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# Comparison of two Bayesian-point-estimation methods in multiple-source localization

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**Abstract**

Environmental uncertainty represents the limiting factor in matched-field localization. Within a Bayesian framework, both the environmental parameters, and the source parameters are considered to be unknown variables. However, including environmental parameters in multiple-source localization greatly increases the complexity and computational demands of the inverse problem. In the paper, the closed-form maximum-likelihood expressions for source strengths and noise variance at each frequency allow these parameters to be sampled implicitly, substantially reducing the dimensionality and difficulty of the inversion. This paper compares two Bayesian-point-estimation methods: the maximum a posteriori (MAP) approach and the marginal posterior probability density (PPD) approach to source localization. The MAP approach determines the sources locations by maximizing the PPD over all source and environmental parameters. The marginal PPD approach integrates the PPD over the unknowns to obtain a sequence of marginal probability distribution over source range or depth. Monte Carlo analysis of the two approaches for a test case involving both geoacoustic and water-column uncertainties indicates that: (1) For sensitive parameters such as source range, water depth and water sound speed, the MAP solution is better than the marginal PPD solution. (2) For the less sensitive parameters, such as, bottom sound speed, bottom density, bottom attenuation and water sound speed, when the SNR is low, the marginal PPD solution can better smooth the noise, which leads to better performance than the MAP solution. Since the source range and depth are sensitive parameters, the research shows that the MAP approach provides a slightly more reliable method to locate multiple sources in an unknown environment.

**Key words:** source localization, Bayesian-point-estimation method, uncertain environment

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## 1 Introduction

Matched-field processing (MFP), an approach for solving inverse problems by matching acoustic fields measured at an array of sensors with solutions of the wave equation, has been developed for localizing acoustic sources (Bucker, 1976) and for estimating acoustic parameters (Tolstoy and Diachok, 1991). For source localization problems, MFP estimates source ranges and depths by comparing acoustic fields with replica fields computed for a grid of possible source locations using an acoustic field model. Localization requires good knowledge of the physical properties of the ocean environment and the tilt of the array, which strongly affect the propagation of acoustic signals. Two challenging issues in MFP involve source localization when properties of the environment and the array are poorly known, and localization of multiple sources. This paper addresses both of these problems.

Variants of MFP have been proposed for localization of multiple sources. Model-based methods in underwater environments were discussed (Greening et al., 1997; Nielson, 2005). These multiple source localization techniques typically rely on eigenvector decompositions, modified Bartlett functions, or combination of these two methods. In a recent work, Michalopoulou (2006) developed a simultaneous approach to multiple-source localization based on formulating the PPD over source locations, noise variance, and complex source strengths, and Gibbs sampling these parameters to provide optimal estimates and uncertainties of the results. Nevertheless, it was also shown that the approach is highly sensitive to environmental uncertainties. Dosso and Wilmut (2011) developed two approaches to source localization called marginalization and focalization. Marginalization includes first integrating the PPD over the environmental unknown parameters to obtain a sequence of joint marginal probability distributions over source range and depth. Focalization includes determining the source location that maximizes the PPD over all source and environmental parameters, that is, the MAP solution. Dosso indicates that marginalization significantly outperforms focalization for source localization in an unknown environment. However, through these paper's analysis, the MAP method provides a slightly more reliable result than the marginal PPD approach, which are some different from Dosso's conclu-

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sions. A major reason is that, focalization and marginalization use different numerical algorithms to this optimization problem, which is the main reason that makes the marginalization approach outperforms the focalization approach significantly. Specifically, the marginalization approach has applied a powerful Markov-chain Monte Carlo method, here which combines Metropolis–Hastings Gibbs sampling (GS) for environmental parameters and heat-bath GS for source ranges and depths. However, the focalization approach has used both differential evolution and adaptive simplex simulated annealing as the numerical optimization. So the difference between focalization and marginalization is that they use different numerical algorithms rather than two different Bayesian-point-estimation methods, that is, the MAP solution and the marginalization PPD solution. Although the MAP approach and the marginalization PPD approach generally provide different solutions to source localization in an uncertain environment, there does not appear to have been any comparison of the two approaches to date, this paper compares the two approaches for the multiple-source localization problem, and shows the physical reason of the MAP approach being better than the marginalization PPD approach.

In this paper, the genetic algorithms (GA) are used for the optimization. To overcome the precocious problem of the GA, a temperature is adopted, as in simulated annealing, giving the opportunity to stretch the probability and enhance the algorithm performance. But, as with simulated annealing, the choice of the temperature $T^*$ is difficult. It must be neither too high nor too low. A good compromise is a temperature of the same magnitude as the objective function $\emptyset(\boldsymbol{m})$, here $T^* = \min[\emptyset(\boldsymbol{m})]$. During optimization, the objective function increases and the temperature reduce. Meanwhile, the closed-form maximum-likelihood expressions for source strengths and noise variance at each frequency allow these parameters to be sampled implicitly, substantially reducing the dimensionality and complexity of the inversion.

This paper compares the MAP approach and the marginal PPD approach to multiple-source localization problem. The MAP approach determines the sources locations by maximizing the PPD over all source and environmental parameters just like focalization. The marginal PPD approach integrates the PPD over the unknowns to obtain a sequence of one-dimension (1D) marginal probability distribution over source range or depth something like marginalization. The MAP method and the marginal PPD approach represent distinct approaches in estimating parameters of interest in the presence of nuisance parameters and generally produce different solutions for nonlinear problems such as acoustic inversion. When considering a nonlinear problem, the solutions via these two methods are not coincident.

The two approaches are illustrated conceptually in Fig. 1, which considers determining the value of a model parameter of interest, $m_1$ and $m_2$. Figure 1a considers the case of a binary Gaussian distribution. It is essentially a probability density function of a binary gaussian distribution:

$$\sigma(m_1, m_2) = 3(1-m_1)^2\,\mathrm{e}^{-m_1{}^2-(m_2+1)^2} - 10\times \left(\frac{1}{5}m_1 - m_1{}^3 - m_2{}^5\right)\mathrm{e}^{-m_1{}^2-m_2{}^2} - \frac{1}{3}\,\mathrm{e}^{-(m_1+1)^2-m_2{}^2}.$$

Figure 1b shows the MAP solution for parameter $m_1$ and $m_2$, which are –0.19 and 1.63. Figure 1c shows the corresponding marginal distribution for $m_1$ obtained by integration over $m_2$, whose solution is –0.38. The marginal PPD solution for parameter $m_2$ in Fig. 1d is 1.58. Obviously, the MAP solution and the marginal PPD approach end up with different results. Although the MAP approach and the marginal PPD approach generally provide different solutions to source localization in an uncertain environment, it is impossible to prove in theory which of these

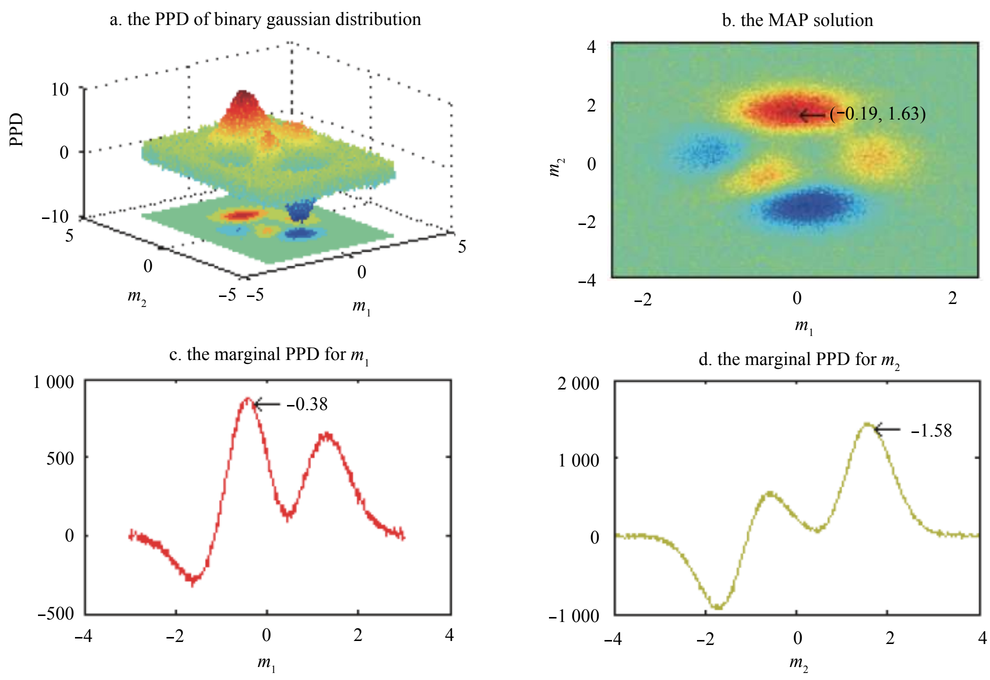


**Fig. 1.** Conceptual illustration of the MAP and marginal PPD approaches. a. The a posteriori distribution for parameter $m_1$ and $m_2$, b. the maximum a posteriori estimation of $m_1$ and $m_2$, c. the corresponding marginal distribution for $m_1$ obtained by integration over $m_2$, and d. the corresponding marginal distribution for $m_2$ obtained by integration over $m_1$.

two methods is more reliable. Monte Carlo studies are employed in both localization approaches which are applied to a large number of noisy synthetic data. Statistical analysis of the results is carried out, and indicates that the MAP approach provides a slightly more reliable approach to locate multiple sources in an unknown environment.

In the paper, the number of sources was assumed known and set equal to two. In practice, there is no such precise information on the number of sources. There are several ways of determining the source numbers, and we studied three source number estimation algorithm based on information theoretic criterion and another method based on Gerschgorin Disks theory, and compares their performance based on numerical simulation results. It is found that the algorithms based on the Bayesian information criterion (BIC) are suitable for the white Gaussian noise. However, due to the limited space of the article, this paper only discusses the multiple-source localization after knowing the number of sources.

## 2 Theory and algorithms

In a Bayesian formulation (Dosso and Wilmut, 2011), the solution to an inverse problem is characterized by its PPD. Before experiment, the information about the models is reflected in the a priori distribution $\rho(\boldsymbol{m})$, but after the experiment, the information about the models is reflected in the a posteriori distribution $\sigma(\boldsymbol{m})$. These two distributions are related through the likelihood function $L(\boldsymbol{m})$, which is a measure of the fitness between the measured data and the data generated using an acoustic field model and the unknown parameters $\boldsymbol{m}$ (Bayesian Theorem):

$$\sigma(\boldsymbol{m}) = L(\boldsymbol{m})\rho(\boldsymbol{m}), \tag{1}$$

where the likelihood function is $L(\boldsymbol{m}) \propto \exp[-E(\boldsymbol{m})]$, and $E(\boldsymbol{m})$ is the data misfit function. The objective function $\varphi(\boldsymbol{m})$ can be written as

$$\varphi(\boldsymbol{m}) = E(\boldsymbol{m}) - \log_{\mathrm{e}}\rho(\boldsymbol{m}). \tag{2}$$

Due to multi-dimensionality, the a posteriori distribution is not fit for graphic display, and therefore mainly integral properties of the a posteriori distribution are of interest, such as the MAP solution, and the marginal PPD for parameter$m^i$ , defined respectively as

$$\widehat{\boldsymbol{m}}^{\mathrm{MAP}} \equiv \arg(\max(\sigma(\boldsymbol{m}))), \boldsymbol{m} \in M, \tag{3}$$

$$\sigma^i(m^i) = \int \sigma(\boldsymbol{m})\mathrm{d}m^1 \cdots \mathrm{d}m^{i-1}\mathrm{d}m^{i+1} \cdots \mathrm{d}m^M, \tag{4}$$

where $M$ is the number of the estimated parameters. The MAP solution is found using the optimization algorithm without integration, which consumes less time. The main advantage of the marginal probability distribution is that it provides a quantitative measurement of localization uncertainty.

To define the data misfit function $E(\boldsymbol{m})$, consider data $\boldsymbol{p}^{\mathrm{e}} = \{\boldsymbol{p}_f^{\mathrm{e}}; f = 1, F\}$ consisting of complex acoustic measurements at $F$ frequencies and $N$ hydrophones. Namely, $\boldsymbol{p}_f^{\mathrm{e}} = \{[\boldsymbol{p}_f^{\mathrm{e}}]_n;\ n = 1,\ N\}$ is a complex vector with $N$ elements. The acoustic measurements at each frequency is assumed to be due to $N_{\mathrm{s}}$ sources at locations $x = \{x_s = (r_s,\ z_s)\,; s = 1,\ N_s\}$ with complex source strengths $\boldsymbol{S}=\{[S_f]_s\}$. The data errors are considered complex Gaussian-distributed random variables with unknown standard deviations $\boldsymbol{v}=v_f$, the likelihood function is given by

$$L(\boldsymbol{m}, \boldsymbol{S}, \boldsymbol{v}) = \prod_{f=1}^{F} (\pi v_f^2)^{-N} \exp\left[ -\frac{\left| \boldsymbol{p}_f^{\mathrm{e}} - \sum_{s=1}^{N_s} (w(x_s, \omega_f) S(x_s, \omega_f)) \right|^2}{v_f^2} \right]$$

$$= \frac{1}{\prod_{f=1}^{F} (\pi v_f^2)^N} \exp\left\{ \sum_{f=1}^{F} \left|\boldsymbol{p}_f^{\mathrm{e}} - \boldsymbol{D}_f \boldsymbol{S}_f\right|^2 / v_f^2 \right\}$$

$$= \exp\left\{ -\sum_{f=1}^{F} [N \ln(\pi v_f^2) + \left|\boldsymbol{p}_f^{\mathrm{e}} - \boldsymbol{D}_f \boldsymbol{S}_f\right|^2 / v_f^2] \right\}, \tag{5}$$

where $w(x_s, \omega_f)$ represents the replica acoustic fields computed for a zero-phase unit-amplitude source at $x_s$, and $\boldsymbol{D}_f$ is an $N \times N_s$ complex matrix defined as $[\boldsymbol{D}_f]_{n,s} = w_n(m_s, \omega_f)$, and $\boldsymbol{S}_f$ is an $N_s \times 1$ complex matrix defined as $[\boldsymbol{S}_f]_s = S(x_s, \omega_f)$.

An unknown source can be treated by maximizing the likelihood over $\boldsymbol{S}$ and $\boldsymbol{v}$ $\left( \dfrac{\partial L}{\partial \boldsymbol{S}_f} = \dfrac{\partial L}{\partial v_f} = 0 \right)$ to give

$$\varphi(\boldsymbol{m}) = \sum_{f=1}^{F} [2N \ln \left| (\boldsymbol{I} - \boldsymbol{D}_f \boldsymbol{D}_f^{-g}) \boldsymbol{p}_f^{\mathrm{e}} \right| + (N + N \ln \pi - N \ln N)], \tag{6}$$

neglecting additive constants leads to

$$\varphi(\boldsymbol{m}) \propto \sum_{f=1}^{F} [2N \ln \left| (\boldsymbol{I} - \boldsymbol{D}_f \boldsymbol{D}_f^{-g}) \boldsymbol{p}_f^{e} \right|]. \tag{7}$$

Hence, by using this equation, the corresponding variability in standard deviations and source strengths is accounted for implicitly. This implicit sampling replaces explicit sampling over these two parameters, substantially reducing the dimensionality of the inversion. For an environmental model with $N_E$ parameters, explicit sampling of all parameters has the dimension $2N_sF$+$F$+$2N_s$+$N_E$, whereas implicit sampling reduces this to $2N_s$+$N_E$. Such as, in the latter example which involves two sources at three frequencies and seven environmental parameters, the dimensionality is reduced from 26 to 11.

Knowing that the likelihood function is usually related to the objective function $\varphi(\boldsymbol{m})$ through an exponential $L = \exp(-\varphi(\boldsymbol{m})/\hat{v})$ (Gerstoft and Mechlenbrauker, 1998), where $\hat{v}$ is the estimated noise power, the following scaling is used

$$L_{\mathrm{emp}} = \exp(-[\varphi(\boldsymbol{m}) - \varphi(\boldsymbol{m}_0)]/T), \tag{8}$$

where $\varphi(\boldsymbol{m})$ is the objective function mentioned in Eq. (7), $\boldsymbol{m}_0$ is the estimated parameter corresponding to the optimal value of the objective function and $T$ is the temperature (Li, 2016). Researches show that a good value for $T$ is the average of the 50 best objective functions obtained during the optimization, minus the best value of the objective function. It should be pointed out that this value of $T$ is not intended to estimate the noise, but rather to provide a reasonable value with which to estimate the uncertainties of the parameters. The advantage is that it works irrespective of the stochastic model for the data.

## 3 Simulation example

This section illustrates multiple-source localization with a simulated example involving two sources with the same strength in a poorly-known environment. The scenario is illustrated in Fig. 2 and parameter values and prior bounds are summarized in Table 1. The locations of the two sources are ($r_1$, $zs_1$)=(7 km, 60 m), and ($r_2$, $zs_2$) =(7 km, 20 m), with corresponding SNRs at the receiver array between –10 dB and 15 dB at each of three frequencies of 200 Hz, 300 Hz, and 400 Hz. Simulated acoustic data were recorded by a vertical line array (VLA) which had 24 elements at 3.5-m spacing, sampling the water column with depths of 4–90 m using the normal-mode propagation model SNAP. The prior information for all source locations is a uniform distribution over 10–90 m in depth and 1–10 km range, and the depth and range steps are 0.5 m and 35 m, respectively. Water column unknowns include the water depth ($D$), and the sound-speed profile represented by three parameters ($c_1$–$c_3$) at depths of 0 m, 9.5 m, and $D$ m. Unknown geoacoustic parameters include the bottom sound speed ($c_b$), density ($\rho_b$), and attenuation ($\alpha_b$). Prior information for the estimated parameters is given in Table 1.

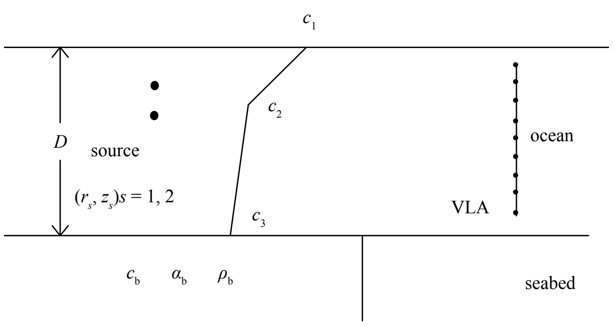


**Fig. 2.** Experimental configuration and shallow water environment.

## 4 The multiple-source localization results

Figure 3 shows inversion results for all parameters to one realization at an SNR of 10 dB. Further, the results are summarized in Table 1. Figure 3 shows that the MAP solution and the marginal PPD solution for range $r$ and depths $z_1$ are the same and slightly smaller than the correct value. While the MAP solution for depths $z_2$ are slightly better than the marginal PPD solution. Figure 3 also shows that the MAP solution and the marginal PPD solution for other environmental parameters are nearly the same and slightly differ from the correct value. In conclusion, Fig. 3 shows that the MAP method and the marginal PPD approach are comparable in source localization and environment inversion. Figure 3 also shows that the MAP or marginal PPD methods have the advantage that these methods have narrower main lobe and lower side lobes than conventional MFP (Li, 2016). This is helpful to solve the problem of localizing a weak target with the presence of stronger interference in an uncertainty ocean environment. The weak source of interest cannot be masked by the loud interfering sources presented in coastal waters.

To evaluate and compare the MAP method and the marginal PPD approach to multiple-source localization in an uncertain environment, a Monte Carlo performance study was carried out, with results summarized in Fig. 4. The performance study considers the two sources at a variety of SNRs between –10 and 15 dB. For each case these two approaches were applied to 50 realizations of noisy acoustic data to allow statistical analysis. Localization performance is quantified in terms of the probability of correct (PCL) localization, defined here as achieving absolute errors in source range and depth of less than 400 m and 6 m, respectively, for both sources. Results are given in Fig. 4. Figure 4 shows that the MAP method slightly produces higher PCL values than the marginal PPD approach for SNRs from –10 dB to 10 dB for the first source and most of SNRs for the second source. At the SNR of 15 dB for the first source, and 5 dB and 15 dB for the second source, similar PCL results are obtained for the two methods.

To further evaluate and compare the MAP method and the marginal PPD approach for estimating ocean acoustic parameters, Fig. 5 gives the 95% confidence intervals (CIs) for the inversion results of the two methods. Figure 5 shows that, as the SNR increases, the CIs become narrow and approach the real value gradually. This indicates that the proportion of the correct inversion rises. Figure 5 also shows that in addition to bottom sound speed, the MAP method has equal or a slight advantage over the marginal PPD approach. The same conclusions are also gained in Fig. 3.

In Dosso's article (Dosso and Wilmut, 2011), the focalization method adopts the MAP solution, and marginalization takes the marginal PPD solution. Dosso indicates that marginalization significantly outperforms focalization for source localization. However, through the above analysis, the MAP method generally produces higher PCL values than the marginal PPD approach, which are some different from Dosso's conclusions. A major reason is that, focalization and marginalization use different numerical algorithms to this optimization problem, which is the main reason that makes the marginalization approach outperforms the focalization approach significantly. So the difference

**Table 1.** Parameter value, prior bounds and inversion results

| Parameter | True values | Bounds | The MAP solution | The marginal PPD solution |
|---|---|---|---|---|
| $r_1$/km | 7 | [1, 10] | 6.8 | 6.8 |
| $r_2$/km | 7 | [1, 10] | 6.8 | 6.8 |
| $zs_1$/m | 60 | [10, 90] | 59.6 | 59.6 |
| $zs_2$/m | 20 | [10, 90] | 19.7 | 19.4 |
| $D$/m | 100 | [98, 102] | 98.7 | 98.5 |
| $t$/m | 0 | [–10, 10] | –0.16 | –0.16 |
| $c_b$/m·s$^{-1}$ | 1 580 | [1 520, 1 700] | 1 592.0 | 1 592.7 |
| $\rho_b$/g·cm$^{-3}$ | 1.5 | [1.2, 2.2] | 1.5 | 1.6 |
| $\alpha_b$/dB·$\lambda^{-1}$ | 0.1 | [0, 0.5] | 0.19 | 0.23 |
| $c_1$/m·s$^{-1}$ | 1 520 | [1 515, 1 525] | 1 519.6 | 1 518.5 |
| $c_2$/m·s$^{-1}$ | 1 518.8 | [1 514, 1 522] | 1 518.4 | 1 519.3 |
| $c_3$/m·s$^{-1}$ | 1 510 | [1 508, 1 512] | 1 509.2 | 1 509.9 |

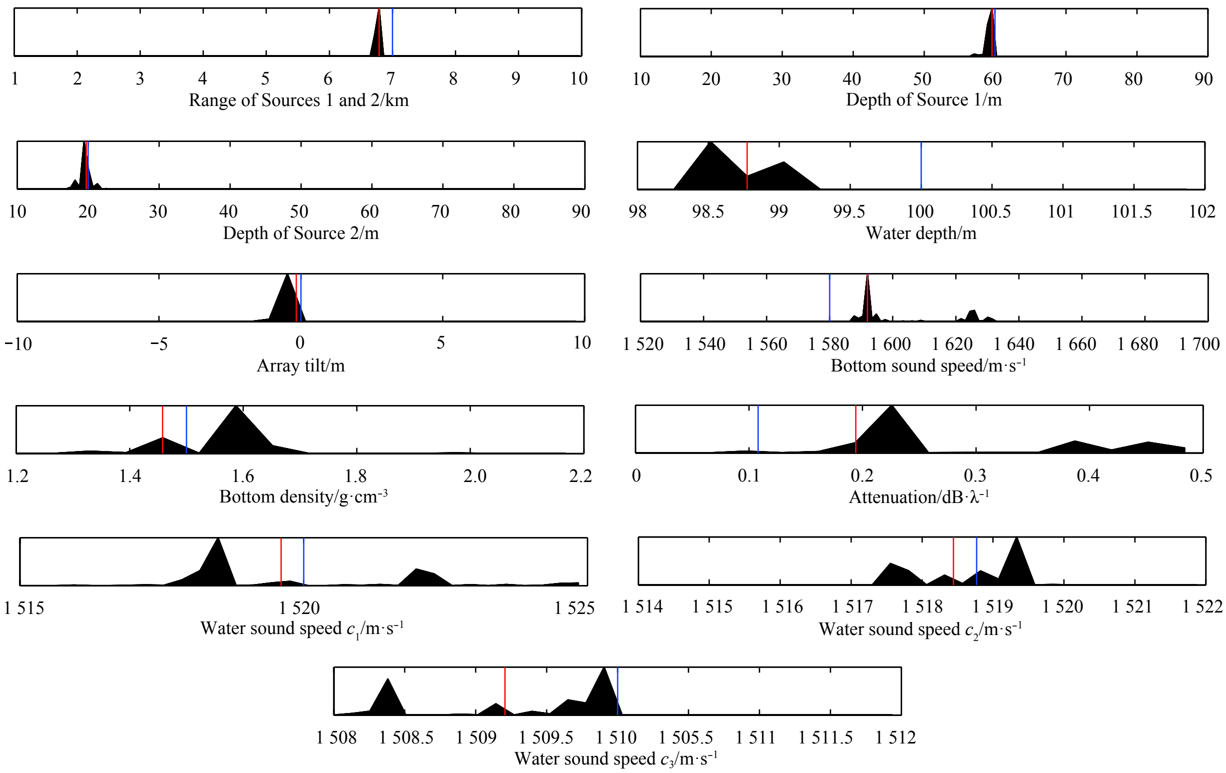


**Fig. 3.** The marginal PPDs for all parameters. The red line indicates the MAP solution, and the blue linethe true value.

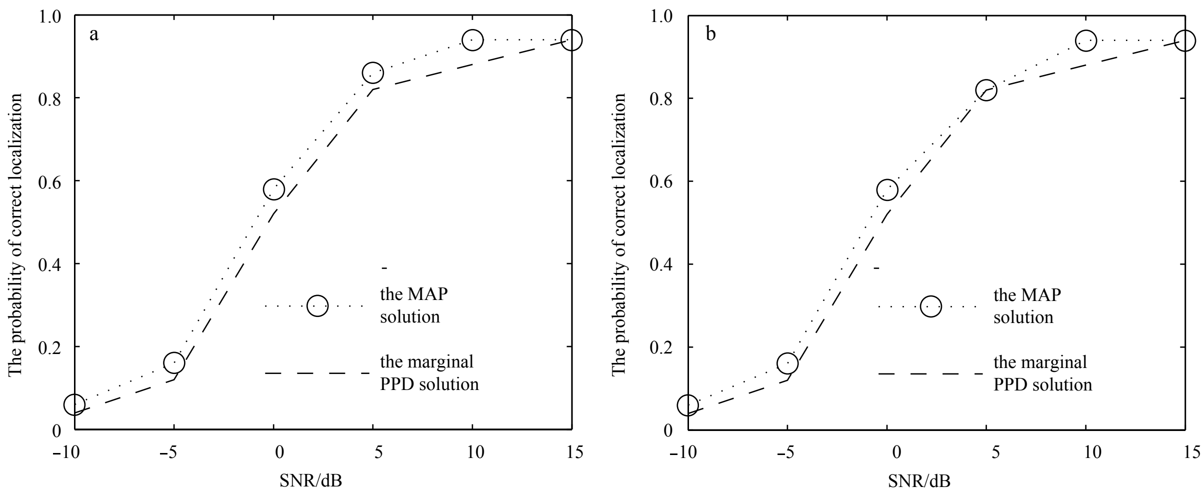


**Fig. 4.** Localization performance study for two sources. Probability of correct localization over 50 noise realizations is shown for the MAP method (open circles) and the marginal PPD approach (dotted line) as a function of the SNR. The first image indicates PCL results for the first source, and the second image indicates PCL results for the second source.

between focalization and marginalization is that they use different numerical algorithms rather than two different Bayesian-point-estimation methods, that is, the MAP solution and the marginalization PPD solution.

In order to eliminate the influence of different numerical algorithms on localization, both of the two methods mentioned in this paper have used the genetic algorithm as optimization algorithm. To further compare the MAP solution and the marginal PPD solution, Fig. 6 gives the root-mean-square error (Rmse) for both methods. The Rmse is defined as

$$\sigma=\sqrt{\frac{\sum_{i=1}^{n}(M_e^i-M_t)^2}{n}}, \tag{9}$$

where $n$ is the number of realization ($n$=50), $M_e^i$ is the estimated value for unknown parameters, and $M_t$ is the true value. The Rmse can well measure the deviation between the estimate and the truth value, and reflect the precision of the method. Figure 6 gives the root-mean-square error of the MAP solution and the marginal PPD solution. The results can be divided into three

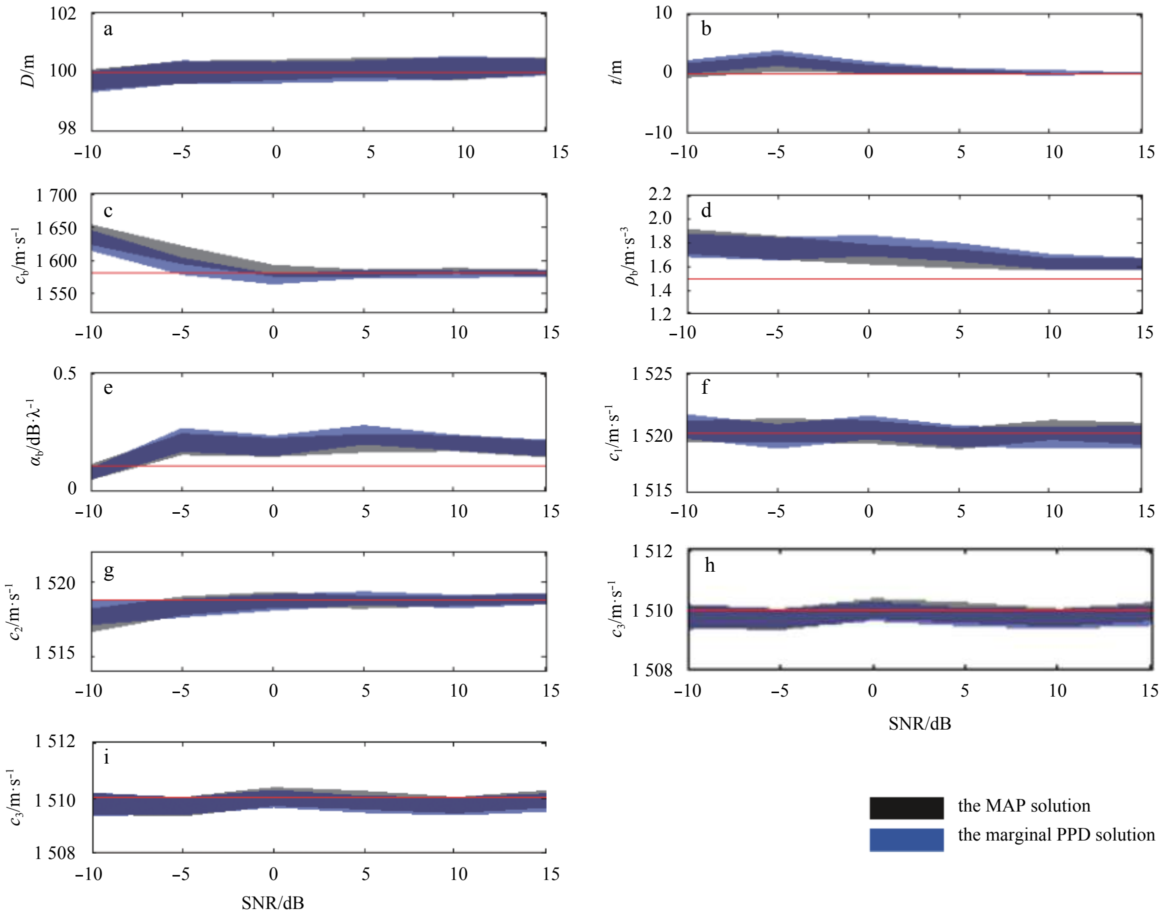


**Fig. 5.** The 95% confidence intervals (CIs) for the inversion parameters. Both approaches were applied to 50 realizations of noisy acoustic data. The black area shows the MAP solution, the blue area the marginal PPD method, and the red line the true value.

parts: (1) As shown in Figs 6a–c, the MAP solution for range $r$, water depth $D$ and water sound speed $c_1$ are better than the marginal PPD solution. These three parameters are most sensitive parameters which could greatly change the objective function (Li et al., 2012). That is, the sensitive parameters can achieve better estimation accuracy. If the marginal PPD solution is adopted, the effect of less sensitive parameters on the objective function is considered, and the inversion precision of the sensitive parameter will be reduced. That is to say, sensitive parameters do not require collective decision, which is the reason that the MAP solution for range $r$, water depth $D$ and water sound speed $c_1$ are better than the marginal PPD solution. (2) As shown in Figs 6d–g, these are the less sensitive parameters, such as, bottom sound speed $c_b$, bottom density $\rho_b$, bottom attenuation $\alpha_b$ and water sound speed $c_2$. When the SNR is low, the noise energy is large, and the marginal PPD solution can better smooth the noise, which leads to better performance than the MAP solution. (3) As shown in Figs 6h–k, these are the medium sensitive parameters, such as, source depths $z_1$ and $z_2$, array tilt $t$, and water sound speed $c_3$. With the change of SNR, the location accuracy of the two methods varies. This is due to the interaction between parameter sensitivity and noise smoothing. In addition, when the SNR is 15, the estimation results of these two methods for all parameters are similar, with the same root-mean-square root. This shows that when the signal-to-noise ratio is high enough, the precision of the two localization methods is the same.

## 5 Discussion and conclusions

This paper considered the MAP method and the marginal PPD approach to multiple-source localization when uncertain environmental parameters are included as unknowns in an augmented inverse problem. The MAP method and the marginal PPD approach represent distinct approaches in estimating parameters of interest in the presence of nuisance parameters and generally produce different solutions for nonlinear inverse problems. Hence, Monte Carlo analysis was applied to compare the two approaches for source localization in an uncertain environment. Both approaches were applied to a large number of noisy synthetic data sets for a test case involving uncertain water-column and seabed parameters.

In the MAP method, the PPD is maximized numerically over all dimensions to provide the most probable set of model parameters, including optimal source ranges and depths. The MAP solution is found using the optimization algorithm without integration, which takes less time but has no measure of uncertainty. In the marginal PPD approach, the PPD is integrated numerically to produce 1D marginal probability distributions over source range or depth, from which source locations can be obtained. In addition, it provides uncertainty analysis, which aids in understanding the information content of the inverse problem. But this method increases computational effort (the marginal PPD approach generally takes about five times as long as the MAP method).

Statistical analysis indicates that: (1) for sensitive parameters

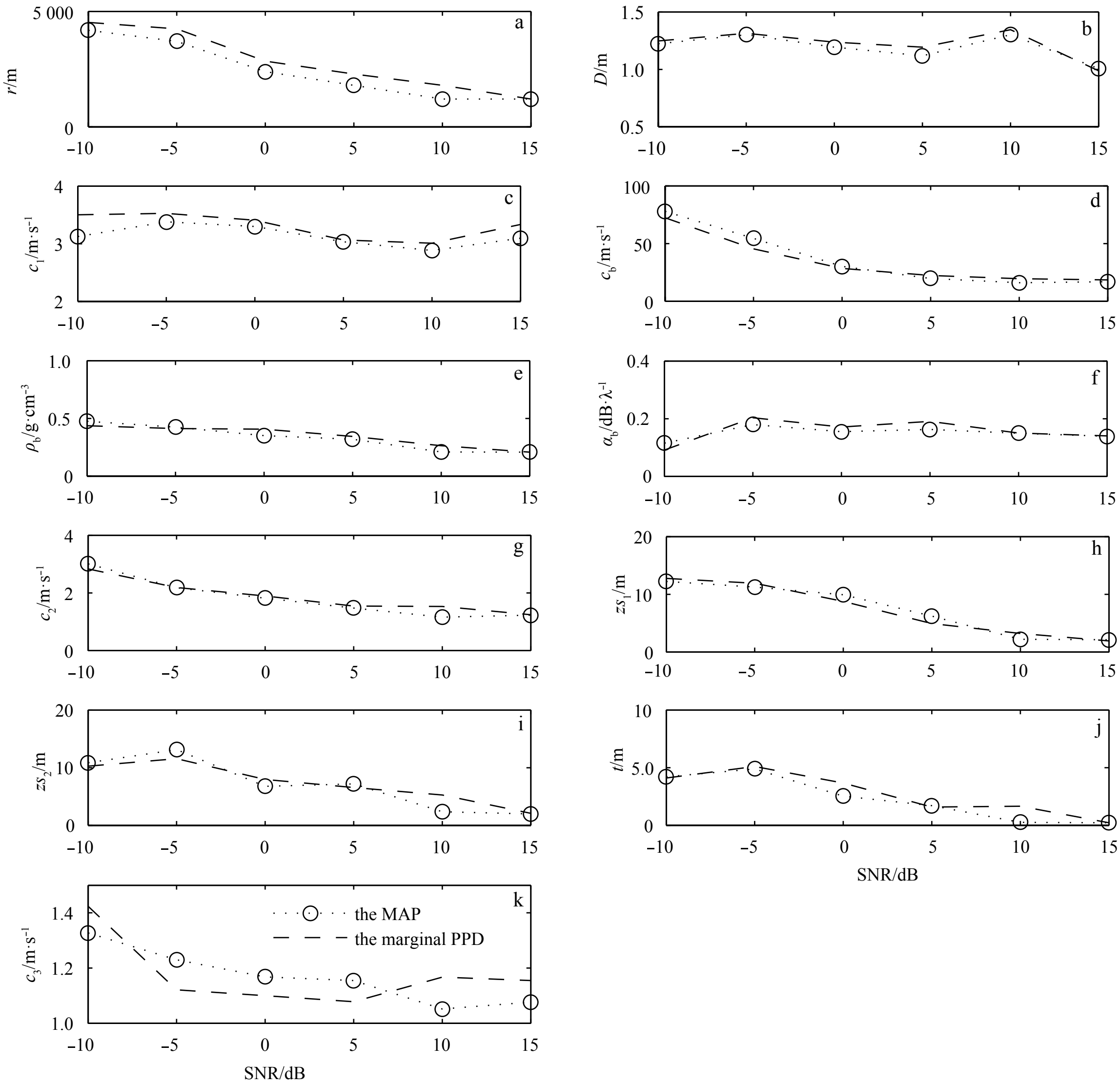


**Fig. 6.** The root-mean-square error of the MAP solution and the marginal PPD solution. Both approaches were applied to 50 realizations of noisy acoustic data.

such as source range, water depth and water sound speed, the MAP solution is better than the marginal PPD solution; (2) for the less sensitive parameters, such as, bottom sound speed, bottom density, bottom attenuation and water sound speed, when the SNR is low, the marginal PPD solution can better smooth the noise, which leads to better performance than the MAP solution; (3) for the medium sensitive parameters, such as, source depths, array tilt, and water sound speed, with the change of SNR, the location accuracy of the two methods varies. In summary, the MAP method slightly out-performed the marginal PPD approach in source localization in an unknown environment. So if there is no need to provide uncertainty analysis, the MAP solution is a time saving and high accuracy method.

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