, including internal solitary waves, could be established. Based on the double-oscillation structure, internal solitary waves could be analyzed effectively.

Overall, this proposed method could be interesting for acoustic monitoring of internal waves. It can be used in poorly known or highly variable areas because the orthogonal basis used in the novel method is less dependent on sample size than EOFs and can provide a good description of SSP. In addition, the double-oscillation structure is a conspicuous sign of an internal solitary wave and therefor can be used as a simple and efficient internal solitary wave detector. Because only the first two order coefficients are needed, the complexity of acoustic inversion is greatly reduced. This method can be used to build an early-warning system for internal solitary waves, to develop situation awareness for undersea surveillance, and for other purposes.

Acknowledgements

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