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The multifractal spectrum of a sea clutter using a random walk model

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Abstract

The radar echo signal of a sea clutter is nonlinear, nonstationary and time varying. A multifractal measure analysis can describe the local singularity of a physics system. The random walk model of a sea clutter scattering is analysed to disclose the intrinsic physical characteristics and laws of the sea clutter. Stochastic differential equations are given for the physical quality of the sea clutter. A diffusion process model is established using $\widehat{Ito}$ formula. The singularity of the random walk model is tested by a multifractal spectroscopy, and the accuracy of this model is proven by the multifractal spectroscopy of a real-life IPIX radar data set. Thus, the random walk model is effective for describing the dynamics mechanism of the sea clutter.

Key words: random walk, sea clutter, multifractal

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1 Introduction

The sea clutter refers to the surface of a radar-scattering echo and is the most complicated form of a radar clutter. After years of testing, researchers have found that the probability distribution of the sea clutter is a Gaussian model. They have proposed a logarithmic normal distribution, Weibull distribution, K-distribution and other distribution models (Rosenberg and Bocquet, 2015; Velotto et al., 2014; Güntürkün, 2015; Jakeman and Pusey, 1976; Li et al., 2014). These models are close to the sea clutter probability distribution form to some extent, but they do not clearly describe the generating mechanism of the sea clutter. Only the statistical models for the first-order and second-order characteristics of the sea clutter are described. The time-varying characteristic of the sea clutter is not depicted (Granström et al., 2015; Suresh et al., 2015; Xing et al., 2014; Xiong et al., 2014). Consequently, precise modelling, analysis and processing of the nonlinear, nonstationary and time-varying characteristics of the sea clutter are difficult (Wu et al., 2014; Zhang et al., 2014; Xu et al., 2014).

The multifractal measure analysis of different physical systems can completely describe the local singularity (Guan et al., 2010; Liu et al., 2012); the statistical properties of such singular measure in the characterisation can reveal complex heterogeneous structure system by determining the singularity spectrum. Jakeman and Tough (1987) proposed electromagnetic scattering of the random walk model to reveal the dynamics mechanism of the sea clutter. Considering the different parameters of the model, the evolution of the sea clutter can be all types of a statistical distribution model (e.g., Rayleigh distribution, Weibull distribution and K-distribution). The random walk model must accurately depict the time-varying characteristic of the sea clutter. On this basis, it utilises the random walk model to describe the time-varying electromagnetic scattering characteristics of the sea clutter using the stochastic differential equation model.

2 Electromagnetic scattering characteristics of sea clutter

The electromagnetic scattering signal of the sea clutter is derived by the stochastic differential equation based on a random walk. According to Jakeman random walk theory, the electromagnetic scattering model of the sea clutter (Jakeman and Tough, 1987; Feng et al., 2007) can be expressed as

$$\begin{aligned}\Psi_t &= \lim_{N_t\to\infty}\left\{\frac{1}{(\overline{N})^{1/2}}\sum_{j=1}^{N_t}\mathrm{e}^{\mathrm{i}\phi_t^{(j)}}\right\}\\ &= \lim_{N_t\to\infty}\left\{\left(\frac{N_t}{\overline{N}}\right)^{1/2}\frac{1}{(N_t)^{1/2}}\sum_{j=1}^{N_t}\mathrm{e}^{\mathrm{i}\phi_t^{(j)}}\right\}\\ &= (x_t)^{1/2}\gamma_t\quad,\end{aligned}\tag{1}$$

with $\phi_t^{(j)}=\Delta^{(j)}+B^{1/2}W_t^{(j)}$. $\Delta^{(j)}$ obeys a uniform random distribution, and $W_t^{(j)}$ denotes the Wiener process. $x_t=\lim\limits_{N_t\to\infty}\left[N_t/\overline{N}\right]$; and $\gamma_t=\lim\limits_{N_t\to\infty}\left[\sum_{j=1}^{N_t}\mathrm{e}^{\mathrm{i}\phi_t^{(j)}}/(N_t)^{1/2}\right]$. In studying the stochastic differential equation of γ_t, the following definition is given:

$$\varepsilon_t^{(N_t)}=\sum_{j=1}^{N_t}\mathrm{e}^{\mathrm{i}\phi_t^{(j)}}.\tag{2}$$

According to the $\widehat{Ito}$ formula,

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$$\begin{aligned} \mathrm{d}\varepsilon_t^{(N)} &= \sum_{j=1}^{N}[\mathrm{i}\mathrm{d}\phi_t^{(j)} - \frac{1}{2}(\mathrm{d}\phi_t^{(j)})^2]\mathrm{e}^{\mathrm{i}\phi_t^{(j)}} \\ &= \sum_{j=1}^{N}[\mathrm{i}B^{1/2}\mathrm{d}W_t^{(j)} - \frac{1}{2}B\,\mathrm{d}t]\mathrm{e}^{\mathrm{i}\phi_t^{(j)}} \\ &= \sum_{j=1}^{N}\mathrm{i}B^{1/2}\mathrm{d}W_t^{(j)}\mathrm{e}^{\mathrm{i}\phi_t^{(j)}} - \frac{1}{2}B\,\mathrm{d}t\sum_{j=1}^{N}\mathrm{e}^{\mathrm{i}\phi_t^{(j)}} \\ &= V - \frac{1}{2}B\varepsilon_t^{(N)}\mathrm{d}t, \end{aligned} \tag{3}$$

where

$$\begin{aligned} V &= \sum_{j=1}^{N}\mathrm{i}B^{1/2}\mathrm{d}W_t^{(j)}\mathrm{e}^{\mathrm{i}\phi_t^{(j)}} \\ &= B^{1/2}[\mathrm{i}\sum_{j=1}^{N}\cos\phi_t^{(j)}\mathrm{d}W_t^{(j)} - \sum_{j=1}^{N}\sin\phi_t^{(j)}\mathrm{d}W_t^{(j)}] \\ &= B^{1/2}[\mathrm{i}\sigma_c\mathrm{d}W_t^{(c)} - \sigma_s\mathrm{d}W_t^{(s)}] \\ &= B^{1/2}\sigma[\mathrm{i}\frac{\sigma_c}{\sigma}\mathrm{d}W_t^{(c)} - \frac{\sigma_s}{\sigma}\mathrm{d}W_t^{(s)}]. \end{aligned} \tag{4}$$

When $N \to \infty$, $\sigma_c/\sigma = \sigma_s/\sigma = 1/\sqrt{2}$, the Wiener process is defined as

$$\mathrm{d}\xi_t = \frac{1}{\sqrt{2}}(\mathrm{i}\mathrm{d}W_t^{(c)} - \mathrm{d}W_t^{(s)}), \tag{5}$$

which satisfies

$$\begin{cases} |\mathrm{d}\xi|^2 = \mathrm{d}t \\ (\mathrm{d}\xi)^2 = 0 \end{cases}. \tag{6}$$

The resulting equation is

$$\mathrm{d}\gamma_t = -\frac{1}{2}B\gamma_t\mathrm{d}t + B^{\frac{1}{2}}\mathrm{d}\xi_t. \tag{7}$$

According to a birth-death-immigration model (Wu et al., 2014), satisfies

$$\mathrm{d}x_t = \mathrm{A}(\alpha - x_t)\mathrm{d}t + (2\mathrm{A}x_t)^{1/2}\mathrm{d}W_t^{(x)}, \tag{8}$$

where α and A are constants.

Let $r_t = (x_t)^{1/2}$, we can obtain

$$\mathrm{d}r_t = \mathrm{A}\left[\frac{2(\alpha - x_t) - 1}{4r_t}\right]\mathrm{d}t + \left(\frac{\mathrm{A}}{2}\right)^{1/2}\mathrm{d}W_t^{(x)}, \tag{9}$$

where r_t is satisfied using the generated stochastic differential equations.

In polar coordinates $\Psi_t = R_t\mathrm{e}^{\mathrm{i}\theta_t}$, according to the theory of stochastic differential, $\mathrm{d}(X_tY_t) = X_t\mathrm{d}Y_t + Y_t\mathrm{d}X_t + \mathrm{d}X_t\mathrm{d}Y_t$; thus,

$$\mathrm{d}z_t = \Psi_t^*\mathrm{d}\Psi_t + \Psi_t\mathrm{d}\Psi_t^* + |\mathrm{d}\Psi_t|^2. \tag{10}$$

According to Eq. (1),

$$\begin{aligned} \Psi_t^*\mathrm{d}\Psi_t + \Psi_t\mathrm{d}\Psi_t^* &= x_t(\gamma_t^*\mathrm{d}\gamma_t + \gamma_t\mathrm{d}\gamma_t^*) + 2|\gamma_t|^2r_t\mathrm{d}r_t \\ &= -Bz_t\mathrm{d}t + B^{1/2}x_t(\gamma_t^*\mathrm{d}\xi_t + \gamma_t\mathrm{d}\xi_t^*) + \frac{2z_t}{r_t}\mathrm{d}r_t. \end{aligned} \tag{11}$$

Combining Eqs (9)–(11), yields

$$\begin{aligned} \mathrm{d}z_t = &\left[B(x_t - z_t) + \frac{Az_t(\alpha - x_t)}{x_t}\right]\mathrm{d}t \\ &+ \left(2Bx_tz_t + \frac{2Az_t^2}{x_t}\right)^{1/2}\mathrm{d}W_t^{(z)}. \end{aligned} \tag{12}$$

We can then obtain the sea clutter scattering signal energy part (square amplitude) of the stochastic differential equation.

3 Numerical results

IPIX radar sea clutter data are provided by the McMaster University and have become a global study of the theory of a sea clutter standard database. The database records all types of a heavy sea condition, all types of a radar frequency and different polarisation modes of the sea clutter data. The database is thus suitable to be used for the in-depth study of the sea clutter. A number of studies have cited the database depicted in Fig. 1.

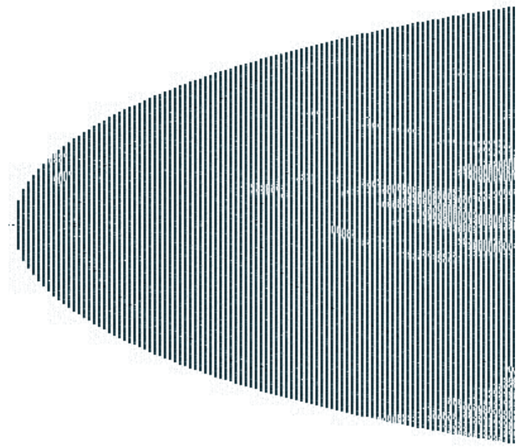

Fig. 1. The sea clutter of PPI from IPIX radar.

Using the public IPIX on the coherent radar sea clutter data by the Canadian McMaster University, this study selects 100 sea clutter data records for the experiments; these data records are shown in Fig. 2.

The multifractal spectrum is analysed, and the result is shown in Fig. 3. From the diagram, the IPIX coherent radar-measured sea clutter data of the minimum fractal index fall in between (0.6, 1.0), and the maximum fractal index falls in between (1.0, 1.4).

The multifractal spectrum analysis must also be conducted to validate the sea clutter stochastic differential equation model. According to Eq. (12) types of the sea clutter simulation data, the parameter settings are as follows: B=1, dt=0.001 and σ=1. The 100 data records are shown in Fig. 4.

The multifractal spectrum analysis results are shown in Fig. 5. The diagram indicates that the sea clutter random walk model data yield a minimum fractal index between (0.6, 1.0) and a maximum fractal index mainly between (1.0, 1.4), with consistent

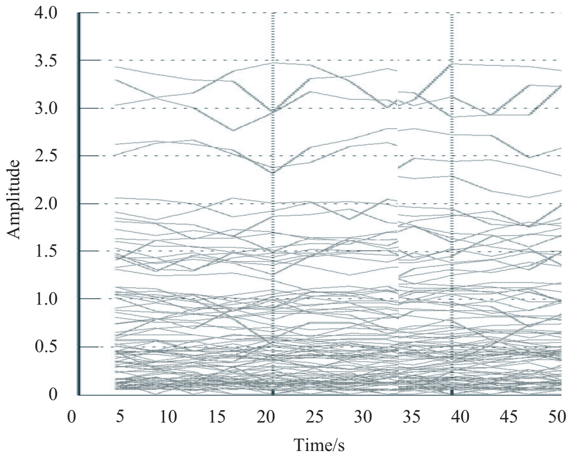

Fig. 2. The sea clutter data records.

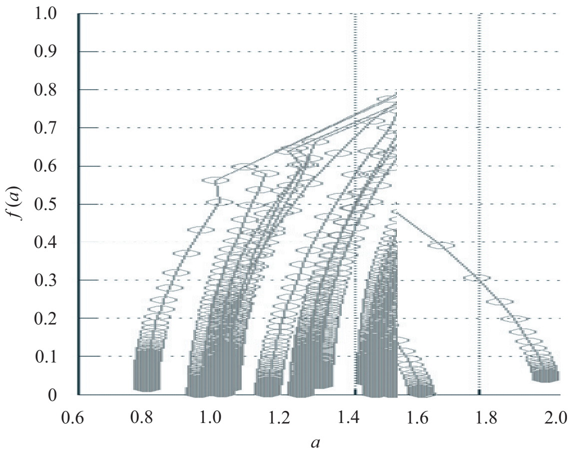

Fig. 3. The sea clutter multifractal spectrum of the measured data.

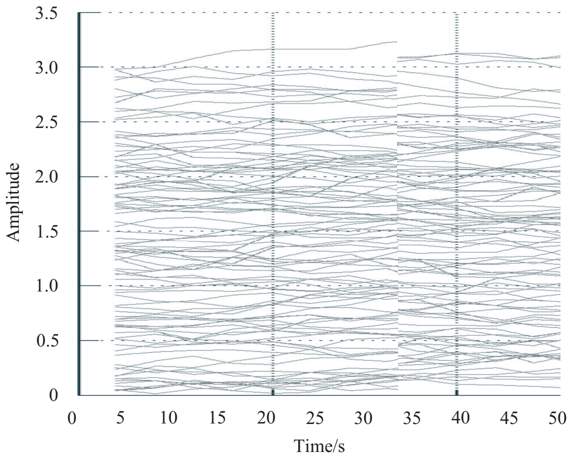

Fig. 4. The sea clutter random differential equation model of the simulated data.

IPIX radar-measured sea clutter data of the multifractal spectrum. Thus, the stochastic differential equation based on the random walk can effectively describe the dynamics mechanism of the sea clutter.

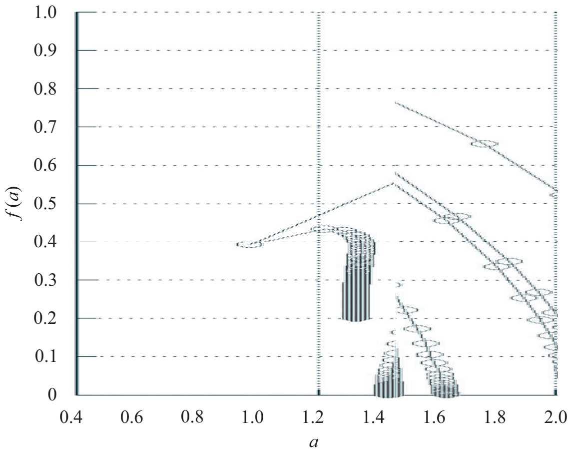

Fig. 5. Sea clutter simulation data of the multifractal spectrum.

4 Conclusion

The sea clutter is nonlinear, nonstationary and time varying. The multifractal measure analysis is the local singularity of the complete description of a physical system. On the basis of the classic model of sea clutter electromagnetic scattering, that is, the random walk model, the characteristics of the sea clutter are systematically analysed using the theory of stochastic differential physical in this study. The sea clutter time-varying electromagnetic scattering characteristics of stochastic differential equations are described. The sea clutter scattering signal amplitude and phase of the diffusion process model are obtained using the generated formulas. The multifractal spectrum is analysed. The accuracy of this model is verified by the Canadian McMaster IPIX radar data set. The experimental results show that the stochastic differential model based on the random walk is an effective algorithm to describe the mechanism of the sea clutter.

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