

## Derivation of the thermal characteristics of mesoscale eddies

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### Abstract

This study aims at explaining the relationship between thermodynamic characteristics and direction of rotation of mesoscale eddies (MEs). The geometric characteristics of the MEs are under the following assumptions: the structure of the MEs is symmetrical, and changes of oceanic physical variables are close to linear features in the radial direction in the ME regions. Based on these assumptions, by using primitive equations without friction under a cylindrical coordinate system, the thermodynamic characteristics of the MEs are derived, showing that the conventional relationship of warm anticyclonic eddies with high sea surface height (SSH) and cold cyclonic eddies with low SSH is not consistent with the SSH and sea surface temperature (SST) observations of eddies. The results show that the symmetrical form is an ideal approximation for the geometric characteristics of MEs. In consideration of the above assumptions, there are advantages for derivation of the characteristics of the MEs under a cylindrical coordinate.

**Key words:** mesoscale eddies, thermodynamic characteristics, symmetry

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### 1 Introduction

Mesoscale eddies (MEs) are some of the most important oceanographic phenomena. They are critical factors for ocean dynamic, thermodynamic and marine chemical processes. The MEs also are of significant impact on the ocean energy development, climate variation and so on (Zheng et al., 2013a, b, 2016; Zheng and Pan, 2014). By analyzing sea surface observations, a large number of eddies are found almost everywhere (Thorpe, 2007; Chelton et al., 2011). The energy transfer from the general circulation to the MEs is largely a consequence of barotropic and baroclinic instabilities with a flux about 1 TW (Thorpe, 2007). The energy from wind is substantial (Zheng and Pan, 2014), and the MEs are further supported by the wind energy flux about 0.2 TW, which can sustain a total energy in the mesoscale field of about 13 EJ (Thorpe, 2007). The MEs have great influence on the spatial distribution of oceanic physical variables (Xu et al., 2014; Wang et al., 2004a) and nearshore vertical velocity structure (Xiu et al., 2002).

In recent years, studies of the generation of the MEs show that the main causes are: the meandering path of the western boundary current, the intersection of cold and warm currents, wind-driven currents and topographical variation. The contributions of

these factors to the ME generation are different in areas; for example, the generation of the MEs in the South China Sea (Wang et al., 2004a, b; Su, 2005). In western boundary current regions, the MEs are related to the western boundary currents, including the Kuroshio (Hu et al., 2007). Ebuchi and Hanawa (2001) tracked the MEs in the Kuroshio recirculation region based on satellite altimeter data. In addition, in a follow-up study (2002), they analyzed variations of the Kuroshio path south of Japan, using the satellite altimeter data and Kuroshio axis position data. Moreover, underway observation data are also important to the MEs (Yuan et al., 2006).

The numerical simulations and observations of the sea surface temperature (SST) and sea surface height (SSH) are usually used in the study of the MEs. Researchers (Hwang et al., 2004; Chelton et al., 2011; Souza et al., 2011; Glorioso et al., 2005) have studied the MEs based on these data or methods. With applications of the satellite altimeter data and the development of numerical models (Johannessen et al., 1989; Wang et al., 2004b), the study of the MEs is becoming more popular than before.

In such studies, the MEs must be identified from the observations or model output. The methods of identifying the MEs are mainly based on their characteristics, such as geometric and dy-

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dynamic features. The geometric features are based on the ME symmetrical structures. Many studies have focused on two main properties to classify the MEs: the thermodynamic characteristics and the rotational direction (Chen et al., 2011; Pearce and Griffiths, 1991; Roemmich and Gilson, 2001; Hamilton et al., 2002). Dong et al. (2009) analyzed the effect on cold eddies from the lee side of Lanai Island, Hawaii. Lou and Yuan (2004) described the circulation on both sides of the Ryukyu Islands. Guan and Yuan (2006) examined two cyclonic cold eddies. In contrast to the mainstream view of the ME classification both in research and textbooks, Shi et al. (2008) noted the existence of anticyclonic cold eddies in the Arctic Ocean. This poses a question: What is the relationship between the thermodynamic characteristics and rotational direction of MEs?

## 2 Eddy characteristics and ME hypotheses

### 2.1 Eddy characteristics

On the basis of the methods of identifying the MEs mentioned above, the geometric characteristics of the MEs are summarized as below:

(1) The MEs have been identified by the observations and model data (Chelton et al., 2011; Lin, 2005) based on methods, such as the SSH-based method. In this method, one can define the structures of the MEs. The characteristics can be ideally simplified in a cylindrical coordinate system to derive ME structures (Xiu et al., 2002).

(2) In contrast to the SSH, the SST is less commonly used in the above-mentioned methods. This poses the question: what is the best way to identify a cold or warm eddy? The general relationship, in which cyclonic eddies tend to be cold and anticyclonic eddies tend to be warm, is often taken into account in the discussion of the thermodynamic characteristics of the MEs, but the reason for this relationship is rarely mentioned, which is a key issue that will be addressed in this paper.

(3) Considering that the ME thermodynamic properties are related to the SSH differences (Figs 1 and 2) and the spatial distribution of oceanic physical variables, we can approximately identify the MEs as concentric closed curves, as shown in Figs 1

and 2.

### 2.2 ME hypotheses

With strong vertical movement in the ME regions, the distributions of oceanic physical variables are nearly even in the tangential direction, but not in the radial direction. Hence, the MEs share properties of certain water masses. On the basis of these factors, the first hypothesis is that the MEs are centrosymmetric, which is an ideal approximation in the deep ocean (not in nearshore areas).

According to the direction of rotation, the MEs can be divided into two classes: cyclonic eddies and anticyclonic eddies. According to this classification (Table 1), the former corresponds to cold MEs with a lower sea level in the eddy center; the latter corresponds to warm MEs with a higher sea level (a warm eddy example is shown in Fig. 3).

The second hypothesis is that changes in oceanic physical variables are nearly linear in the radial direction, and changes in the tangential direction can be neglected.

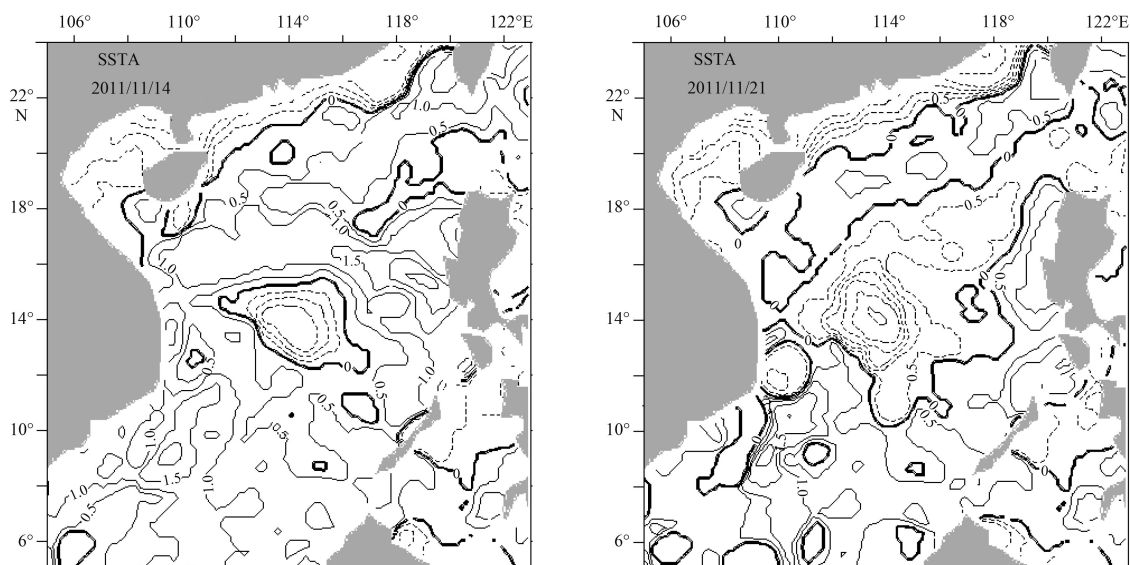
The second hypothesis is based on the following two points: (1) there is no tangential variation of the isolines; and (2) the SSH difference between the center and the boundary of the ME has a small order of 0.1 m, whereas the radius of a ME is more than 10 km. In addition, the horizontal velocity of ME has a typical order of 1 m/s.

Consequently, the ME hypotheses are stated here:

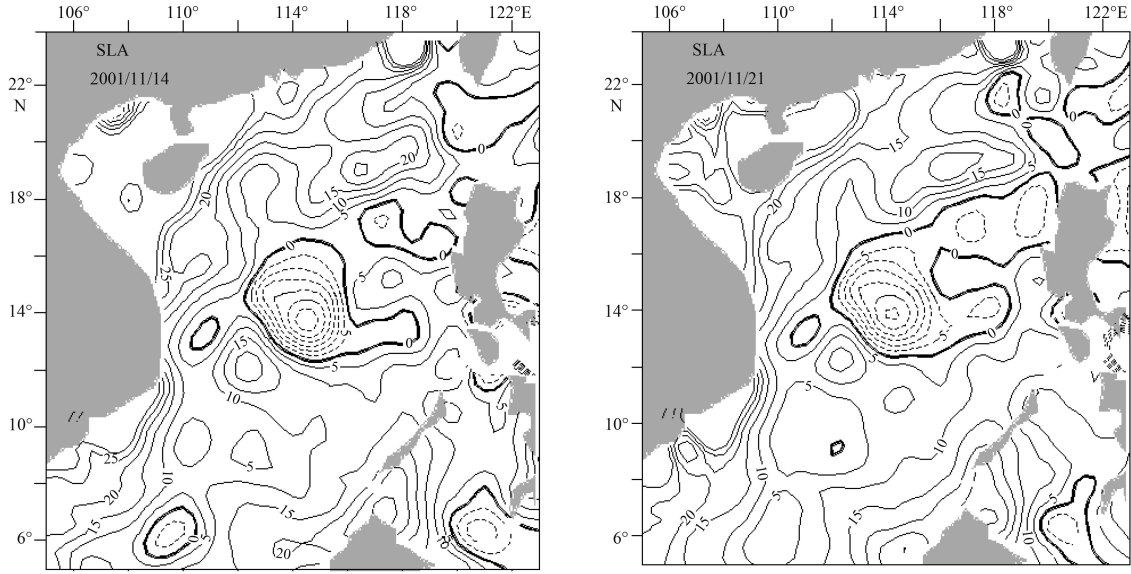
- (1) corresponding to rotational movement, the MEs are approximately centrosymmetric;
- (2) oceanic physical variables vary linearly in the radial direction but remain nearly unchanged in the tangential direction; and
- (3) in this study, we do not discuss the MEs in nearshore areas, where the distributions of oceanic physical variables are deeply affected by topography and other coastal factors.

## 3 Analysis of MEs

On the basis of the above assumptions and using primitive equations (Stewart, 2009), we can obtain some simple features of the MEs.



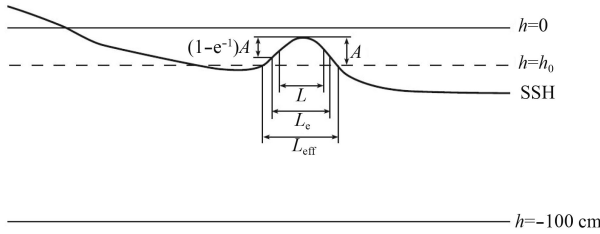
**Fig. 1.** Sea surface temperature anomaly ( $^{\circ}\text{C}$ ) from NOAA data (Lin, 2005). The dotted contours represent negative values, the bold solid line is the 0 contour, and the solid contours represent positive values.



**Fig. 2.** Sea level anomaly (SLA) (cm) from TOPEX/Poseidon plus ERS altimeter data (Lin, 2005). The contour lines are the same as that in Fig. 1.

**Table 1.** Classification of MEs (Stewart, 2009; Chen et al., 2011; Pearce and Griffiths, 1991; Roemmich and Gilson, 2001)

Type of eddy	SSH in the eddy center	Direction of rotation
Cold eddy	low	cyclonic
Warm eddy	high	anticyclonic



**Fig. 3.** Schematic of an anticyclonic eddy (Chelton et al., 2011). The dashed line is the basal value  $h_0$  of SSH around the outermost closed contour of SSH defining the boundary of the eddy,  $A = h_{\max} - h_0$ ,  $L_e$  is the e-folding scale,  $L_{\text{eff}}$  is the effective radius, and  $L = 2^{-1/2}L_e + 0.44L_{\text{eff}}$ .

### 3.1 The derivation of basic dynamic characteristics

Horizontal motion equations are as follows:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} - f v = -\frac{1}{\rho} \frac{\partial p}{\partial x} + F_x, \quad (1)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + f u = -\frac{1}{\rho} \frac{\partial p}{\partial y} + F_y, \quad (2)$$

$$\frac{d\rho}{dt} + \rho \frac{\partial u}{\partial x} + \rho \frac{\partial v}{\partial y} + \rho \frac{\partial w}{\partial z} = 0, \quad (3)$$

where  $\rho$ ,  $p$ ,  $f$ ,  $F_x$ ,  $F_y$ ,  $(u, v)$ , and  $w$  are density, pressure, the Coriolis parameter, friction force components, horizontal velocity components and vertical velocity, respectively.

Differentiating Eq. (2) with respect to  $x$  and Eq. (1) with respect to  $y$ , and subtracting yields the following:

$$\frac{d(\xi + f)}{dt} = (\xi + f) \frac{\partial w}{\partial z} - \nabla_h \frac{1}{\rho} \times \nabla_h p + \nabla_h \times F - \left( \frac{\partial w}{\partial x} \frac{\partial v}{\partial z} - \frac{\partial w}{\partial y} \frac{\partial u}{\partial z} \right), \quad (4)$$

where  $\xi$  is the vertical vorticity,  $F = (F_x, F_y)$ ,  $\xi = \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$ ,  $\nabla_h = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right)$ , and  $\frac{d}{dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y}$ .

If Margules' method based on geostrophic equilibrium is applied to the MEs (Stewart, 2009), one can obtain the ME velocity based on the geostrophic equilibrium:

$$v_\theta = \frac{-fr + \sqrt{f^2 r^2 + \frac{4r}{\rho} \frac{\partial p}{\partial r}}}{2}. \quad (5)$$

Equation (5) is expressed in the cylindrical coordinate system, in which  $r$  is the radial coordinate and  $v_\theta$  is the tangential velocity. Transforming Eq. (4) to the cylindrical coordinate system yields:

$$\frac{\partial A}{\partial t} + v_r \frac{\partial A}{\partial r} + \frac{v_\theta}{r} \frac{\partial A}{\partial \theta} + w \frac{\partial A}{\partial z} = A \rho \frac{\partial w}{\partial z} - \nabla_h \frac{1}{\rho} \times \nabla_h p + \nabla_h \times F + \frac{1}{r} \frac{\partial w}{\partial \theta} \frac{\partial v_r}{\partial z} - \frac{\partial w}{\partial r} \frac{\partial v_\theta}{\partial z}, \quad (6)$$

where  $A = \frac{(\xi + f)}{\rho}$ . Using  $\frac{4r}{\rho} \frac{\partial p}{\partial r} < f^2 r^2$ , Eq. (5) can be rewritten as follows:

$$v_\theta \approx \frac{1}{f\rho} \frac{\partial p}{\partial r}. \quad (7)$$

In a steady state, the vertical velocity and a turbulent viscous stress can be neglected, thus Eq. (6) can be simplified to

$$\frac{v_\theta}{r} \frac{\partial A}{\partial \theta} = -\nabla_h \frac{1}{\rho} \times \nabla_h p, \quad (8)$$

substituting Eq. (7) into Eq. (8) gives

$$\frac{\partial A}{\partial \theta} = - \left( \nabla_h \frac{1}{\rho} \times \nabla_h p \right) r \frac{f \rho}{\left( \frac{\partial p}{\partial r} \right)}. \quad (9)$$

On the basis of the hypotheses presented in Section 2, the density can be expressed as  $\rho = \rho_0 - Dr$ , where  $\rho_0$  is the central density, and  $D$  is the change of the density in the radial direction and is taken to be a constant. Thus, the baroclinic term in Eq. (9)

$-\nabla_h \frac{1}{\rho} \times \nabla_h p$  can be written as

$$-\nabla_h \frac{1}{\rho} \times \nabla_h p = - \frac{1}{r \rho^2} D \frac{\partial p}{\partial \theta}. \quad (10)$$

In the vicinity of the eddy's center and based on the above hypotheses,  $p$  can be written as  $p = p_0 + Hr + \varepsilon_r(\theta)$ , where  $p_0$  is the pressure in the center,  $H$  is the change of sea surface pressure in the radial direction, and  $\varepsilon_r(\theta)$  is the change of sea surface pressure in the tangential direction. On the basis of the second hypotheses, the change of  $\varepsilon_r(\theta)$  can be neglected:

$$A - \frac{2f}{\rho} \propto \frac{Df^2 \varepsilon_r'(\theta)}{\rho H \beta r \cos \theta},$$

where  $\beta$  is the latitudinal variation of the Coriolis parameter, so  $\frac{\partial A}{\partial \theta} \propto - \left( A - \frac{2f}{\rho} \right)$ . Corresponding to the second hypotheses,  $\varepsilon_r'(\theta)$  should be proportional to  $\beta r \cos \theta$ :

$$A - \frac{2f}{\rho} \propto k \frac{Df^2}{H\rho}, \quad (11)$$

where  $k$  is positive; and Eq. (11) can be converted into  $\xi - f \propto k \frac{Df^2}{H}$ .

Equation (11) shows that  $\xi - f$  is inversely proportional to  $H$  and proportional to  $Df^2$ . For the cyclonic case, the center height of the ME is lower, i.e.,  $H > 0$ ; meanwhile, the center density is relatively higher, i.e.,  $D > 0$ . Here, the special case in which the center height of the ME is higher while the density is lower still exists. This is true for the cold anticyclonic eddies mentioned by [Shi et al. \(2008\)](#) in the Arctic Ocean.

The MEs share properties of water masses, and thus, their thermal properties are more profound than their surroundings. Conventional theory states that the cyclonic eddies tend to be cold, and the anticyclonic tend to be warm. Eddies identified only from satellite altimeter data are related to the SLA value in the center, with a warm or cold core ([Table 1](#)), and the thermodynamic characteristics of the eddy can be confirmed based on this relationship. In the warm eddies, the center value of the SLA is large and positive, whereas in the cold eddies, the value is small and negative. This is consistent with the mainstream view that the cyclonic eddies correspond to the cold eddies with low SSH, the anticyclonic eddies correspond to the warm eddies with high SSH ([Figs 1 and 2](#)) ([Lin, 2005](#)).

Through the analysis of density, the density of the center region in the cyclonic eddies ( $\xi > f$ ) is higher, which may be due to the lower central temperature and/or higher central salinity. Because the salinity is homogeneously distributed, especially within a water mass and in strongly mixed areas, in our opinion, the main factor affecting the density distribution is the temperature. When the direction of eddy rotation is determined, the ME tem-

perature properties are nearly determined. However, the above relationship between the thermodynamic characteristics of the MEs and the directions of eddy rotation is not the only relationship for the SSH and eddy temperature properties. Equation (11) indicates that when the rotational direction is determined, the temperature properties for a given ME may remain uncertain. Cold cyclonic (warm anticyclonic) eddies and geostrophic balance are in agreement; presumably, this may be the reason why so many studies have not considered alternate cases (i.e., warm cyclonic and cold anticyclonic eddies).

### 3.2 Discussion

In the stationary state, because the distributions of the SST and the salinity are inhomogeneous, the baroclinic term  $\left( -\nabla_h \frac{1}{\rho} \times \nabla_h p \right)$  in Eq. (4) will lead to the change of the absolute vorticity. As the ideal assumptions and approximations taken into consideration, the result obtained from the equations and the actual observations may not be in agreement.

If  $A$  is considered as the eddy characteristics,  $A$  also holds the main characteristics of Rossby wave propagation. The vertical velocity can be removed from Eq. (4):

$$\frac{\partial A}{\partial t} + u \frac{\partial A}{\partial x} + v \frac{\partial A}{\partial y} = -\nabla_h \frac{1}{\rho} \times \nabla_h p. \quad (12)$$

By defining a stream function  $\psi$   $\left( u = -\frac{\partial \psi}{\partial y}, v = \frac{\partial \psi}{\partial x} \right)$  and substituting the relationship  $A = (\Delta \psi + f) / \rho$  into Eq. (12):

$$\begin{aligned} & \alpha \frac{\partial \Delta \psi}{\partial t} - \frac{\partial \psi}{\partial y} \alpha \frac{\partial \Delta \psi}{\partial x} - \frac{\partial \psi}{\partial x} \Delta \psi \frac{\partial \alpha}{\partial x} + \\ & \frac{\partial \psi}{\partial x} \alpha \frac{\partial \Delta \psi}{\partial y} + \frac{\partial \psi}{\partial x} \Delta \psi \frac{\partial \alpha}{\partial y} + \frac{\partial \psi}{\partial x} f \frac{\partial \alpha}{\partial y} + \\ & \frac{\partial \psi}{\partial x} \beta \alpha = \frac{1}{r \rho^2} D \left( \frac{\partial p}{\partial x} \frac{r^2}{y} - \frac{\partial p}{\partial y} \frac{r^2}{x} \right). \end{aligned} \quad (13)$$

Substituting  $\psi = a_\psi e^{i(kx+ly-\omega t)}$ ,  $p = a_p e^{i(kx+ly-\omega t)}$  into Eq. (13):

$$\begin{aligned} & \omega \alpha (k^2 + l^2) \psi + l (k^2 + l^2) \psi^2 \alpha^2 D \frac{x}{r} - \\ & k (k^2 + l^2) \psi^2 \alpha^2 D \frac{y}{r} - k \psi f \alpha^2 D \frac{y}{r} + k \psi \beta \alpha = \\ & \frac{1}{r \rho^2} D \left( k p \frac{r^2}{y} - l p \frac{r^2}{x} \right). \end{aligned} \quad (13a)$$

Substituting  $\psi = a_\psi e^{i(kx+ly-\omega t)}$ ,  $p = a_p e^{i(kx+ly-\omega t)}$  (where  $k$ ,  $l$ , and  $\omega$  are the wave number components and the wave frequency in radians per second, respectively) into Eq. (13a) and eliminating the index terms:

$$\begin{aligned} & \omega \alpha (k^2 + l^2) a_\psi + (k^2 + l^2) \\ & \alpha^2 \frac{D}{r} a_\psi (lx - ky) \psi - k a_\psi f \alpha^2 D \frac{y}{r} + \\ & k a_\psi \beta \alpha = \frac{1}{r \rho^2} D \left( k a_p \frac{r^2}{y} - l a_p \frac{r^2}{x} \right), \end{aligned} \quad (13b)$$

and integrating Eq. (13b) over a period of  $t$ , then we can eliminate  $(k^2 + l^2) \alpha^2 \frac{D}{r} a_\psi (lx - ky) \psi$ . The other items in Eq. (13b) will be multiplied by an integral constant period in the index terms in  $\psi$  or  $p$ ; eliminating the integral constant yields

$$\omega\alpha(k^2 + l^2)a_{\psi} - ka_{\psi}f\alpha^2D\frac{y}{r} + ka_{\psi}\beta\alpha = \frac{1}{r\rho^2}D\left(ka_p\frac{r^2}{y} - la_p\frac{r^2}{x}\right), \quad (14)$$

from Eq. (14), we can obtain the wave frequency:

$$\omega = \frac{r^2D(ka_px - la_py) + ka_{\psi}f\alpha^2Dxy^2\rho^2}{xyr\rho^2\alpha(k^2 + l^2)a_{\psi}} - \frac{ka_{\psi}\beta\alpha xy r\rho^2}{xyr\rho^2\alpha(k^2 + l^2)a_{\psi}}. \quad (15)$$

By substituting the geostrophic balance for  $a_{\psi}l = \alpha a_p k$ ,  $a_{\psi}k = \alpha a_p l$  in Eq. (15), the following is obtained:

$$\omega = \frac{r^2D(lx - ky) + kf\alpha Dxy^2 - k\beta xy r}{xyr(k^2 + l^2)}. \quad (16)$$

Equation (16) can be simplified as follows by omitting the first item in the numerator (the baroclinic term):

$$\omega = \frac{kf\alpha Dy - k\beta r}{r(k^2 + l^2)}, \quad (17)$$

where  $-\frac{k\beta}{k^2 + l^2}$  is the typical characteristic of a Rossby wave.

The phase velocity is  $c_x = \frac{f\alpha Dy - \beta r}{r(k^2 + l^2)}$ , which has properties of westward propagation; meanwhile, due to the existence of  $y$ , the velocity distribution is symmetrical about the  $x$  axis. The approximation of Eq. (17) actually simplifies the expression of  $\omega$ .

Now, reconsidering  $\omega = \frac{rD(lx - ky)}{xy(k^2 + l^2)} + \frac{kf\alpha Dy}{r(k^2 + l^2)} - \frac{k\beta}{k^2 + l^2}$

and converting it into a polar coordinate system, i.e.,

$\omega \approx \frac{kf\alpha^2 D \cos\theta \sin^2\theta + kf\alpha D \sin\theta - k\beta}{k^2 + l^2}$ , we can obtain the

phase velocity:  $c_x = \frac{f\alpha^2 D \cos\theta \sin^2\theta + f\alpha D \sin\theta - \beta}{k^2 + l^2}$ . Disturbances of the phase velocity typically are related to eddy location, volume and changes in density in the radial direction, which suggests that stronger eddies may have a higher propagation velocity. Simultaneously, the MEs also have properties of Rossby waves in the presence of perturbations. Here, assuming that the eddy rotation is determined by Eq. (7), and setting  $\frac{U}{c}$  as the rate of the maximum rotating speed of the internal eddy and propagation speed, when its value is greater than 1, the eddy has typical properties of water masses.

The assumption for eddies is that the MEs are geometrically symmetrical, but its shape is mainly influenced by the ME movement, the Coriolis parameter and other factors. Therefore, in realistic conditions, eddies are asymmetric or anisotropic. From the phase velocity and the frequency, it can be observed that the anisotropy is caused by ocean baroclinicity.

#### 4 Conclusions

The MEs are some of the most remarkable oceanographic phenomena. Studies of the MEs are mostly based on the analysis of observational data and simulation output of numerical models with sufficiently high resolution for resolving eddies. These methods revealed many features of the MEs, and this paper summarized some of the main characteristics of the MEs based on these methods. As there are some differences in the distribution

of variables in the center and boundaries of eddies, we can neglect changes in the tangential direction and apply the hypothesis that changes in oceanic physical variables are close to linear features in the radial direction.

On this basis, by taking the centrosymmetric form as the geometric characteristics of eddies and using the primitive equations without viscous force, this paper theoretically deduced that the MEs rotational direction and thermal properties are not in agreement. The reason for the dominance of cold-cyclonic and warm-anticyclonic eddies may be that they are in agreement with the geostrophic balance.

The main conclusions of this paper are summarized as follows:

(1) On the basis of the primitive equations and reasonable hypotheses, the relationship between the temperature properties and SSH of eddies is deduced.

(2) By using the traveling wave method, this paper demonstrated that the ME location, density distribution and changes of density in the radial direction impact the eddy phase velocity.

(3) According to Eq. (11), we do not exclude the existence of the anticyclonic cold eddies and the warm cyclonic eddies.

The influences of external forcing and topography are not considered in this work, and it does not provide an explanation for the eddy formation. The features of eddy movement and the characteristics of the phase velocity are given only by the traveling wave solutions. These problems will be addressed in future research.

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